A Power Efficient Algorithm for Power/Rate Control in Wireless Transmission

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Abstract

We propose the Lagrangian relaxation power and rate control (LRPRC) algorithm, which achieves relatively higher average system throughput, but with substantially lower power consumption and comparable outage probability when compared to two previously published schemes in [5]. We find that these are achieved by lowering the data rates of users with unfavorable channel conditions, instead of driving their powers to the maximum as in [5]. We conclude that if power efficiency is a major concern, the LRPRC is a relatively strong alternative among the three schemes, for light to heavy traffic load conditions.

Index terms: Power/rate control algorithm, variable rate transmission, power efficiency, wireless channels, Lagrangian relaxation.

1. Introduction

In [1], the authors combine the power control and adaptive modulation schemes, and derive adaptive algorithms that maximize the total throughput. A combined power and rate adaptation scheme is studied for fading channels in [2]; while variable processing gain is used to adapt transmission rate with the objective to maximize the average transmission rate in [3]. In [4], a more general approach is adopted to maximize the system throughput, by taking into account base station assignment, handover and call dropping.

Continuous throughput and transmission rate are considered in most of the previous works. However, in practice, various transmission modes such as the modulation level, processing gain, are limited to several discrete values. A recent work that considers discrete transmission rate is [5], in which two distributed algorithms are proposed. The first scheme, based on the Lagrangian relaxation technique, is called the Lagrangian relaxation power control (LRPC). The second scheme is called selective power control (SPC), and it is based on heuristics.

In this paper, we propose a power/rate control algorithm that has the desirable property of power efficiency. We present simulation results and compare the proposed algorithm with two schemes in [5]. Our results show that the proposed algorithm achieves relatively good performance in terms of system throughput and outage probability, but with substantially lower average transmitted power, when compared to these two known schemes.

2. Background

2.1 System Model

Consider a cellular radio system with $N$ mobiles accessing the same frequency channel. Without loss of generality, we consider the uplink throughout this paper. Each $i$-th mobile ($1 \leq i \leq N$) can transmit using power $0 \leq p_i \leq \overline{p}$, where $\overline{p}$ is the maximum power limit. Let $G = \{ g_{ij} \}$ denote the link gain matrix, where $g_{ii}$ is the link gain from $j$-th mobile to $i$-th base station. We shall use bold face letter to denote either a matrix or a vector throughout this paper. Thus, the $g_{ii}$’s are the gain factors of the desired communication links; while the $g_{ij}$’s, $i \neq j$, represent gain factors of the links that cause interference. Given a power vector $P = (p_1, p_2, \ldots, p_N)$, the received SIR for $i$-th mobile is defined as

$$ SIR_i(P) = \frac{g_{ii}p_i}{\sum_{j=1}^{N} \theta_{ij}g_{ij}p_j + v_i}, \ 1 \leq i \leq N $$

where $v_i > 0$ is the background additive noise power at the $i$-th base station receiver; $0 \leq \theta_{ij} \leq 1$ denotes the normalized cross-correlation between $p_i$ and $p_j$ at the $i$-th base station.
2.2 Combined Transmission Rate Selection
and Power Control

In this section, we briefly summarize the LRPC algorithm developed in [5], which is based on the Lagrangian relaxation technique.

**Problem formulation:** Let $\mathbf{p} = (p_1, p_2, \ldots, p_N)$ be the power vector, and $\overline{\mathbf{p}} = (\overline{p}_i)$ denotes the maximum power limit, therefore $0 \leq p_i \leq \overline{p}_i$, and $0 \leq \mathbf{p} \leq \overline{\mathbf{p}}$. Let $r^{(i)}_1 > r^{(i)}_2 > \ldots > r^{(i)}_K$ be the set of rates that i-th mobile can utilize. Let $Y = \left[ y^{(i)}_k \right]$ be a matrix in which, for every i-th mobile and rate $r^{(i)}_k$,

$$y^{(i)}_k = \begin{cases} 1, & \text{if mobile } i \text{ is transmitting with rate } r^{(i)}_k, \\ 0, & \text{otherwise.} \end{cases}$$

The objective is to maximize the system throughput $T$, i.e., the sum of effective data rates of all users at

$$T = \max_{Y \in \mathcal{Y}, \mathbf{p} \in \mathcal{P}} \sum_i \sum_k y^{(i)}_k r^{(i)}_k$$

subject to the constraints that for every $i$ and $k$,

$$y^{(i)}_k \in \{0, 1\}, \quad \sum_{i=1}^K y^{(i)}_k \leq 1, \quad \text{and } 0 \leq p_i \leq \overline{p}_i$$

(4a)

$$p_i + (1 - y^{(i)}_k) M^{(i)}_k \geq \frac{p_i y^{(i)}_k}{\text{SIR}(\mathbf{p})}$$

(4b)

