Frequency offset correction for space-time block coded OFDM systems based on maximum likelihood estimation

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Abstract - This paper discusses the Maximum Likelihood (ML) frequency offset estimation and subsequent correction for a Space-Time Block Coded Orthogonal Frequency Division Multiplexing (STBC-OFDM) system. Carrier frequency offset in OFDM system destroys the orthogonality between subcarriers resulting in performance deterioration. Also, it considerably reduces the diversity gain for STBC. Accurate frequency offset estimation and subsequent correction will significantly improve the performance of a STBC-OFDM system. This paper proposes an averaging approach for frequency offset estimation. The proposed scheme has been simulated with parameters in conformity with WLAN standards in a Rayleigh fading environment. Simulation results show that after the frequency offset correction, the STBC system may acquire the same level of performance as the system that possesses perfect frequency synchronization.

Key Words - Maximum Likelihood Estimation (MLE), Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM), Space-Time Block Coding (STBC), Frequency Offset Correction.

I. INTRODUCTION

Multi-carrier systems such as OFDM-based WLAN can support high data rate transmission in multi-path fading channel with the presence of large delay spread. This is due to the fact that a high data rate stream of information can be split into paralleled lower data rate stream, thus providing the stronger immunity against channel distortion caused mainly by Doppler shift in the context of wireless communication. Simultaneously, channel efficiency is obtained by overlapping subcarriers in the frequency domain [1]. Performance of wireless link can be further improved by introducing spatial diversity with the help of multiple antennas at both transmitters and receivers. A simple space-time block coding method is proposed in [2] as an approach of spatial-time diversity with two antennas at both ends of the link. This scheme has relatively low system complexity but results in good performance and therefore is attractive for practical implementation. The combination of OFDM and space-time block coding is the next logical step in obtaining further gain [3].

Cyclic Prefix (CP) has been exploited in OFDM systems to combat Inter-Symbol Interference (ISI), which is totally eliminated given this guard time is longer than the expected multi-path delay spread [4]. On the other hand, OFDM is sensitive to frequency offset because the orthogonality is destroyed in such a case, leading to Inter-Carrier Interference (ICI). Consequently, estimation and correction of frequency offset is extremely important for OFDM systems. Many different frequency offset estimation schemes have been proposed in literature including both frequency-domain approach [5] and time-domain approach [6]. Maximum likelihood estimation is the key idea underlying these proposals and correlation operation is the common structure for most of them. It is in this context, this paper exploits the concept further for the frequency offset estimation and correction of space-time block coded OFDM.

The rest of the paper is organized as follows: Section II describes a system model for STBC OFDM system with carrier frequency offset. Section III introduces the proposed ML method for frequency offset correction. Section IV discusses the simulation result and shows the improvement brought by the novel approach. Section V concludes the paper.

II. SYSTEM MODEL

The OFDM system with two transmit and two receive antennas is shown in Figure 1. The system embeds an STBC module before the IFFT in the transmitter and an ML detector is added in the receiver. There exist two independent cyclic prefix (CP) appending/removing block in the two antenna branches just before and after the channel but are not shown in Figure 1 for simplicity. The received signal vector for k-th subcarrier can be expressed as,

\[
R(k) = \begin{bmatrix}
    r_{00}(k) & r_{01}(k) \\
    r_{10}(k) & r_{11}(k)
\end{bmatrix}
= \begin{bmatrix}
    t_0(k) & t_1(k) \\
    -t^*_1(k) & t^*_0(k)
\end{bmatrix} \ast \begin{bmatrix}
    h_{00}(k) & h_{01}(k) \\
    h_{10}(k) & h_{11}(k)
\end{bmatrix}
+ \begin{bmatrix}
    n_{00}(k) & n_{01}(k) \\
    n_{10}(k) & n_{11}(k)
\end{bmatrix}
\]

where * denotes convolution, \(t_0(k)\) and \(t_1(k)\) are the STBC-OFDM symbol, \(h_{00}(k)\) through \(h_{11}(k)\) denote the channel impulse response at the k-th subcarrier of the OFDM block corresponding to the \(i\)-th transmit and the \(j\)-th receive antenna \((i = 0, 1; j = 0, 1)\) respectively. The channel state information (CSI) for at least two successive OFDM symbol remains fixed according to the traditional “quasi-static” channel model. The additive complex Gaussian noise plus the interference through
the channel are denoted by $n_{00}$ through $n_{11}$ and are assumed to be zero mean with variance $\sigma^2_n$. The noise is uncorrelated for different $i,j$ and $k$.

The OFDM symbol is given by $N$ point complex sequence

$$t(k) = \left(1/N\right) \sum_{n=0}^{N-1} X_n e^{2\pi j nk/N}, \quad k = 0, 1, \ldots, N-1$$

where $X_n$ is the input symbols and $N$ is the number of subcarriers.

