The Stable Tracking Adaptive Fuzzy Control of Nonlinear Dynamic Systems Using the Takagi-Sugeno Fuzzy Logic

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ABSTRACT

Adaptive linearization controllers have been shown to have nice performance. However, two functions in the controllers are derived from the considered system. In other words, they can only work for known systems. In this paper, the considered dynamic nonlinear model can be unknown. Next, we proposed an affined Takagi-Sugeno-type (TS-type, for short) fuzzy modeling approach to model those two unknown functions. The proposed approach is called the model reference indirect adaptive fuzzy control (MRIAFC). In this approach, a reference linear model can be used not only to characterize the desired goal for performance but also to reflect the possibility of achieving them. The Lyapunov’s stability theorem is used to derive controller parameters update laws, which ensure that the system states remain bounded and the plant output asymptotically tracks an arbitrary piecewise reference trajectory. Finally, the proposed method is successfully applied to a second-order nonlinear inverted pendulum system to verify superiority in tracking control.

Keyword: Lyapunov’s stability, adaptive fuzzy control, Takagi-Sugeno.

1. INTRODUCTION

Adaptive control schemes for nonlinear systems via feedback linearization concept have been employed for decades [1]. The idea of feedback linearization approaches is to transform a nonlinear dynamic system into a linear system through state feedback mechanisms. With such transformations, those well-explored linear control skills can then be applied to meet the desired control specifications. Several primitive results have been reported in [2].

Fuzzy logic control systems using conditional linguistic expresses and approximation reasoning have been widely used to apply many plants that are either mathematically poorly understood or described by the experienced human operators [3]. Therefore, fuzzy controllers are assumed to use in situations that the consistency of performance of a system should be maintained in the presence of uncertainties. As a result, fuzzy system usually equips with an adaptation law [4].

It is well known that most of fuzzy controller structures fall into two categories: the Mamdani [5] and the Takagi-Sugeno [6]. The stability and convergence analysis for an adaptive fuzzy control system concerning the Mamdani-type has been addressed in the literature [7,8]. However, they only focus on the adaptation of membership functions of the THEN-part. Consequently, many more rules are required to establish knowledge base. Traditionally, when using a TS-type fuzzy model to approximate an unknown function, a huge number of parameters need to be searched, thus, usually results in a complicated and time-consuming task. An obvious solution to overcome the problem is to introduce a simplified linear TS-type fuzzy rule scheme in the adaptive control algorithm [9,10]. Likewise, a lot of useful information cannot be exerted. In the direct adaptive fuzzy control [11], the consequent part of the fuzzy control rules which is only combination of the input variables without offset term; namely; a homogeneous TS-type model, presents easily to fail to learn modeling an optimal controller for tracking control.

In this paper, a novel model reference indirect adaptive fuzzy controller (MRIAFC) is developed. The statement of rules with an affined linear TS-type model is formed as an analytic parameter equation explicitly; therefore, the method of adaptive control can be utilized successfully. Then, the MRIAFC scheme uses a reference model to provide closed-loop performance for regulating the fuzzy controller’s parameters. Based on Lyapunov’s stability criterion, two fuzzy rule adaptive laws are derived to modify the knowledge base’s rules that are used to learn a desired control in which the mathematical model of the controlled system is unknown. Thus, the
fuzzy controller based on the proposed adaptive law has the ability to adjust the control rules during on-line operation.

The paper is organized as followed. First, the problem is formulated in section 2. A brief description of the TS-type fuzzy logic system is presented in section 3. The updating algorithms and stability analysis for the proposed controller are given in section 4. In section 5, simulation results are illustrated to confirm the feasibility and superiority of the proposed method. Finally, conclusion remarks are included in section 6.

2. PROBLEM FORMULATION

Consider the dynamic equation of a class of nth-order nonlinear continuous time dynamic system of the form

\[ \dot{x}_1 = x_2 \]
\[ \dot{x}_2 = x_3 \]
\[ \vdots \]

\[ \dot{x}_n = f(x_1, x_2, \ldots, x_n) + g(x_1, x_2, \ldots, x_n)u \]
\[ y = x_1 \]

where \( x_1, \ldots, x_n \) are system states, which are assumed measurable, \( f(.) \) and \( g(.) \) are nonlinear smooth functions, \( u \) is the scalar manipulated input variable, and \( y \) is the output variable. It is assumed that the nonlinear functions governing the nonlinear dynamic system are not known, but with \( g(.) \neq 0 \) in order to be controllable.

