Performance Analysis of Variable Bit Rate Multiclass Services in a Multirate DS-CDMA System

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Abstract – An analytical formulation of the outage probability in terms of bit error rate specification for variable bit rate (VBR) multiclass services in the uplink of a multirate DS-CDMA-based cellular system is presented. The analytical framework is formulated for the general case in which different traffic classes have different spreading gains and within a class the users can have different spreading gains. Each user has multiple spreading codes with different data rates. The analytical work leads to the determination of the capacity region of a multirate DS-CDMA system with VBR traffic. Numerical results of the capacity region corresponding to typical parameter values are also presented.

Index Terms – analytical formulation, bit error rate, variable bit rate, multiclass services, multirate DS-CDMA, quality of service

1. Introduction

In the present development of wireless personal communication networks, there are already papers that address the capacity in wireless cellular CDMA networks [1-4]. In [1], the system capacity for a single traffic class using code division multiple access (CDMA) is considered, while [2] considers the system capacity for voice and data traffic in multicode CDMA. In [3], the system capacity for two traffic classes in wideband CDMA (WCDMA) is considered, while in [4], the system capacities for a single voice traffic class using narrowband CDMA and a single data traffic class using WCDMA are considered separately. These papers consider only one or two traffic classes.

For third generation (3G) and beyond cellular systems, many types of connections are anticipated, not just voice and data traffic. The connections could be voice, video, data, multimedia, web browsing, etc. That is, we have multiclass traffic. In [5], Wong et al. generalize the analysis in [3] for two traffic classes to \( K \) traffic classes. In numerous performance analyses of the system capacity [1-5], the traffic source is often assumed to be an on/off process. Such an on/off process enables simple closed-form solutions in the performance analyses. However, such an assumption is valid only for voice or data traffic but not for video or other traffic which can be modeled as a multirate traffic [6-8]. Thus, the performance of cellular DS-CDMA networks with a new traffic model that can account for multirate traffic as well should be studied, rather than with the extensively used on/off traffic model. Since the multirate traffic model is more complex than a simple on/off traffic model, closed-form solution is more difficult, though not impossible. To our knowledge, there has been no cellular system capacity analysis with multirate sources, except [9], to date. In [9], Wong et al. consider a multirate traffic model for variable bit rate traffic to study the capacity of the uplink of a WCDMA system with multiclass traffic, where the spreading gain is assumed to be variable for different traffic classes. The spreading gain is assumed to be the same within the same class. In this paper, we generalize the results in [9] such that the spreading gain can also be different within the same class. That is, each user has multiple spreading codes with different data rates. Thus the main contribution in this paper is the analytical formulation which is applicable to variable bit rate multiclass traffic where each user has a number of spreading codes with different spreading gains or data rates for a multirate DS-CDMA system. The probabilities of bit error rate for different classes are formulated in terms of the numbers of different class users, the states of the active spreading codes, the intra-cell received powers for all classes, the inter-cell interference for all classes, the spreading gains for all classes, the bit energy-to-interference ratio requirements for all classes and the background noise. The analytical formulation leads to the determination of the system capacity for connection admission control (CAC) of variable bit rate multiclass traffic in a multirate DS-CDMA system.

The rest of the paper is organized as follows. Section 2 describes the system model, assumptions and system parameters. Section 3 states the problem to be addressed in this paper. In Section 4, we present an analytical model for multiclass services in a multirate DS-CDMA system. Numerical results for three traffic classes are presented in Section 5. Finally, concluding remarks are made in Section 6.

2. System Model

We consider VBR multiclass traffic in the uplink of a wireless cellular multirate DS-CDMA network with multiple variable spreading gains spreading codes for each user. The cell-site (base station) supports \( K \) classes of services that can originate from the mobile users in the cell.
The following system parameters are used throughout the paper.

**System parameters**

- $K$: number of traffic classes
- $W$: spread-spectrum bandwidth
- $R_i$: basic transmission rate of class $i$ traffic using one class $i$ spreading code in state 1 of the source model, $i=1,2,\ldots,K$
- $G_i(m)$: spreading gain of class $i$ traffic in state $m$ of the source model
- $n_i$: number of class $i$ users per sector
- $BER_{i,m}$: bit error rate QoS of class $i$ in state $m$
- $SIR_{i,m}$: signal-to-interference ratio QoS of class $i$ in state $m$
- $BER_{i,m}^*$: bit error rate QoS requirement of class $i$ in state $m$
- $SIR_{i,m}^*$: signal-to-interference ratio QoS requirement of class $i$ in state $m$
- $\gamma_{i,m}$: $E_b/I_0$ QoS requirement of class $i$ traffic in state $m$
- $S(m)$: received power of class $i$ traffic in state $m$
- $I(m)$: inter-cell interference power of class $i$ traffic in state $m$
- $\eta$: power spectral density of ambient noise
- $\rho$: density of class $i$ users per unit area
- $\psi_j$: random variable for the source’s state of a user $j$ belonging to class $i$ traffic
- $M_i$: maximum number of active spreading codes used by a class $i$ user
- $\alpha_i$: increase rate of a two-state mini-source
- $\beta_i$: decrease rate of a two-state mini-source

The following assumptions are made to facilitate the analytical formulation.

