DEVELOPING A GEOMETRIC DEFORMABLE MODEL FOR RADAR SHAPE INVERSION

Alper Yildirim, Anthony Yezzi

Georgia Institute of Technology
School of Electrical and Computer Engineering
Atlanta, GA

ABSTRACT
In this paper, we develop a radar-based dense scene reconstruction model that extracts shape information embedded in the radar return signal. Our method uses a deformable shape evolution approach which seeks to match the received signal to a computed forward model based on the evolving shape. This allows us to directly incorporate geometric considerations of the shape into the problem formulation, such as smoothness and self-occlusions. Iterations start with an initial shape which is gradually deformed until its image under the forward model gets sufficiently close to the actual measured signal. For this purpose, we employ the technique of stretch processing to extract geometric properties of the shape from radar return signal. This yields a smooth and purely geometric cost functional by which shape inversion can be robustly performed via gradient-based minimization algorithms. Synthetic simulations with a polygonal shape model show the promise of this type of approach on some challenging shapes.

Index Terms—Shape inversion, Radar imaging

1. INTRODUCTION
Vision systems are becoming more and more essential in robotic systems because of the rich information they can provide about their environment. Especially for robots that are to navigate in cluttered environments, awareness of the scene structure is of great importance as it is usually the main limiting factor on robot motion. Inferring such structure using visual cues from camera images is a natural approach which mimics the way we sense the world with our eyes. This is an old and well established area of research in computer vision known as structure from motion or multiview stereo reconstruction, including methods based on deformable shape models [1][2]. However, stereo vision systems have inherent fragility in the case of low ambient light or in the presence of obstructing factors for the visible light spectrum, such as rainy and foggy weather or smoke. As a result, alternative sensing modalities can often be required for the applications where these conditions are present.

Radar systems are immune to many factors which challenge visual sensors as they have good penetration capabilities through certain mediums, air, water etc. [3] These systems are essential especially for airborne and spaceborne imaging applications where light rays can easily be blocked by the clouds or the thick layer of air between the antenna and ground scene. Radar imaging is mostly performed with an apparatus known as Synthetic aperture radar (SAR)[4][5][6][7]. A SAR system is usually composed of a small antenna or antennas attached to a moving platform which takes measurements of the scene from different viewpoints. These measurements are then used to synthesize a high resolution image of the scene. The scene is modelled as an array of scatterers where a reflectivity (intensity) value is assigned to each scatterer as a result of the synthesis.

SAR imaging does not presently have an explicit notion of shape that can easily be leveraged to exploit geometric prior information about object shape. For example, we know that the surfaces of scene objects usually exhibit some level of smoothness which could potentially be used to regularize the problem. Another important aspect of object geometry is self-occlusion where certain parts of the object can block the view of other parts. Occlusion modelling is especially critical for close range applications where the visible parts of the object can drastically change with respect to the view-point. Modelling such considerations can be of great help to enhance the quality of estimation.

In this paper, we propose a generative model based evolution approach for radar by which we can directly incorporate object geometry into the problem formulation. In Section 2, we formulate the forward model to be used to estimate the received signal given the shape. Second, we outline an iterative inversion scheme based on a deformable shape model. In section 3, simulations for different cases are presented. In section 4, results are discussed.

2. DEFORMABLE SHAPE MODEL
Reflected radar signal is a highly non-linear function of shape where the inversion problem can only be attacked by an iterative approach. We first need a forward model to compute the expected return signal given a candidate shape for the object. We then measure the discrepancy between this computed signal and actual measured signal and use the residual error value to update our guess for the object so that we have a decreased
2.1. Forward Model

Our forward model computes what is expected from the receiver antenna given a transmitted waveform and an object shape (and reflectivity). On the antenna side, we model the transmitter and receiver as point antennas with directional gains. For the object, we assume Lambertian surface reflectivity where incoming radiation is scattered via a cosine power law. From a small surface patch of area \(dS\), the received signal is modelled as:

\[
dQ_c(t) = \frac{G'G (\mathbf{u}' \cdot \mathbf{n}) (\mathbf{u} \cdot \mathbf{n})}{R'^2R^2} f \left( t - \frac{R + R'}{c} \right) dS \quad (1)
\]

where \(G'\) and \(G\) are antenna gain values for the transmitter and receiver in a given direction, \(\mathbf{u}'\) and \(\mathbf{u}\) are the unit ray directions connecting transmitter and receiver antennas to the object, \(R'\) and \(R\) are the lengths of these rays and \(\mathbf{n}\) is the unit normal vector of the object at the given point, respectively. \(f(t)\) is the transmitted waveform which is mostly a frequency-modulated complex exponential. Squared ray lengths in the denominator model the decay of power density with wave propagation (inverse-square law). Our forward model is depicted in the Fig. 1.

For a given antenna position, all of the visible points on the surface will contribute to this measurement which makes the total measured signal:

\[
Q_c(t) = \int_S \frac{G'G (\mathbf{u}' \cdot \mathbf{n}) (\mathbf{u} \cdot \mathbf{n})}{R'^2R^2} f \left( t - \frac{R + R'}{c} \right) dS \quad (2)
\]

where \(S\) is the set of visible points visible to both transmitted and receiver antenna.