For simplicity, we focus only on the signals received on the first antenna. Expand Eqn.(1) and consider the first column, the two successive symbols that arrive on $k$-th subcarrier form the received signal vector and is as follows:

$$\begin{bmatrix} r_{00}(k) \\
 r^*_1(k) \end{bmatrix} = \begin{bmatrix} h_{00}(k) & h_{10}(k) \\
 h^*_{10}(k) & -h^*_{00}(k) \end{bmatrix} \begin{bmatrix} t_0(k) \\
 t_1(k) \end{bmatrix} + \begin{bmatrix} n_{00}(k) \\
 n^*_1(k) \end{bmatrix}$$

Notice that the conjugate operation on transmit symbol has been shifted to the channel impulse response.

Thus, one element of the received signal vector can be expressed as

$$r_{00}(k) = \left(1/N\right) \sum_{n=0}^{N-1} X_{0,n} H_{00,n} e^{2\pi j (n+\epsilon_0)/N} + (1/N) \sum_{n=0}^{N-1} X_{1,n} H_{10,n} e^{2\pi j (n+\epsilon_1)/N} + n_{00}(k)$$

where $H_{00,n}$ and $H_{10,n}$ are the transfer functions of the channel at subcarrier $n$ from the two transmit antennae towards the first receive antenna respectively, $\epsilon_0$ and $\epsilon_1$ are the normalized frequency offsets of the channel (the ratio of frequency offset to the inter subcarrier spacing) and $n_{00}$ is the corresponding channel noise. The signal $r^*_1(k)$ can also be derived using similar methods.

At the receiver, $r_{00}(k)$ and $r^*_1(k)$ represent the signal in time domain. The demodulated symbol is then obtained by the DFT of these received signals and can be written as

$$\begin{bmatrix} R_0(m) \\
 R^*_1(m) \end{bmatrix} = \begin{bmatrix} H_0(m) & H_1(m) \\
 H^*_1(m) & -H^*_0(m) \end{bmatrix} \begin{bmatrix} S_0(m) \\
 S_1(m) \end{bmatrix} + \begin{bmatrix} N_0(m) \\
 N^*_1(m) \end{bmatrix}$$

where $R,H,S$ and $N$ are the corresponding frequency domain counterparts with respect to those in Eqn.(3). The index in subscript has also been modified for simplicity. Combining Eqn.(4) and (5) we may get the expression for the received signal sequence as:

$$R_0(m) = X_{0,n} H_{00,n} \xi(m,\epsilon_0) + X_{1,n} H_{10,n} \xi(m,\epsilon_1) + N_0(m)$$

where $\xi$ is the distortion of the channel caused by frequency offset and is expressed by [7]:

$$\xi(m,\epsilon) = (1/N) \sum_{n=0}^{N-1} e^{2\pi j \epsilon n/N} = \frac{1}{N} e^{j\pi \epsilon/N} \sin(\pi \epsilon/N)$$

$R^*_1(m)$ and the received information on the second receive antenna can be derived by analogy.

### III. FREQUENCY OFFSET CORRECTION

With the system model established in previous section, frequency offset can be corrected through maximum likelihood estimation. The ML estimation is achieved by the maximization of the log-likelihood function denoted by $\Lambda(\epsilon)$:

$$\Lambda(\epsilon) = \ln(\Pr\{R(\cdot)|\epsilon\}) = \ln f(R(\cdot)|\epsilon)$$

The estimation can be found through the solution of

$$\frac{\partial \Lambda(\epsilon)}{\partial \epsilon} = 0$$

Generally, the frequency offset is approximately constant over several received OFDM symbols. With this assumption, Eqn.(6) can be treated as a generic function with the form as shown in Eqn.(10):

$$R(n) = \Phi(n, \epsilon) + W(n)$$
where \( n = 1, 2, \cdots, N - 1 \) and \( W(n) \) are independent and identically distributed Gaussian random variables with zero mean and variance \( \sigma^2_{W} \).