**Assumptions**

- The transmission rates of all users are integer multiples of that for the user with the basic rate.
- The processing gain, $G_i(m)$ for class $i$ users in state $m$ of the source model are given by $W/(mR_i)$.
- The system is made up of hexagonal cells.
- The mobile users have omni-directional antennas.
- The base station antenna has three sectors in each cell.
- The sectorization in the cells is perfect.
- Users are uniformly distributed in each cell.
- There are equal numbers of users from each class in every cell.
- There is perfect power control in each cell.
- The spreading gain can be varied for different traffic class.
- The spreading gain can be varied within the same class.
- The channel is modeled as a combination of path loss and log-normal shadowing, represented by $r^{4+10\sigma^2}$, where $r$ is the distance between the mobile and the serving base station and $\sigma$ is a Gaussian random variable with zero mean and variance $\sigma^2$. The path loss exponent, which is normally determined from measurements and is in the range 2-5, is assumed to be 4 in this paper.

### 3. Problem Statement

We are concerned with the uplink capacity of the multirate DS-CDMA system in terms of the number of users, $n_i$, that can be supported for the $i$th class. The capacity region for $K$ classes is derived by considering the outage probability in terms of the signal-to-interference ratio (SIR) specification. These probabilities are expressed in terms of the number of class $i$ users, the states of the active spreading codes, the intra-cell received powers for $K$ classes, the inter-cell interference for $K$ classes, the spreading gains for $K$ classes, the $E_b/I_0$ requirements for $K$ classes and the background noise.

The capacity of the $K$-class system is defined by $(n_1,\ldots,n_K)$. The aim here is to determine the maximum number of users for the $K$ classes that are allowable in the system while maintaining the required QoS.

### 4. Analytical Model

From [6-8], a variable bit rate source can be modeled by a continuous-time Markov chain with finite states. Each state represents the discrete level of bit rate that is generated by a single source. We assume that the highest level is state $M_i$. If $M_i = 1$, the source is an on/off source. Each level can be modeled by a two-state mini-source with an increase rate of $\alpha_i$ and a decrease rate of $\beta_i$. The Markov chain for this mini-source is shown in Fig. 1.

**Fig. 1. Two-state Markov chain for a mini-source**

Thus the continuous-time Markov chain for a single source at state $m$ has an increase rate of $(M_i - m)\alpha_i$ and a decrease rate of $m\beta_i$. This Markov chain is shown in Fig. 2.

**Fig. 2. Continuous-time Markov chain for a single variable bit rate source**

The steady-state probability of being in state $m$, denoted by $P_m$, is given by

$$P_m = \left(\frac{M_i}{m}\right)p_i^m(1-p_i)^{M_i-m}, m = 0,1,\ldots,M_i, \quad (1)$$

where

$$p_i = \frac{\alpha_i}{\alpha_i + \beta_i}, \quad (2)$$
and its mean, second moment and variance are $M_\rho$, $M_\rho [1 + (M_f - 1)\rho_f]$ and $M_\rho[1 - \rho_f]$, respectively. We assume that each state uses one spreading code with the required data rate for a class $i$ user. This means that state $m$ has a data rate of $mR_i$ corresponding to one class $i$ spreading code with this rate, where $R_i$ is the basic data rate in state 1.

Next, let $\gamma_{im}$ denote the $E_o/I_o$ for class $i$ in state $m$. It is given by

$$
\gamma_{im} = \frac{G_i(m)}{\sum_{j=1}^{n_i-1} S_i(\psi_{ij}) + \sum_{k=1}^{M_f} \sum_{k=1}^{M} S_i(\psi_{kj})}.
$$

where $\psi_{ij} \in \{0, 1, 2, \ldots, m, M_i\}$ is a binomial random variable indicating the data rate state and its corresponding spreading code used by the $j$th user of class $i$. The probability that $m$ active spreading codes are used by a source, denoted by $Pr[\psi_{ij} = m] = P_m$, $m = 0, 1, 2, \ldots, M_i$. (4)