Then select a reference model whose linear dynamic system to follow:

\[ \dot{x}_r1 = x_r2 \]
\[ \dot{x}_r2 = x_r3 \]
\[ \vdots \]

\[ \dot{x}_rn = -\alpha_1x_r1 - \alpha_2x_r2 - \cdots - \alpha_nx_rm + r \]
\[ y_r = x_1 \]

We can rewrite (2) as

\[ \dot{\bar{y}} = A\bar{y} + Br, \bar{y} = \begin{bmatrix} x_{r1}, x_{r2}, \ldots, x_{rn} \end{bmatrix}^T \] (3)

where

\[ A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 & \cdots & -\alpha_{n-1} & -\alpha_n \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \]

The elements \( a_1, a_2, \ldots, a_n \) of \( A \) are chosen such that \( A \) is a Hurwitz matrix. Thus Eq. (1) can be rewritten as

\[ \dot{x} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \end{bmatrix} \]
\[ \dot{x} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \end{bmatrix} \]

\[ \dot{x} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \end{bmatrix} \]

then

\[ \ddot{x} = A\ddot{x} + Bf(x, u) \]

(5)

By Subtracting (3) from (5), the tracking error between the dynamic system and the reference model can be derived as

\[ \ddot{e} = \ddot{x} - \ddot{x}_r = A\ddot{e} + B\dddot{x} + g(x)u - r \]

(6)

If system is known i.e. \( f(.) \) and \( g(.) \) are known, the control input can be obtained as

\[ u = -\frac{\dddot{x} + g(x)}{g(x)} \]

(7)

Which can make Eq. (6) to be

\[ \ddot{e} = A\ddot{e} \]

(8)

Because \( A \) is reasonably chosen as a Hurwitz matrix, the tracking error will asymptotically converge to zero.

However, when the nonlinear system to be controlled is unknown, i.e. \( f(.) \) and \( g(.) \) are unknown. Obviously, the above controller Eq. (7) cannot be obtained directly. So the TS-type fuzzy logic system will be introduced to learn the unknown functions.

3. DESCRIPTION OF THE USED TS-TYPE FUZZY LOGIC SYSTEM

The TS-type fuzzy logic system consists of some fuzzy IF-THEN rules and a fuzzy inference engine. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from the current states to an output variable, which is linearly composed of the current states with offset constant; namely, an affined TS model. The i-th fuzzy IF-THEN rule is expressed as

\[ R^{(i)}: \text{If } x_1 \text{ is } A_1^i \text{ and } x_2 \text{ is } A_2^i \text{ and } \ldots \text{ and } x_n \text{ is } A_n^i \text{ then } y^i = C_{i0} + C_{i1}x_1 + C_{i2}x_2 + \cdots + C_{in}x_n \] (9)

Where \( x_1, \ldots, x_n \) are the state variables defined in Eq. (1), \( A_1^i, \ldots, A_n^i \) are the corresponding fuzzy labels, for \( i=1, \ldots, L \). \( y^i \) is the local output variable for the fuzzy system, and \( C_{i0}, C_{i1}, \ldots, C_{in} \) are constant coefficients of the consequent part of the corresponding output value for the i-th rule.

By using the product operations for the conjunction relations in the premise parts of fuzzy rules, the output of a fuzzy system consisting of \( L \) rules is obtained as:

\[ y(x) = \frac{\sum_{i=1}^{L} \prod_{j=1}^{n} \mu_{A_j^i}(x_j)(C_{i0} + C_{i1}x_1 + \cdots + C_{in}x_n)}{\sum_{i=1}^{L} \prod_{j=1}^{n} \mu_{A_j^i}(x_j)} \] (10)

where \( \mu_{A_j^i}(x_j) \) is the membership degree of \( x_j \) belonging to the fuzzy label \( A_j^i \) for \( i=1, \ldots, L \), and \( j=1, \ldots, n \).
If let $\mu_i(x_i)$'s be fixed and $C_i$'s be viewed as adjustable parameters, consequently Eq. (10) can be given as

$$y(x) = 0^T \eta(x)$$

(11)

where $0 = (C_0, C_1, \cdots, C_n, C_{n+1}, \cdots, C_m)^T$ is a parameter vector and

$$\eta(x) = (\eta_{i0}(x), \eta_{i1}(x), \cdots, \eta_{in}(x))$$

is referred as the regressor vector. Here the superscript $T$ for a vector is the transpose of the vector, and $\eta_{ik}(x)$ is defined as

$$\eta_{ik}(x) = \left( \prod_{j=1}^{n} \mu_{A_j}(x_j) \right) x_k$$

(12)

is called the fuzzy basis function. Where $i=1, \ldots, L$, and $k=0,1,\ldots,n$, and $x_k=1$. So, Eq. (11) can be used to learn the unknown functions in the following sections.

