Retrieving the instantaneous frequency from images by wavelet ridges

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Abstract

Instantaneous frequency (IF) is very important in signal communications and methods, such as Hilbert transform and more recently wavelet ridges, have been developed to extract them. The importance of IF for 2D images has often been ignored. For example, in moiré interferometry, IF can directly provide strain contours, which usually is the desired parameter in experimental mechanics. In this paper, the method of wavelet ridges is first extended from a 1D signal to 2D images with carrier fringes. In cases where the carrier fringes is not available, the phase shift approach is proposed, to extract the Instantaneous Frequency (IF) using wavelet ridges. Both methodologies are introduced and applied to real fringe patterns.

1. Introduction

Instantaneous frequency (IF), very important in signal communications, can be extracted effectively by Hilbert transform and wavelet ridges. For the following amplitude-modulated and frequency-modulated (AM-FM) signal

\[ f(t) = b(t) \cos[\phi(t)] \] with \( b(t) \geq 0 \)  

(1)

IF is defined as a positive derivative of the phase [1]:

\[ \omega(t) = \phi'(t) \geq 0 \]  

(2)

In optical interferometry, the 2D interferometric fringe patterns can be written as [2]

\[ f(x, y) = a(x, y) + b(x, y)\cos[\varphi(x, y)] \]  

(3)

where \( f(x, y) \), \( a(x, y) \), \( b(x, y) \) and \( \varphi(x, y) \) are the recorded intensity, background intensity, fringe amplitude and phase distribution, respectively. The phase is proportional to displacement component and the desired derivative of displacement (phase) gives, for example in moiré interferometry, the normal strain, as:

\[ \varepsilon_{xx} = c \frac{\partial \varphi(x, y)}{\partial x} \]  

(4)

where \( c \) is a constant. Comparing Eq. (4) to Eq. (2), it is easy to recognize that the strain in moiré interferometry is analogous to IF in communication terminology except for a constant. Another example is in speckle shearography [3]. The shearographic patterns can also be written in the form of Eq. (3). But this time the phase is proportional to the slope of the deformation and IF thus provides the curvature of deformation.

This recognition helps to bridge the field of system communication and optical interferometry. As the method of wavelet ridges is very effective for IF extraction, Section 2 extends this approach from 1D signal to 2D images. This is called the Carrier Wavelet Ridges (CWR) method, since a carrier frequency is necessary. However, unlike for signal communications, in 2D images, carrier frequency is not always available. To overcome this problem, we proposed a new method - Phase Shifting Windowed Fourier Ridges (PSWFR). Multiple phase shifted images rather than a single image with a carrier frequency are used. This is described in Section 3. The conclusions of this study are detailed in Section 4. This study is restricted to processing of optical interferometric patterns obtained using any of the many existing optical methods.

2. Carrier Wavelet Ridges (CWR) [4]

For brevity, we give the principle of CWR for 2D images, as a direct extension of that for 1D signal [1].

A 2D wavelet is a function \( \psi \in L^2(\mathbb{R} \times \mathbb{R}) \) with a zero average:

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(x, y) dx dy = 0 \]  

(5)

It is normalized as \( \|\psi(x, y)\| = 1 \) and centered in the neighborhood of \((x, y) = (0,0)\). A family of time-frequency atoms is obtained by scaling \( \psi \) by \( s_1 \) and \( s_2 \) and translating it by \( u \) and \( v \) along the two directions \( x \) and \( y \):

\[ \psi_{u,v,s_1,s_2}(x, y) = \frac{1}{\sqrt{s_1 s_2}} \psi \left( \frac{x-u}{s_1}, \frac{y-v}{s_2} \right) \]  

(6)