The numerator in the right hand side of equation (3) is the class $i$ processing gain. In the denominator, the first term is due to the intra-cell interference from other users in class $i$, the second term is due to the intra-cell interference from users from other classes, the third term is due to the inter-cell interference from all classes and the last term is due to background noise. Rearranging and taking the average of equation (3), we have

$$(n_i - 1) \sum_{\psi_i = 1}^{M_f} S_i(m) G_i(m) \gamma_{im} = - \sum_{k=1}^{M_f} \sum_{\psi_i = 1}^{M} I_k(\psi_{ik}) - \eta_i, \quad i = 1, 2, \ldots, K. \quad \text{(5)}$$

Note that $S_i(0) = 0$ and $I_k(0) = 0$. For $x, y = 1, 2, \ldots, M_n$, and manipulating (5), we have

$$S_i(x) = \frac{G_i(y)}{\gamma_{i,y}} \frac{G_i(x)}{\gamma_{i,x}}. \quad \text{(6)}$$

For $i \neq k$, the power ratio can be expressed as

$$\frac{S_k(x)}{S_i(y)} = \left( \frac{G_i(y)}{\gamma_{i,y}} \sum_{\psi_i = 1}^{M_f} \frac{G_i(\psi_{ik})}{\gamma_{i,\psi_{ik}}} + 1 \right) \left( \frac{G_k(x)}{\gamma_{k,x}} \sum_{\psi_k = 1}^{M_f} \frac{G_k(\psi_{ik})}{\gamma_{k,\psi_{ik}}} + 1 \right), \quad x = 1, \ldots, M_k, \quad y = 1, \ldots, M_i. \quad \text{(7)}$$

From (3), the inter-cell interference-to-signal ratio for a class $i$ user with interference power, $I_n$ and received power, $S_n$ is given by

$$I_i = \frac{r_m}{r_d} 10^{(\varepsilon_d - \varepsilon_m)/10}, \quad \text{(8)}$$

where $r_d$ is the distance between the inter-cell mobile that is causing interference and the intra-cell base station, $r_m$ is the distance between the inter-cell mobile and its own base station, and $\varepsilon_d$ and $\varepsilon_m$ are Gaussian random variables with zero mean and standard deviation $\sigma$. Since $\varepsilon_d$ and $\varepsilon_m$ are independent, $(\varepsilon_d - \varepsilon_m)$ is a Gaussian random variable with zero mean and $2\sigma^2$ variance.

Treating the probability of the state that a source is at (e.g., state $x$) with a corresponding spreading code and grouping together the other probabilities of the states that the source is not at this particular state (states $0, 1, \ldots, x-1, x+1, \ldots, M_i$), we have a Bernoulli random variable, $\phi_{ix}$, for the indication of the usage of the spreading code. The mean and variance of $I_i(x)/S_i(x)$ are upper bounded by [3]

$$E \left[ \frac{I_i(x)}{S_i(x)} \right] = Pr[\psi_{ij} = x] \rho_i \int \frac{r_m}{r_d} dA \leq \mu_{ii,xx}, \quad \text{(9)}$$

and

$$Var \left[ \frac{I_i(x)}{S_i(x)} \right] \leq Pr[\psi_{ij} = x] \rho_i \int \left( \frac{r_m}{r_d} \right)^2 dA + \rho_i^2 \int \left( \frac{r_m}{r_d} \right)^2 dA = \sigma^2_{ii,xx}, \quad \text{(10)}$$

where $\mu_{ii,xx}$ is the upper bound on the mean of $I_i(x)/S_i(x)$, $\sigma_{ii,xx}$ is the upper bound on the variance of $I_i(x)/S_i(x)$, and $\rho_i$ is the density of class $i$ users per unit area and is given by $\rho_i = 2n_i / \sqrt{3}$. 

$$f \left( \frac{r_m}{r_d} \right) = \frac{r_m}{r_d} e^{\frac{\sigma \ln 10^2}{10}} \times \left[ 1 - Q \left( \frac{40\log(r_d / r_m) - \sqrt{2\sigma^2 \ln 10}}{10} \right) \right], \quad \text{(11)}$$

and

$$g \left( \frac{r_m}{r_d} \right) = \frac{r_m}{r_d} e^{-\frac{\sigma \ln 10^2}{5}} \times \left[ 1 - Q \left( \frac{40\log(r_d / r_m) - \sqrt{2\sigma^2 \ln 10}}{5} \right) \right], \quad \text{(12)}$$

and

$$Q(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx. \quad \text{(13)}$$

The random variable, $I_i(x)/S_i(y)$, can be expressed as

$$\frac{I_i(x)}{S_i(y)} = \frac{I_i(1)}{S_i(1)} \frac{S_i(1)}{S_i(y)} \frac{S_i(y)}{S_i(x)} \frac{I_i(x)}{I_i(1)} \frac{I_i(1)}{S_i(1)} \frac{I_i(1)}{S_i(1)}, \quad i, k = 1, 2, \ldots, K. \quad \text{(14)}$$