2.2. Inverse Model

Our purpose is to estimate the surface shape from the measured signal, which requires inverting the forward model. However, this model is highly nonlinear and inverting it is an inherently ill-posed operation. That’s why we start with an initial shape and let this shape evolve such that its image under the forward model gets closer to the actual measured signal, balanced by additional geometric priors under a designed cost functional, with successive iterations.

Design of the cost functional to be used in an iterative minimization procedure is tricky for radar applications, especially when high frequency waveforms are used. We need a cost functional that is not only rich in shape information but at the same time independent as possible from the structure of the waveform being used. Naive choices for this design can bring the oscillatory structure of the waveform into the cost functional. This causes the energy manifold to be full of local minima, making robust shape inversion impossible or impractical. Accordingly, the design of the cost functional is the key component of our scheme. For this purpose we employ the technique known as stretch processing [8].

2.2.1. Stretch Processing

Stretch processing is a commonly used technique in the radar community to process large bandwidth signals using low sampling rates. It is a requirement for some applications as signals of large bandwidths cannot be processed directly on hardware due to the sampling rate limitations of A/D converters [9][10][11]. This is done by mixing the return signal with a time shifted replica of the transmitted signal, which results in a lower frequency signal at the mixer output that can then be sampled properly. However, a more important aspect of this process for us is that each frequency component residing in the output signal can directly be linked to a subset of surface points all of which have equal round-trip distance values. This property of stretch processing will be highly beneficial for our purposes as this indirectly gives us distribution of signal strength along the object range. For a point on the object surface, we express the received signal in Eq. 1. Choosing \(f(t)\) as an LFM pulse and transforming this signal using a stretch processor, at the mixer output we obtain:

\[
dh(t) = \frac{G'G (\mathbf{u}' \cdot \mathbf{n}) (\mathbf{u} \cdot \mathbf{n})}{R'^2R^2} \Phi (t_r) e^{i2\pi(f_c(t_r - t_h))} dS \quad (3)
\]

\[
\Phi (t_r) = e^{i2\pi(f_c(t_r - t_h) - \alpha(t_r^2 - t_h^2))} \quad (4)
\]

\[
t_r = \frac{(R' + R)}{c} \quad (5)
\]

where \(\alpha\) is the chirp rate, \(f_c\) is the carrier frequency, \(t_r\) and \(t_h\) are the delays of received and replica signals with respect to.
the transmitted signal, respectively. Since the output signal of
the stretch processor will be the linear combination of these
infinitesimal components coming from visible points along
the object surface, the total output for the object becomes:
\[ h(t) = \int_S \frac{G' G (u' \cdot n) (u \cdot n)}{R^2 R'^2} \Phi(t_r) e^{i \omega (t_r - t_h) t} \, dS \]  
(6)
where \( S \) denotes the visible part of the object surface with re-
spect to the transmitter and receiver. It should be noted that
\( \Phi(t_r) \) is a pure phase term that does not have any time depen-
dency.

2.2.2. Feature Extraction
The stretch processor output gives us another time dependent
oscillatory signal with respect to both \( t \) (time) and \( t_r \) (round-
trip delay). Our purpose is to remove these dependencies such
that we can formulate our cost functional in terms of purely
geometric quantities. For this purpose, we will find it useful
to express Eq. 6 in terms of another integral with a frequency
measure. Assuming an infinitely long transmitted pulse, we
would have a finite support frequency spectrum where mini-
mum (\( f_{\text{min}} \)) and maximum (\( f_{\text{max}} \)) frequencies are specified
by minimum and maximum round-trip distance values of the
visible portion of the object surface. \( h(t) \) could then be ex-
pressed as a Fourier synthesis:
\[ h(t) = \int_{f_{\text{min}}}^{f_{\text{max}}} H(\zeta) e^{i 2 \pi \zeta t} \, d\zeta \]  
(7)
At this point we define a new function \( H(x) \) which will hap-
pen to be the envelope of the frequency spectrum. \( H(x) \) is
defined such that \( H(x) \) can be decomposed as:
\[ H(x) = H(x) \Phi(t_r) \]  
(8)
\[ t_r = \frac{x}{2 c_t} + t_h \]  
(9)
Integrands in Eq. 6 and 7 can be related to each other after
cancellation which then becomes:
\[ H(\zeta) = \int_{S_{\zeta}} \frac{G' G (u' \cdot n) (u \cdot n)}{R^2 R'^2} \left\| \frac{dS}{d\zeta} \right\| \, ds \]  
(10)
where the \( S_\zeta \) is the set of iso-round-trip distant points that
induce a signal with a constant frequency of \( \zeta \) at the mixer
output.
\( H(x) \) is almost always a smoothly changing function of
object shape when certain level of regularity is assumed for
the object surface. That's why we choose to define our cost
functional in terms of this expression. It should be noted that
this expression is almost never a physically measurable quan-
tity (as the set \( S_\zeta \) is most of the time composed of finite num-er of curves lying on the object surface) so we compute its
integral over a frequency range and then use the average value
of the integral as our feature. We partition our frequency
spectrum \([f_{\text{min}}, f_{\text{max}}]\) into frequency bins and integrate the
expression in Eq.10 over each frequency interval.
\[ H_j = \frac{1}{\Delta f_j} \int_{f_j-\Delta f_j}^{f_j} \int_{S_\zeta} \frac{G' G (u' \cdot n) (u \cdot n)}{R^2 R'^2} \left\| \frac{dS}{d\zeta} \right\| \, dS \, d\zeta \]  
(11)
\[ \Delta f_j = f_j - f_{j-1} \]  
(12)
where superscript \( j \) denotes frequency bin index. It should
be emphasized that feature \( H_j \) does not have any waveform
dependency as we desire.