To ensure a small SIR for non-selected rates (i.e., with $y^{(i)}_k = 0$), $M^{(i)}_k$ is set to be an arbitrarily large number such that

$$M^{(i)}_k \geq \min_p \frac{p y^{(i)}_k}{\text{SIR}(\mathbf{p})}$$

(5)

**Optimization using the Lagrangian multipliers:** For given $Y$ and $\mathbf{p}$, define

$$d^{(i)}_k(Y, \mathbf{p}) = \left[ p_i + (1 - y^{(i)}_k) M^{(i)}_k \right] - \frac{p_i y^{(i)}_k}{\text{SIR}(\mathbf{p})}$$

(6)

For a given nonnegative matrix of Lagrangian multipliers $\Lambda(n) = \left[ \lambda^{(i)}_k(n) \right]$, the Lagrangian relaxation problem is,

$$L(\Lambda) = \max_{Y, \mathbf{p}} \left\{ \sum_{i=1}^K \sum_{k=1}^K y^{(i)}_k r^{(i)}_k + \sum_{i=1}^K \lambda^{(i)}_k \cdot d^{(i)}_k(Y, \mathbf{p}) \right\}$$

(7)

and subject to the constraints in (4).

**Optimal solution:** For a given $\Lambda$, the optimal solution, $\hat{Y}(\Lambda) = \left[ \hat{y}^{(i)}_k(\Lambda) \right]$, to the above Lagrangian relaxation problem is [5]

$$\hat{y}^{(i)}_k(\Lambda) = \begin{cases} 1, & \text{if } r^{(i)}_k - \lambda^{(i)}_k M^{(i)}_k > 0, \\ \max_m \left\{ r^{(m)}_i - \lambda^{(m)}_k M^{(m)}_k \right\} > 0, & \text{otherwise.} \end{cases}$$

We then select the largest rate $r^{(i)}_k$ among those fulfilling $r^{(i)}_k - \lambda^{(i)}_k M^{(i)}_k = \max_m \left\{ r^{(m)}_i - \lambda^{(m)}_k M^{(m)}_k \right\}$, as the target rate for $i$-th mobile, provided that the value $r^{(i)}_k - \lambda^{(i)}_k M^{(i)}_k$ is positive.

Next, we describe the rate assignment scheme, and then present the proposed algorithm.

3. Proposed Power/Rate Control Algorithm

3.1 Data Rate Assignment

Suppose each $i$-th mobile can utilize a set of discrete transmission rates $r_i \in \{ r^{(i)}_1 > r^{(i)}_2 > \ldots > r^{(i)}_K \}$ by varying its processing gain. Let $\gamma_i$ be the target SIR of $i$-th mobile such that $r_i \in \{ r^{(i)}_1 > r^{(i)}_2 > \ldots > r^{(i)}_K \}$. To ensure reliable signal transmission at a data rate $r^{(i)}_k$, the SIR$(\mathbf{p})$ of the $i$-th mobile should be greater than $\gamma^{(i)}_k$, $k = 1, \ldots, K$. Each target SIR $\gamma^{(i)}_k$ corresponds to a target rate $r^{(i)}_k$.

The bit energy-to-interference power spectral density ratio of $i$-th mobile is given by

$$\frac{E_b}{I_0} = \frac{r^{(i)}_k}{\text{SIR}(\mathbf{p})}$$

(9)

where $W$ denotes the spreading bandwidth. We assume that the target $(E_b / I_0) = \Gamma$ is required at the receiver to maintain a constant bit error probability for $i$-th user.

3.2 Proposed Power Control Algorithm

We propose the following distributed constrained power and rate control (DCPRC) algorithm,

$$p_i(n+1) = \max_k \left\{ \frac{p_i(n) \gamma^{(i)}_k}{\Gamma(n)}, \quad \text{if } \frac{p_i(n) \gamma^{(i)}_k}{\text{SIR}(\mathbf{p}(n))} \leq \overline{p}_i, \\ \frac{\text{max}}{\text{SIR}(\mathbf{p}(n))} \chi \left( \text{SIR}(\mathbf{p}(n)) \geq \gamma^{(i)}_k \right), \quad \text{otherwise.} \right\}$$

(10)

where $\chi(E)$ is the indicator function of the event $E$.

The main motivation behind our proposed power control algorithm (10) is to avoid using the maximum power when the channel quality is poor, and therefore to improve power efficiency. Since each target SIR $\gamma^{(i)}_k$ corresponds to a target rate $r^{(i)}_k$, the algorithm (10) attempts to reduce overall undesirable interference by
lowering \( \gamma_i^{(k)} \) (and the corresponding \( r_i^{(k)} \)) of users with poor channel conditions. Such users will not transmit if they cannot be supported even with their minimum rates within their power limits. Such variable rate transmission is applicable for non-real time data services that can tolerate varying delay.