These atoms remain normalized: \( \|\psi_{u,v,s_1,s_2}(x, y)\| = 1 \). The continuous wavelet transform of a signal \( f \in L^2(\mathbb{R} \times \mathbb{R}) \) at coordinates \((x, y)\) and scale \( s_1 \) and \( s_2 \) is...
\[ Wf(u,v,s_1,s_2) = \left\langle f, \varphi_{u,v,s_1,s_2} \right\rangle \]
\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \varphi^*_{u,v,s_1,s_2}(x,y) \, dx \, dy \] (7)

where \( \langle \cdot, \cdot \rangle \) denotes an inner product. A separable 2D Gabor wavelet is chosen for simplicity as
\[ \varphi(x,y) = \psi(x)\psi(y) = g(x)g(y)\exp(j \eta_1 x + j \eta_2 y) \] (8)

where \( j = \sqrt{-1} \) and the window functions \( g(t) = \frac{1}{(\sigma^2 \pi)^{1/4}} \exp\left(-\frac{t^2}{2\sigma^2}\right) \) (9)

For the fringe pattern described in Eq. (3), its continuous wavelet transform is
\[ Wf(u,v,s_1,s_2) = \left\langle f(x,y), \varphi_{u,v,s_1,s_2}(x,y) \right\rangle \]
\[ = \left\langle a(x,y), \varphi_{u,v,s_1,s_2}(x,y) \right\rangle \]
\[ + \frac{1}{2} \left\langle b(x,y)\exp[j \varphi(x,y)], \varphi_{u,v,s_1,s_2}(x,y) \right\rangle \]
\[ + \frac{1}{2} \left\langle b(x,y)\exp[-j \varphi(x,y)], \varphi_{u,v,s_1,s_2}(x,y) \right\rangle \]
\[ = I(a) + I(\varphi) + I(-\varphi) \]

(10)

If \( a(x,y), b(x,y), \frac{\partial \varphi(x,y)}{\partial x} \) and \( \frac{\partial \varphi(x,y)}{\partial y} \) vary slowly, it can be derived that
\[ I(a) \approx 0 \]
\[ I(\varphi) \approx \frac{\sqrt{s_1 s_2}}{2} b(u,v) \exp[j \varphi(u,v)] \]

(11) (12)

\[ \hat{g}\left\{ s_1 \left[ \xi_1 - \frac{\partial \varphi(u,v)}{\partial u} \right] \right\} \hat{g}\left\{ s_2 \left[ \xi_2 - \frac{\partial \varphi(u,v)}{\partial v} \right] \right\} \]

\[ I(-\varphi) \approx \frac{\sqrt{s_1 s_2}}{2} b(u,v) \exp[-j \varphi(u,v)] \]
\[ \hat{g}\left\{ s_1 \left[ \xi_1 + \frac{\partial \varphi(u,v)}{\partial u} \right] \right\} \hat{g}\left\{ s_2 \left[ \xi_2 + \frac{\partial \varphi(u,v)}{\partial v} \right] \right\} \]

(13)

where \( \hat{\cdot} \) denotes a Fourier transform, \( \xi_1 = \eta_1 / s_1 \), and \( \xi_2 = \eta_2 / s_2 \).

When a carrier is present, i.e., \( \frac{\partial \varphi(u,v)}{\partial u} \) or \( \frac{\partial \varphi(u,v)}{\partial v} \), or both of them are large enough, \( I(\phi) \) and \( I(-\phi) \) can be separated. When \( \xi_1 \) and \( \xi_2 \) is near \( \frac{\partial \varphi(u,v)}{\partial u} \) or \( \frac{\partial \varphi(u,v)}{\partial v} \), \( I(-\phi) \) is near zero since the effective support of function \( \hat{g} \) is limited. Thus normalized scalogram can be expressed as
\[ \left\| \frac{1}{s_1 s_2} Wf(u,v,s_1,s_2) \right\|^2 \]
\[ = \frac{1}{s_1 s_2} |I(a) + I(\varphi) + I(-\varphi)|^2 \approx \frac{1}{s_1 s_2} |I(\varphi)|^2 \] (14)

It can be seen that when \( \xi_1 = \frac{\partial \varphi(u,v)}{\partial u} \) or \( \xi_2 = \frac{\partial \varphi(u,v)}{\partial v} \), the normalized scalogram reaches the maximum value, which is called wavelet ridge. Alternately, if the wavelet ridge is found, the corresponding \( \xi_1 \) and \( \xi_2 \) are the desired IFs, among all other possible frequencies.