2.3. Inversion Scheme
We will perform shape inversion mainly by using the set of
geometric features we extract from the signal. There will be
two sets of these, one extracted from the actual return sig-
nal (\( H_j \)) and the other one from the evolving object (\( \hat{H}_j \)).
For the following discussion, we will use a tailored version of
our formulation to a 2D case with a discrete polygonal shape
representation since our simulation results will be based on
this formulation. We will model our scene as a polygonal
shape which is parametrized by the set of the vertex coordi-
nates (\( (v_k) \)). Object surface is then composed of line segments
connecting these vertices together. A given line segment can
have intersection with more than one frequency bin in which
case we slice it into pieces such that each piece is contained in
one frequency bin. As a result, a given frequency bin can be
contributed by portions of different line segments. For \( n \)
vertices, a given feature \( \hat{H}_j \) is computed as:
\[ \hat{H}_j = \sum_{k=1}^{n} H_j^k \]  
(13)
where \( H_j^k \) denotes the contribution to the \( \hat{H}_j \) brought by the
line segment that connects \( v_k \) to \( v_{k+1} \) (takes zero value if not
intersecting).

2.3.1. Cost Functional
Our cost functional depends mainly on these two sets of fea-
tures. We also add a curvature based regularizer penalty to
make the evolving shape favor a certain level of smoothness.
The ability to directly regularize the shape estimate in this
manner is a powerful advantage of this approach. Combining
these two terms yields:
\[ E(v_1, \cdots, v_n) = \sum_{j=1}^{N_F} \left( \hat{H}_j - H_j \right)^2 + \lambda \sum_{k=1}^{n} \kappa_k^2 \]  
(14)
where \( N_F \) is the number of frequency bins. In the case of
multiple antenna sets, residual term is additionally summed
over all antenna sets. For the regularizer term, \( \lambda \) is a tun-
able regularization coefficient and \( \kappa_k \) is the discrete curvature
value computed around the \( k^{th} \) vertex. We define \( \kappa_k \) as:
\[ \kappa_k = \left\| v_{k+1} - v_k \right\| - \left\| v_k - v_{k-1} \right\| \]  
(15)
where \( v_{k-1}, v_k, v_{k+1} \) are the 2D coordinates of consecutive
vertices of our polygonal shape in counter-clockwise order.

2.3.2. Initialization
Using an iteration based approach brings the question of how
to choose an initial parameter set. Luckily our design of the
feature set gives an easy way to come up with a close initialization. The feature set extracted from the actual measured signal naturally reveals which frequency bins have an intersection with the object where a nonzero feature value implies intersection. Triangulation of this information coming from other antennas give us a good estimate for actual object shape and placement.

2.3.3. Minimization
We use Nesterov’s accelerated gradient descent algorithm [12][13][14] minimize our cost functional over the vertex coordinates \((v_1, \cdots, v_n)\). This yields a faster convergence rate and increases robustness against shallow local minima. At each iteration, we perform a visibility analysis on the evolving shape which defines our domain of integration (different for each antenna).

3. SIMULATION RESULTS
We apply our inversion model to three progressively challenging 2D shapes that are modelled as polygonal objects. For each polygonal shape, we fix the angular positions of the vertices and let the shape evolve by changing their radii. Multiple antennas are placed around the object as a circular pattern with equal angular intervals between consecutive antennas.

Simulations are run for three different cases with different initial and actual shapes. For each simulation, we use 20 antennas that are circularly placed around the object where origin-antenna distance is taken as 6 meters. For each shape, we use a polygonal model with 100 vertices.

4. CONCLUSIONS
We propose a new model for radar based shape inversion that is using a forward model based approach since such an approach allows us to introduce geometric properties of the shape into the problem formulation. However, such an approach can be tricky when cost functional is naively chosen in terms of radar signals as these oscillations can be easily transferred to the cost functional which would make the iterative approach impractical. Thus we propose new way to design features for radar based inversion that are purely geometric and independent of the waveform. Because of the easiness in the implementation and visualization, we choose to tailor our approach to 2D case. Simulation results are presented for different cases where even for large initial parameter error values, we see our method can effectively recover the shape of the scene.

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5. REFERENCES


