ASYNCHRONOUS BLIND NETWORK DIVISION MULTIPLE ACCESS

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Abstract—We present a blind collision resolution algorithm in slow fading channels based on retransmission diversity. The algorithm neither assumes packet nor symbol synchronization of the different users and it does not demand estimates of the arrival times of the colliding signals. The proposed scheme works independently of the relative alignment of the packets, and so it can also resolve synchronous collisions. The decoding complexity does not scale with the packet size and thus does not burden the receiver. Having forgone synchronization, the penalty paid is a longer queueing delay of data at the transmitters. Still the algorithm achieves high throughputs similar to synchronous network division multiple access (NDMA) protocols.

Index —blind collision resolution, asynchronous, random access, NDMA, packet-switched

I. INTRODUCTION

The maximum achievable throughput in wireless networks is reduced by interference. Since the wireless medium is shared, concurrent utilization of the same network resources lead to packet collisions and the packets are typically discarded. Modular designs in the medium access control (MAC) and physical (PHY) layers approach the collision problem from different perspectives. For example, transmissions could be based on fixed allocation of communication resources in order to avoid collisions as in TDMA, and they could be contention-based as in ALOHA and rely on the MAC layer functionality to retransmit the colliding packets [1]. The former schemes are poor in bursty data networks while the latter schemes suffer under heavy network load. PHY approaches like CDMA spreading [2] and interference alignment [3] [4] rely on signal processing to separate colliding signals but do not exploit the MAC capabilities.

Cross-layer designs for collision resolution consider both the randomness of data arrival at the transmitters and the multi-reception capability enabled by signal processing as an attempt to improve the system performance [5]. For example, [6] jointly optimizes the MAC and PHY layer by combining interference alignment for transmit beamforming with opportunistic packet transmission for interference management but assumes all nodes have multiple antennas. Two MAC protocols are advised in [7] and [8] for successive interference cancellation (SIC). The random access protocol in [7] is only applied to establish a connection between the nodes and the base station using power domain multiplexing of preamble transmissions. In [8] messages are exchanged before data transmission in order to determine if the receiver can support additional interference from currently inactive nodes and still decode the desired signal using SIC.

We present a collision-resolution algorithm in the many-to-one communication scenario via retransmission diversity. This is first explored in [9]. In the case of \( K \) colliding signals, the receiver stores the collided data and requests \( K - 1 \) retransmissions from the colliding transmitters. The \( K \) mixtures are then used to resolve the individual packets. Orthogonal codes are used to detect \( K \) from the first collision, which renders the resolution algorithm highly sensitive to any lack of synchronization. In [10] and [11], \( K \) is detected using rank tests and collisions are blindly resolved using parallel factor analysis and independent component analysis respectively. In both works, all new packet transmissions have to be stopped until the collision is resolved. [12] extends [10] andallows the transmitters that are not involved in the collision to contact the same receiver before the collision is resolved. The mentioned schemes [9–12] assume that all network users are perfectly synchronized with the slot timing, which is challenging in general and particularly difficult to implement in large networks [13].

In this paper we neither assume packet nor symbol synchronization. Transmitters follow the same transmission scheme as in [12]. Colliding signals may be arbitrarily aligned but they can still be blindly separated by the receiver. The receiver does not need to know nor estimate the arrival times of desired signals on contrary to [13], and this does not computationally burden the receiver. All nodes including the receiver have single antennas as opposed to [13] and [14]. Moreover, we do not rely on particular patterns of arrival of the colliding packets for successful collision resolution. For instance, the colliding packets may be perfectly aligned and still could be separated. This is distinct from zigzag decoding [15] and variants as employed in [16] and [17]. The latter two works are also only applicable in wireless networks that can afford augmenting each transmitted packet with its replica.

Section II presents the system model. The collision resolution algorithm is described in Section III. In Section IV we show numerical results on throughput analysis of the algorithm. Section V concludes the paper.