We use the power update algorithm (10) to replace the original distributed constrained power control (DCPC) in the power update step of LRPC [5] scheme. The proposed scheme shall be called as the Lagrangian relaxation power and rate control (LRPRC), and is summarized below.

3.3 The LRPRC Algorithm

This algorithm updates the target SIR \( \gamma_i^{(k)} \) (and therefore the target rate \( r_i^{(k)} \)) for every mobile in each iteration. Each step requires only local information that is available in each i-th mobile, therefore it can be implemented in a distributed way [5].

1. Initialize: Set the iteration number \( n = 1 \). Fix all users’ powers, \( P(n) \), and Lagrangian multipliers, \( \Lambda(n) \).

2. Select data rate for every i-th mobile: For every i-th mobile, solve \( \hat{\gamma}_i^{(k)}(\Lambda(n)) \) using (8). If \( \hat{\gamma}_i^{(k)}(\Lambda(n)) = 0 \) for all \( k \), then i-th mobile is not transmitting during iteration \( n \) and its power and \( \gamma_i^{(k)} \) are set to zero. Otherwise, i-th mobile uses the target \( \gamma_i^{(k)} \) that corresponds to rate \( r_i^{(k)} \) with \( \hat{\gamma}_i^{(k)}(\Lambda(n)) = 1 \).

3. Update power for every i-th mobile: For every i-th mobile, execute the proposed DCPRC algorithm (10) by one step with the target \( \gamma_i^{(k)} \) that has been selected in Step 2.

4. Update the Lagrangian multiplier for every i-th mobile: Update \( \lambda_i^{(k)}(n) \) by the modified subgradient method given by [5]

\[
\lambda_i^{(k)}(n+1) = \max \left\{ 0, \lambda_i^{(k)}(n) - \frac{d_i^{(k)}}{n}(\hat{\gamma}_i^{(k)}(\Lambda(n)), P(n)) \right\} \tag{11}
\]

where \( d_i^{(k)} \) is defined in (5).

5. Continue: When the predetermined criterion is met, stop. Otherwise, set \( n = n + 1 \) and go to Step 1.

Next, we compare the proposed LRPRC algorithm with the algorithm in [5], in which Step 3 is being replaced by the DCPC scheme of [6].

\[
p_i(n+1) = \min \left\{ \frac{p_i(n) \cdot \gamma_i^{(k)}}{\text{SIR}(P(n))}, \frac{p_i(n) \cdot \gamma_i^{(k)}}{\text{SIR}(P(n))} \leq P_i \right\} \tag{12}
\]

The proposed algorithm will also be compared with the SPC scheme suggested in [5]. According to the SPC, which does not rely on the formulation of the optimization problem in Section 2.2, the transmission power is updated as follows,

\[
p_i(n+1) = \max \left\{ \frac{p_i(n) \cdot \gamma_i^{(k)}}{\text{SIR}(P(n))}, \frac{p_i(n) \cdot \gamma_i^{(k)}}{\text{SIR}(P(n))} \leq P_i \right\} \tag{13}
\]

The SPC behaves in a way that it selects the maximum feasible rate in each iteration. Any mobile that cannot be supported within the power range constraint, even with its minimum rate, will not transmit.

4. Simulation Results and Discussion

We simulate a CDMA system with 25 base stations, with omni-directional antenna, located in the centers of two-tiers 25 square cells. The distance between two nearest base stations is 2 km. The uplink of the system is considered and a chip rate of \( W = 1.2288 \text{ Mcps} \) is assumed. We also assume that the radio link for each i-th mobile can support 4 transmission rates given by,

\[
r_i^{(k)} = 9.6/2^{k-1} \text{ kbps}, k = 1, 2, 3, 4 \tag{14}
\]

or no transmission if the i-th mobile is not supported. At any given instance, a total of \( N = 50 \) mobiles are generated and they are randomly, uniformly distributed over the 25 cells.

The link gain, \( g_s \), is modeled by the path-loss model, \( g_s = s_v \cdot d_i^{-4} \), where \( s_v \) is the shadowing gain factor and \( d_i \) is the distance between i-th base station and j-th mobile. The lognormal processes \( s_v \) are assumed to be independently and identically distributed with a mean of 0 dB and a standard deviation of 6 dB. Without loss of generality, the average power of Rayleigh fading process is set to unity.

The receiver noise is set to be -150 dB and the maximum mobile power is set to be -60 dB. At each instance, the initial transmission power of each mobile is randomly generated within the power range constraints. We assume that each mobile is connected to only one base station that is associated with the smallest signal attenuation.

We consider a system in which multiple rates are realized by varying the processing gain. The required minimum SIR before despreading is assumed to be \( \gamma_i^{(k)} = \gamma / 2^{k-1} \), which corresponds to each transmission rate \( r_i^{(k)} \) in (14). In our simulation, three values of \( \gamma \) are used to model, respectively, the light, medium and heavy load scenarios.