4. THE ADAPTIVE FUZZY CONTROLLER DESIGN USING THE TS-TYPE FUZZY LOGIC SYSTEM

In the section, we will introduce how to design a model reference indirect adaptive controller via the TS-type fuzzy logic system with on-line updating rule that guarantee the Lyapunov stability.

The TS-type fuzzy logic system is used to learning the unknown functions $f(.)$ and $g(.)$ for the unknown system estimation. Because of the linear-in-parameter form of the TS-type fuzzy logic system output expression in Eq. (11), the powerful fuzzy system is properly used here.

The estimator of the real unknown system can be expressed as:

$$\dot{\hat{x}} = A\hat{x} + B\hat{\theta}_f \hat{\eta}(\hat{x}) + B\hat{\theta}_g \hat{\eta}(\hat{x}) u$$

(13)

From (13) and (3), the error between the estimated output and the reference model output is defined as

$$\hat{e}_y = \hat{x} - x = A\hat{x} + B\hat{\theta}_f \hat{\eta}(\hat{x}) \bar{u}$$

(14)

Let the control input $u$ be

$$u = -\overline{\theta}_f \hat{\eta}(\hat{x}) + r$$

(15)

then

$$\hat{e}_y = A\hat{x}$$

(16)

Moreover, since $A$ is the Hurwitz matrix, then $\lim_{t \to \infty} \hat{e}_y = 0$. As a result, the estimated model can approximate the reference model that has been shown from the above discussion.

Furthermore, from (13) and (5), the error between the estimated output and the real unknown system output is defined as

$$\hat{e}_y = \hat{x} - x = A\hat{x} + B\hat{\theta}_f \hat{\eta}(\hat{x}) - \hat{\theta}_f \hat{\eta}(\hat{x}) + B\hat{\theta}_g \hat{\eta}(\hat{x}) - g(\hat{x}) u$$

(17)

Assumption 1: given two arbitrary small positive constants $\sigma_1$ and $\sigma_2$, there exist two optimal parameter vectors $\hat{\theta}_f$ and $\hat{\theta}_g$, such that the outputs of the optimal TS-type fuzzy system satisfy

$$\hat{\theta}_f \hat{\eta}(\hat{x}) - \hat{\theta}_f \hat{\eta}(\hat{x}) < \sigma_1$$

(18)

$$\hat{\theta}_g \hat{\eta}(\hat{x}) - g(\hat{x}) < \sigma_2$$

(19)

From assumption, the minimum approximation error can be defined as

$$d = [\hat{\theta}_f \hat{\eta}(\hat{x}) - \hat{\theta}_f \hat{\eta}(\hat{x})] + [\hat{\theta}_g \hat{\eta}(\hat{x}) - g(\hat{x})] u$$

(20)

We can rewrite (17) as

$$\hat{e}_y = A\hat{x} + B\hat{\theta}_f \hat{\eta}(\hat{x}) + B\hat{\theta}_g \hat{\eta}(\hat{x})$$

(21)

Since the $A$ is chosen a stable matrix, by the Lyapunov lemma, given any a symmetric positive define matrix $Q$, there exists a unique symmetric positive define matrix $P$ that satisfies

$$A^T P + P A = -Q$$

(22)

In order to guarantee the convergence of the adaptive system, the proposed update laws for the TS-type fuzzy system are chosen as:

$$\hat{\theta}_f = -\gamma_1 \hat{e}_y \hat{\eta}(\hat{x})$$

(23)

$$\hat{\theta}_g = -\gamma_2 \hat{e}_y \hat{\eta}(\hat{x}) u$$

(24)

where $\gamma_1$ and $\gamma_2$ are two learning constants.

Theorem 1: Consider a nonlinear dynamic unknown system as Eq. (1), and whose estimator defined as Eq. (13). The control input being used is Eq. (15). Then if the updating laws are Eqs. (23) and (24), then $e(t)$ converges to zero as $t \to \infty$ and all signals in the closed system are bounded.