II. SYSTEM MODEL

Consider a set of \( K \) transmitters and a single receiver in a single-carrier system. A subset of \( K \) transmitters, \( K \leq \tilde{K} \), may contact the receiver during the same time, on the same frequency and with no use of orthogonal codes. Moreover, all nodes have single antennas. Still, the receiver manages to listen to each of the \( K \) active transmitters by leveraging the diversity created by the transmission scheme. The receiver solves this communication problem in three stages. First, it detects the number of active transmitters \( K \). Second, it identifies which \( K \)-subset of the \( \tilde{K} \) transmitters is currently the active set of transmitters. Third, it decodes the signal of each of the \( K \) transmitters. Although the receiver has to identify the \( K \) active transmitters, we assume the receiver knows the population of \( K \) transmitters beforehand. In particular, each transmitter \( k \) of the \( \tilde{K} \) transmitters is assigned a unique complex exponential \( r_k = e^{j \angle r_k} \) lying on the unit circle, \( 0 \leq \angle r_k < \pi \), and the receiver is aware of this.
We consider packet-switched networks. Active transmitter \( k \) wants to send packet \( \vec{s}_k \) to the receiver. A packet has \( P' \) symbols. After each packet transmission, a transmitter waits for \( G \) symbol durations for an acknowledgement from the receiver. We often abstract the packet as having \( P = P' + G \) symbols of which \( G \) symbols are zeros. A symbol may be real or complex and its duration is \( \tau \). A packet (including the guard interval) occupies one slot duration, so 1 slot = \( P\tau \).

We assume there is a reference clock at the receiver that indicates the start of a time slot. The transmitters are not necessarily synchronized to the receiver, so packet \( \vec{s}_k \) of transmitter \( k \) may not be totally received within a single time slot but might partially overlap in time with two consecutive slots. It is thus unnecessary to define slot boundaries at the receiver. We only do so for two reasons. First, we derive the collision resolution algorithm for asynchronous transmissions based on the solution in [12] for synchronous collisions, so slotted time is assumed for analytical convenience. Second, the slotted time formulation proves that the proposed algorithm in this paper resolves the synchronous collisions as a special case. The algorithm thus also applies in a hybrid network in which only a subset of the transmitters are synchronized to the receiver such as those in its proximity. Without loss of generality, we assume the first packet is always received at \( t = 0 \). The receiver identifies whether \( K = 1 \) by simply checking the cyclic redundancy check (CRC) bits of the collected packet. In the case \( K > 1 \), the receiver does not know the arrival times of the individual colliding packets. It could happen that the receiver may not have decoded these packets yet and then another transmitter sends a packet. In this case \( K \) refers to the total number of active transmitters at the instant of successful decoding. \( N \) refers to the collision resolution interval measured in packet durations (slots). Since data availability at the transmitters is random, so are \( K \) and \( N \). The channels between the transmitters and the receiver are slow fading with respect to \( N \), and there is additive complex Gaussian noise \( CN(0,\sigma^2I) \) of mean 0 and covariance \( \sigma^2I \) at the receiver.

All vectors \( \vec{v} \) are column vectors and have arrow symbols on top. The transpose and conjugate transpose of \( \vec{v} \) are \( \vec{v}^T \) and \( \vec{v}^H \) respectively. Similar notation holds for matrices. The \( l \)th element of \( \vec{v} \) is \( \vec{v}[l] \).

III. ASYNCHRONOUS TRANSMISSIONS

Transmitters that have data to send access the channel without waiting for an idle channel state. The transmitters are not necessarily synchronized to the receiver, so in general packet \( \vec{s}_k \) of transmitter \( k \) is first received at \( t_k = (n_k - 1)P + p_k)\tau \), i.e. in time slot \( n_k \) after \( p_k \) symbol durations relative to the slot start time, \( 1 \leq n_k \leq N, 0 \leq p_k < P \). Shift \( p_k \) should be a decimal. However, we assume \( p_k \in \{0,1,\ldots,P - 1\} \) since the decimal case can simply be modeled as a phase shift that is subsumed under channel effects. Resolving synchronized collisions \( (n_k = 1, p_k = 0)_k \) is presented in [10], [12]. The algorithm is extended to the case of synchronized transmissions \( (n_k \leq N, p_k = 0)_k \) in [12] at the cost of increased decoding complexity of the order of \( K \). In both settings all transmitters are synchronized to the receiver. We now consider resolving collisions in the general case \( (n_k \leq N, p_k < P)_k \). We emphasize that the decoding complexity does not scale with the packet size \( P \). Otherwise it becomes prohibitive since \( P \) could be orders of magnitude larger than the number of colliding packets \( K \) or decoding time \( N \).