We generate 100 independent samples of mobile locations and shadowing processes. In each n-th iteration, each i-th mobile is allocated the effective transmission rate defined by, \( r_i(n) = \max \left\{ \gamma_i^{(k)}: \text{SIR}(P(n)) \geq \gamma_i^{(k)} \right\} \). The corresponding average throughput per user is then computed for every index \( n \).

Two other performance measures for the system are: (i) out age probability and (ii) average transmitted power per
user. To compare with the result in [5], the outage probability in each iteration is evaluated by computing the percentage of mobiles that cannot attain even the minimum rate of 1.2 kbps.

### 4.1 Comparing LRPRC with Other Schemes

We compare the performances of the proposed LRPRC, the LRPC and SPC schemes under various traffic load conditions. For the light load scenario, our results show that these three schemes achieve negligible probability of outage.

![Figure 1](image1.png)

Figure 1. Comparing the proposed LRPRC, the LRPC and the SPC schemes under medium traffic load condition ($\gamma = -10$ dB).

In Figure 1, we present performances of these three schemes under the medium traffic load condition with $\gamma = -10$ dB. We observe fluctuation in the average throughput for the SPC and LRPRC. In relative to other two schemes, the LRPRC requires the lowest average transmitted power while achieves a moderate level of average throughput and outage probability.

We observe in Figure 2 that the LRPRC achieves the highest average throughput but the lowest average transmitted power among all schemes under heavy traffic load condition with $\gamma = 0$ dB. For LRPRC, the outage probability is much higher than the SPC, but is comparable to the LRPC.

As highlighted in Figure 3, the LRPRC algorithm has the lowest average transmitted power for all load conditions. It even outperforms the SPC in terms of power efficiency. Therefore, compare to the other two schemes, the LRPRC scheme trades off a slightly higher outage probability for a much higher throughput plus a substantially lower average transmitted power.

### 4.2 Comparison for Heavy Load Scenario

In this section, we highlight the behaviors of LRPC, SPC and LRPRC.

In Figure 4, the difference between the behaviors of these three schemes becomes obvious under the heavy traffic load condition, which is modeled by setting $\gamma = 20$ dB. We use an example of 5 users associated with the following link gain matrix $G$, which is expressed in decibels,
\[
\mathbf{G} = \begin{bmatrix}
-84.1 & -120.0 & -109.6 & -83.5 & -98.0 \\
-108.2 & -89.5 & -108.2 & -97.2 & -108.0 \\
-100.9 & -101.2 & -84.6 & -120.0 & -99.0 \\
-92.7 & -104.3 & -97.5 & -85.4 & -80.6 \\
-93.7 & -114.0 & -91.3 & -93.4 & -61.8 
\end{bmatrix}
\] (15)

As highlighted in Figure 4(a), the LRPC scheme forces more users to operate at the maximum power (solid line) when their channel conditions are unfavorable. This could result in undesirable high interference to all system users.

![Figure 4](image1.png)

Figure 4. Comparison of transmitted power and throughput for (a) LRPC, (b) SPC and (c) proposed LRPRC scheme, for a CDMA system with 5 users under heavy traffic load condition (\(\gamma = 20\ \text{dB}\)).

In contrast, the proposed LRPRC algorithm behaves quite similar to the SPC. In Figures 4(b) and 4(c), we highlight that unlike the LRPC, both LRPRC and SPC do not drive the transmitted powers (of users with worse channel conditions) to the maximum level. Instead, both attempt to reduce the target rate (and therefore the target SIR) of users with worse channel conditions. This reduces interference level in the system. Because of this, system users only need to transmit relatively lower power to meet their target SIRs. However, due to the varying (but now smaller) interference and the data rate control that is imposed on all users, all users in the system experience fluctuations in their power levels and throughputs.

From Figures 4(b) and 4(c) we highlight that for the given link gain matrix (15), the proposed LRPRC assigns smaller or even zero power to more users compared to the SPC over the same observation interval. The throughputs of users with LRPC tend to switch between the maximum and minimum levels. We believe that the proposed LRPRC’s frequent switching within the maximum and minimum data rates (therefore the throughputs) leads to an overall higher average throughput than the SPC; while the LRPRC’s frequent assignment of smaller or zero power to users with unfavorable channel condition results in higher outage probability than the SPC. This could explain the difference between the results of LRPRC and SPC in Figure 2.

5. Conclusions

We propose a power efficient scheme, so-called the LRPRC, for power/rate control in wireless transmission. The proposed algorithm achieves relatively higher average system throughput, but with substantially lower power consumption and comparable outage probability. We find that these are achieved by lowering the data rates of users with unfavorable channel conditions, instead of driving their powers to the maximum as the DCPC does.

References