Proof: Define a Lyapunov function as

$$V = \frac{1}{2} \hat{e}_y^T \hat{e}_y + \frac{1}{2\gamma_1} \hat{\theta}_f^T \hat{\theta}_f + \frac{1}{2\gamma_2} \hat{\theta}_g^T \hat{\theta}_g$$

(25)

By taking the time derivative of $V$ and using Eqs. (21) and (22) lead(s) to

$$\dot{V} = \frac{1}{2} \hat{e}_y^T \hat{e}_y + \frac{1}{2} \hat{\theta}_f^T \hat{\theta}_f + \frac{1}{2} \hat{\theta}_g^T \hat{\theta}_g$$

(26)

$$\dot{V} = \frac{1}{2} \left( A^T \hat{e}_y^T \hat{e}_y + \hat{\theta}_f^T \hat{\eta}(\hat{x}) + \hat{\theta}_g^T \hat{\eta}(\hat{x}) u + d \right)^T \hat{e}_y$$

(27)

where $\gamma_1$ and $\gamma_2$ are two learning constants.
= \frac{1}{2} e^T \left( A^T P + PA \right) e + e^T PB + \left( \phi_2^T T f + \phi_1^T T \phi_1 \right) + \frac{1}{2} \phi_1^T \phi_1 + \frac{1}{2} \phi_2^T \phi_2 \\
abla = -\frac{1}{2} e^T Q \phi_2^T P B d + \frac{1}{2} \phi_1^T \phi_1 + 1) \phi_1^T T \phi_1 + \phi_2^T P B d \tag{26}

Let the updating law of \( \tilde{\theta}_f \) and \( \tilde{\theta}_g \) be Eqs. (23) and (24), thus:
\[
\tilde{\theta}_f = -\gamma_1 e^T P B \phi_1(x) \\
\tilde{\theta}_g = -\gamma_2 e^T P B \phi_2(x) u .
\]
Then
\[
V = -\frac{1}{2} e^T Q \phi_2^T + \gamma_2^T P B d
\tag{27}
\]
According to the good approximation capability of the powerful TS-type fuzzy system, \( |d| \) in (20) can be expected to be small. Consequently, if \( Q \) is properly chosen to be large enough, then \( V \leq 0 . \)

When \( V > 0 \) and \( V \leq 0 \), it can easily be found that \( \tilde{e} \), \( \tilde{\theta}_f \), and \( \tilde{\theta}_g \) are all bounded from Eq. (25), and, thus \( \tilde{\theta} \) is also bounded from Eq. (21). It follows from Barbalat’s lemma [12] that \( \lim_{t \to \infty} e(t) = 0 . \)

Moreover, since the chosen reference model is stable, then \( \bar{x}, \tilde{x} , \text{ and } \bar{x} \) is also bounded. So far, all signals in the closed system are bounded. \( \Box \)

From theorem 1, \( V \leq 0 \) thus that \( e(t) \to 0 . \)

The TS-type fuzzy system estimated model well approximates the unknown dynamic system so that the error between them will converge to zero asymptotically. Then, the controlled unknown dynamic system output well track the reference model output asymptotically.

5. Simulation results

In this section, an inverted pendulum system is used to verify the performance of the proposed model reference indirect adaptive fuzzy controller. The simulation results illustrate that Lyapunov’s stability for the closed-loop system, and all signals involved are bounded.

Example1: the dynamic equation of the inverted pendulum system is
\[
q = \begin{bmatrix} x_1 \ &= \ x_2 \\
\frac{m v^2 \cos x_1 \sin x_1}{M+m} & \frac{\cos x_1}{M+m} \ &= \ u \end{bmatrix}
\]
where \( x_1 \) is the rod angle velocity of the inverted pendulum with respect to the vertical line, \( x_1 \) is the rod angle of the inverted pendulum with respect to the vertical line, \( M \) is the mass of the cart, \( m \) is the mass of the rod, \( g = 9.8 \text{m/sec}^2 \) is the acceleration due to gravity, \( l \) is the half length of the rod, and \( u \) is the applied force. The system is unstable if the control input \( u \) is set to zero.

In the following simulation, it is assumed that \( M=1 \text{kg}, m=0.1 \text{kg}, \) and \( l=0.5 \text{m}. \)

Let a reference linear model be
\[
\bar{x} = Ax + Br
\]
Where
\[
A = \begin{bmatrix} 0 & 1 \\
-1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\
1 \end{bmatrix} .
\]

By selecting \( Q = \begin{bmatrix} 10 & 0 \\
0 & 30 \end{bmatrix} \), according to Eq. (22), we can obtain
\[
P = \begin{bmatrix} 20 & 5 \\
5 & 10 \end{bmatrix}
\]
which will be applied in the updating rule in Eqs. (23) and (24).