The log-likelihood function can then be written as
\[
\Lambda(\epsilon) = \ln\left( \exp\left( -\sum_{n=0}^{N-1} \frac{|R(n) - \Phi(n, \epsilon)|^2}{2\pi \sigma^2_{W}} \right) \right)
\]
\[
(11)
\]

Differentiating and applying the rule given in Eqn. (9), the correlation-based solution can be obtained as shown in Eqn.(12):
\[
\Lambda(\epsilon) \propto \text{Re}\{ \sum_{n=0}^{N-1} \Phi(n, \epsilon) R^*(n) \}
\]
\[
(12)
\]

where \( \text{Re} \) denotes the real part of a complex expression. The ML estimation of \( \epsilon \) can be obtained using Eqn.(13):
\[
\hat{\epsilon} = \max_{\epsilon} \text{arg}\{ \text{Re}\{ \sum_{n=0}^{N-1} \Phi(n, \epsilon) R^*(n) \} \}
\]
\[
(13)
\]

It is straightforward to see that the frequency offset estimation is done on the basis of \( N \) received symbols. Considering again the assumption of slowly varying frequency offset and two receive antennae, an averaging approach can be realized. Supposing that the frequency offset remains constant during \( T \) symbol duration, the estimation with respect to this period on the first antenna is given as:
\[
\hat{\epsilon}_{1,1} = \max_{\epsilon} \Lambda_{1,1}(\epsilon)
\]
\[
(14)
\]

similarly, the expression for the second antenna is:
\[
\hat{\epsilon}_{1,2} = \max_{\epsilon} \Lambda_{1,2}(\epsilon)
\]
\[
(15)
\]

The mean of the estimation value with respect to both antenna numbers and symbol period is acquired as follows,
\[
\hat{\epsilon}_a = \frac{1}{2T} \sum_{l=1}^{T} (\hat{\epsilon}_{1,1} + \hat{\epsilon}_{1,2})
\]
\[
(16)
\]

The number of \( T \) is determined by the number of pilot symbols used in the OFDM system and the latency at initialization step is inevitable when a moving averaging scheme is utilized.

It is worth noticing that normally, there will be co-channel interference (CCI) in a MIMO system with more than one transmit antenna. The proposed averaging scheme solves this problem in two ways: firstly, it assumes a flat fading channel that guarantees a slowly varying frequency offset, which is reasonable in practical implementation and secondly, the averaging process takes into account the received symbols in both antennae during a certain period of time, thus mitigating CCI to some extent. A diversity order of four can be obtained considering four antennae have been employed totally. In addition, this method can be easily extended to a more general case of more transmit and receive antennae as described in [8].

IV. SIMULATION RESULT

The algorithm described is simulated comprehensively in a STBC based OFDM system. The system parameters are selected in conjunction with the existing WLAN standard such as IEEE802.11a. Table 1 gives the simulation parameter in detail.

The simulation assumes perfect synchronization and perfect channel knowledge. Two antennae at both transmitter and receiver are employed to implement STBC. The frequency offsets are assumed to be identical between the two transmitter antennae and the receiver (i.e. \( \epsilon = \epsilon_1 = \epsilon_2 \)). The simulation evaluates the performance of single antenna, STBC based system with single and double receive antenna respectively. Three different cases have been tested and their performances have been compared with different value of frequency offsets. It is seen that the proposed method can significantly override frequency offset errors and can improve the system performance, especially when the frequency offset is not large.

![Fig. 2. Performance of STBC-OFDM system with normalized frequency offset: 0.5%](image)

Figures 2 and 3 show the system performance with a relative frequency offset of 0.5% and 1% respectively. In the smaller frequency offset case, e.g. \( \epsilon = 0.005 \), the STBC scheme with single receive antenna outperforms the single antenna system by 5dB at the BER level of \( 10^{-5} \). Furthermore, with two
receive antennas, the system experiences another 3dB gain. The system performance with no frequency offset are plotted as dash dot lines in both the figures to serve as a reference in comparing with other results. It can be seen that even very small frequency offset deteriorates the system to a considerable extent, 3dB with $\varepsilon = 0.005$ and 8dB with $\varepsilon = 0.01$. However, when the STBC is used in the transmitter, a 6dB gain is still available when compared to the no frequency offset system. This implies that with the averaging approach, the STBC is almost unaffected with small frequency offset because a 6dB gain can normally be acquired with the same configuration under no frequency offset circumstance. [2]

The mean square estimation error versus SNR is shown in Figure 4. Note that with transmit diversity, the estimation accuracy exceeds the normal system under small SNR but increasing the number of receive antennae does not necessarily give rise to a significant improvement until the SNR reaches a certain value.

V. Conclusion

This paper discusses the frequency offset correction in an OFDM system with STBC. The trade off between complexity and performance improvement is a key issue for multiple antenna systems. STBC makes use of spatial diversity to solve this problem with low extra overhead. There’s a relatively high demand for frequency offset estimation in multi-carrier system because of the strict orthogonality that it must hold. Maximum likelihood frequency offset estimation has been examined in the context of STBC based OFDM system and an average approach has been proposed. Simulation results show that the scheme works well especially with small carrier frequency offset and system degradation caused by frequency offset can be even ignored when STBC is employed in transmitter.

REFERENCES