Thus, the mean and variance of $I_i(x)/S_i(y)$ satisfy the following inequalities:

$$E \left[ \frac{I_i(x)}{S_i(y)} \right] \leq \frac{S_i(1) Pr[\psi_{ik} = x] \rho_k}{S_i(y) Pr[\psi_{il} = x] \rho_l} \mu_{kl,11}, \quad \text{(15)}$$

and

$$Var \left[ \frac{I_i(x)}{S_i(y)} \right] \leq \left( \frac{S_i(1)}{S_i(y)} \right)^2 \sigma^2_{kk,xx}. \quad \text{(16)}$$
Let \( BER_{im}^* \) denote the BER requirement for class \( i \) users in state \( m \) and \( SIR_{im}^* \) denote the SIR requirement for class \( i \) users in state \( m \). The system capacity is defined as the maximum \((n_{1},...,n_{i},...,n_{K})\) that can be supported such that the achieved SIR is greater than or equal to the required SIRs \((K=3)\) 99% of the time for all classes. That is, the outage probability is defined as

\[
Pr[BER_{im} \geq BER_{im}^*] = Pr[SIR_{im} \leq SIR_{im}^*] = \Pr_{i,m} = \Pr_{i,m} \quad (\text{for } i = 1,2,\ldots,K). (21)
\]

5. Numerical Results

In this section we present results for the system capacity with 3 classes \((K=3)\) as illustrative examples. The parameter values used in the numerical examples are tabulated in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Symbol</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>(M_1)</td>
<td>1 (\sigma)</td>
<td>8 dB</td>
<td></td>
</tr>
<tr>
<td>(M_2)</td>
<td>2 (\gamma_{1,1})</td>
<td>7 dB</td>
<td></td>
</tr>
<tr>
<td>(M_3)</td>
<td>3 (\gamma_{2,1})</td>
<td>7 dB</td>
<td></td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>0.125 (\gamma_{1,2})</td>
<td>7 dB</td>
<td></td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>0.875 (\gamma_{3,1})</td>
<td>7 dB</td>
<td></td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>0.1 (\gamma_{3,2})</td>
<td>7 dB</td>
<td></td>
</tr>
<tr>
<td>(\beta_2)</td>
<td>0.1 (\gamma_{3,3})</td>
<td>7 dB</td>
<td></td>
</tr>
<tr>
<td>(\delta_3)</td>
<td>0.2 (S_1(1)/\eta)</td>
<td>-1 dB</td>
<td></td>
</tr>
<tr>
<td>(\delta_2)</td>
<td>0.2 (S_2(1)/\eta)</td>
<td>(0.078 \times 10^{-3})</td>
<td></td>
</tr>
<tr>
<td>(\delta_1)</td>
<td>0.2 (S_3(1)/\eta)</td>
<td>(0.078 \times 10^{-3})</td>
<td></td>
</tr>
</tbody>
</table>

The bounds on \(\sigma_{22,\{\psi_{ij}\}}^2\) and \(\sigma_{33,\{\psi_{ij}\}}^2\) are valid for (see Appendix)

\[
Pr[\psi_{ij} = x] \geq p_1, x = 1,2,\ldots (22)
\]

and

\[
Pr[\psi_{ij} = x] \geq p_1, x = 1,2,3. (23)
\]

With these numerical parameters, we can express equations (20) and (21) as functions of the triplet \((n_{1},n_{2},n_{3})\). The admissible region for the system capacity at \(W=5\) MHz is shown in Fig. 3. Fig. 3 shows that the capacity, \((n_{1},n_{2},n_{3})\), of the system at this spread-spectrum bandwidth is on a “planar” surface. That is, the elements of the triplet \((n_{1},n_{2},n_{3})\) are bounded by this “plane”. Thus the combinations of the numbers of users of different classes that can be admitted to the system are possible only when these numbers are on or below this “plane”. Numerical results can be obtained for \(K\)}
traffic classes and the system capacity is in a $K$-dimensional space. $(n_1, \ldots, n_K)$ are admissible as long as they are within its system capacity.

![Fig. 3. System capacity at W=5 MHz](image)

### 6. Concluding Remarks

An analytical formulation of the outage probability in terms of bit error specifications for variable bit rate multiclass services in a cellular multirate DS-CDMA system is presented in Section 4. This analysis enables the determination of the system capacity region for connection admission control, by treating the general case where different traffic classes have different spreading gains and each user has multiple spreading codes of different spreading gains or data rates. The capacity of three classes at each user has multiple spreading codes of different traffic classes have different spreading gains and admission control, by treating the general case where determination of the system capacity region for connection is presented in Section 4. This analysis enables the multiclass terms of bit error specifications for outage probability in space. ()

### References