A control system of the inverted pendulum system will be developed to track the reference model with a reference input signal \( r(t) = \\sin(2/3pt) \), so that the rod angular displacement will follow the trajectory of a sin wave. The learning constants are selected as \( \gamma_1 = 35, \gamma_2 = 1 . \)

Because the system states are two, and three membership functions for each one, which are shown in Fig. 1, the total fuzzy rules are nine. So, the adjustable parameters are twenty-seven. Then, the initial values of adjustable parameters are chosen to be \( \bar{\theta}_f (0) = [0.65, \ldots, 0.65] \), \( \bar{\theta}_g (0) = [0.8, \ldots, 0.8] \).

The following membership functions for \( \mu_j, j=1,2 \) are given as
\[
\mu_A(x_j) = \mu_A^1(x_j) = \mu_A^2(x_j) = \mu_A^3(x_j) = \begin{cases} 0, x_1 > 0 \\
1, x_1 \leq -\frac{x}{6} \\
-6 x_1 \leq x_1 \leq 0 \\
0, x_1 > 0 \end{cases}
\]
\[
\mu_B(x_2) = \mu_B^1(x_2) = \mu_B^2(x_2) = \mu_B^3(x_2) = \begin{cases} 0, x_2 > 0 \\
1, x_2 \leq -\frac{x}{6} \\
-6 x_2 \leq x_2 \leq 0 \\
0, x_2 > 0 \end{cases}
\]
\[
\mu_C(x) = \mu_C^1(x) = \mu_C^2(x) = \mu_C^3(x) = \begin{cases} 0, x \leq \frac{x}{6} \\
\frac{x}{6} \leq x < \frac{x}{6} \\
-\frac{x}{6} \leq x \leq \frac{x}{6} \\
0, x > \frac{x}{6} \end{cases}
\]
The use of these laws, the stability of the closed law and adaptive update laws are derived. With the proposed adaptive laws, then, the control system. Based on the indirect adaptive approach, scheme into the nonlinear unknown control parameter equation explicitly such that we can incorporate an adaptive control fuzzy controller’s knowledge base. An affined closed-loop performance feedback for adjusting a control (MRIAFC). The MRIAFC scheme successfully.

6. CONCLUSION

In this paper, the proposed novel approach is called the model reference indirect adaptive fuzzy control (MRIAFC). The MRIAFC scheme employs a reference linear model to provide closed-loop performance feedback for adjusting a fuzzy controller’s knowledge base. An affined linear TS-type model is formed as an analytic parameter equation explicitly such that we can successfully incorporate an adaptive control scheme into the nonlinear unknown control system. Based on the indirect adaptive approach, two fuzzy rules are regulated in on-line scheme by the proposed adaptive laws; then, the control law and adaptive update laws are derived. With the use of these laws, the stability of the closed systems is guaranteed. This method has been applied to control the second-order nonlinear inverted pendulum system to track a sinusoidal reference trajectory. The computer simulation results show that the proposed scheme can perform a successful and superior control.

7. REFERENCES


Table 1

<table>
<thead>
<tr>
<th>Cases</th>
<th>Case1</th>
<th>Case2</th>
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</thead>
<tbody>
<tr>
<td>Initial states</td>
<td>$x(0) = [0.5 \ 0.5]^T$</td>
<td>$x(0) = [-0.25 \ -0.25]^T$</td>
</tr>
<tr>
<td></td>
<td>$v(0) = [0.1 \ 0]^T$</td>
<td>$v(0) = [0.1 \ 0]^T$</td>
</tr>
<tr>
<td></td>
<td>$\tau(0) = [0.35 \ 0.13]^T$</td>
<td>$\tau(0) = [0.13 \ -0.35]^T$</td>
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Fig. 1. Fuzzy membership functions defined over the state-space

Fig. 2. Trajectories of the system output, the reference model output, and the estimated output of the case 1

Fig. 3. Trajectory of the control input $u$ of the case 1

Fig. 4. Trajectories of the errors of the case 1

Fig. 5. Trajectories of the system output, the reference model output, and the estimated output of the case 2

Fig. 6. Trajectory of the control input $u$ of the case 2

Fig. 7. Trajectories of the errors of the case 2