Fig. 1(a) shows a moiré interferometric fringe pattern, depicting contours of displacement component in the vertical direction. Using conventional "moiré of moiré" [5], contours of derivative of displacement in the horizontal direction, is visualized (Fig. 2(a)). The whole field IF in horizontal direction by the CWR, which is also proportional to the whole field strain, is shown in Fig. 1(c) in grayscale and Fig. 1(d) in 3D view, from which contour can be easily extracted, as shown in Fig. 1(e). The advantages of CWR are apparent: (1) the whole field quantitative strains are obtained, rather than visual contours and (2) the results are better than the moiré of moiré method.
3. Phase shifting Windowed Fourier Ridges (PSWFR) [6]

In Eq. (10), $I(\phi)$ and $I(-\phi)$ must be separated in order to extract IF and hence a carrier frequency is necessary. However, in optical interferometry, a carrier is not always available. Phase shifting is thus proposed to replace the carrier frequency and to separate the two terms. Phase shifting is a standard tool in optical metrology and can be easily realized. The principle of PSWFR is as follows.

If a phase-shifted fringe pattern with a phase shift of $\pi / 2$ is provided, i.e.

$$f_1(x, y) = a(x, y) + b(x, y) \cos[\phi(x, y) + \pi / 2]$$

$$= a(x, y) - b(x, y) \sin[\phi(x, y)]$$

(15)

then its wavelet transform is

$$Wf_1(u, v, s_1, s_2) = I(a) + j \cdot I(\phi) - j \cdot I(-\phi)$$

(16)

With Eq. (11), $I(\phi)$ can be separated as

$$I(\phi) = \frac{1}{2} \left[ Wf(u, v, s_1, s_2) - j \cdot Wf_1(u, v, s_1, s_2) \right]$$

(17)
and the technique of wavelet ridges can then be applied to retrieve the IF.

However in practice, when the carrier frequency is not present, the IF is usually very low and hence wavelet atoms with very large scales are need to find the wavelet ridges, which is not practical. Hence the windowed Fourier transform, which has a fixed duration of all atoms, is adopted [1].

In real applications, several phase-shifted fringe patterns are recorded and combined into a complex field. The phase-shifted fringe patterns can be written as

\[
f_i(x, y) = a(x, y) + b(x, y) \cos[\phi(x, y) + (i-1)\times \phi_0],
\]

\(i = 1, 2, 3, \ldots\) (18)

where \( (i-1)\times \phi_0 \) are the phase shifts. For simplicity, we chose \( i = 1, 2, 3, 4 \) and \( \phi_0 = \pi/2 \), then we have

\[
h(x, y) = [f_i(x, y) - f_{i-1}(x, y)] + j[f_i(x, y) - f_{i-1}(x, y)]
\]

\( = b(x, y) \exp[j\phi(x, y)]\) (19)

As \( h(x, y) \) is an analytic signal, its wavelet transform or windowed Fourier transform is simply \( I(\phi) \), the other two terms will not appear. Thus wavelet ridges can be directly applied.

Fig. 2(a) and (b) are two of the four phase-shifted fringe patterns recorded from a phase shifting speckle shearographic interferometer. The traditional approach is to compute the phase \( \phi(x, y) \) directly from \( h(x, y) \), and then numerically differentiated it to yield IF [7], Fig. 3(c). IF extracted using the proposed PSWFR, is shown in Fig. 3(d) as a grayscale image and in Fig. 3(e) in 3D perspective. The extracted phase derivative is much better than that in Fig. 3(c) and indeed better than any other existing methods [8,9].
4. Conclusions

The method of wavelet ridges is applied to 2D fringe patterns to extract the instantaneous frequencies. Either a single pattern with a carrier frequency or multiple phase-shifted patterns are needed to process the fringe pattern. For this the Carrier wavelet ridges (CWR) and the phase shifting windowed Fourier ridges (PSWFR) are proposed accordingly. Their principles as well as applications are demonstrated. The methods are very robust to the noise images.

References