A. Transmission scheme

All transmitters follow the same transmission scheme as in [12]. This is important so that a decoding scheme that blindly resolves asynchronous collisions perfectly applies to the synchronous case as a special case. Therefore, an active transmitter \( k \) sends packet \( \vec{s}_k \) of length \( P \) (includes the guard interval). In case of a collision, transmitter \( k \) sends \( r_k \times \vec{s}_k \), then \( r_k^2 \times \vec{s}_k \), and so on. This persists until the receiver manages to decode the colliding packets.

![Fig. 1: Transmission scheme of \( K = 4 \) packets](image)

Figure 1 illustrates an example scenario of \( K = 4 \) colliding signals which we will use to build a decoding algorithm for a general collision setting. Assume for now there is no fading. In Figure 1a, packet \( \vec{s}_1 \) arrives at the receiver at \( t = 0 \), which is the start of the first time slot. Unfortunately, packets \( \vec{s}_3 \) and \( \vec{s}_4 \) arrive within the first slot at \( t = 2\tau \) and \( t = (P - 3)\tau \) respectively and collide with \( \vec{s}_1 \). The CRC is corrupted and the receiver awaits new packet arrivals. At \( t = P\tau \) the second packet \( r_1 \vec{s}_1 \) of transmitter 1 and the first packet \( r_2 \vec{s}_2 \) of transmitter 2 are received. Upon their second transmission, packet \( r_3 \vec{s}_3 \) of transmitter 3 and packet \( r_4 \vec{s}_4 \) of transmitter 4 are received at \( t = (P + 2)\tau \) and \( t = (2P - 3)\tau \) respectively. At \( t = 2P\tau \), packets \( r_1^2 \vec{s}_1 \) and \( r_2 \vec{s}_2 \) are received, and so on. Clearly, all four transmitters follow the same transmission scheme.

B. Expressions of collected packets

Denote by \( \vec{y}_n \) the overall received packet within time slot \( n \). In the example of Figure 1a, whole packet \( \vec{s}_1 \), the first \( P - 2 \) symbols of packet \( \vec{s}_3 \) and the first three symbols of packet \( \vec{s}_4 \) contribute to \( \vec{y}_1 \). Six signal components (beyond noise) contribute to \( \vec{y}_2 \):

- whole packets \( r_1 \vec{s}_1 \) and \( \vec{s}_2 \)
- the last two symbols of packet \( \vec{s}_3 \) and the last \( P - 3 \) symbols of packet \( \vec{s}_4 \)
- the first \( P - 2 \) symbols of packet \( r_3 \vec{s}_3 \) and the first three symbols of packet \( r_4 \vec{s}_4 \)

Suppose in the example of Figure 1a the guard interval is \( G = 5 \) symbol durations and \( G < P - 3 \). In this case the last two
symbols of packet $\overrightarrow{y}_3$ are zero and may be disregarded. Thus
five signal components contribute to $\overrightarrow{y}_2$. In a similar manner,
five signal components contribute to $\overrightarrow{y}_3$; $r_1^2 \overrightarrow{y}_1$, $r_2^2 \overrightarrow{y}_2$, last $P - 3$ symbols of $r_3^2 \overrightarrow{y}_3$, first $P - 2$ symbols of $r_4^2 \overrightarrow{y}_4$ and first three symbols of $r_4^2 \overrightarrow{y}_4$. This continues to be true for all received packets $\overrightarrow{y}_n$, $n > 1$. For convenience, we introduce a new piece of notation. For an arbitrary vector $\overrightarrow{v}$ of length $L$, define

$$
\overrightarrow{v}^{(d)} = \begin{cases} 
\text{d zeros} & \text{if } d = 0 \\
0, \ldots, 0, \overrightarrow{v}[1], \ldots, \overrightarrow{v}[L-d] & \text{if } 1 \leq d \leq L-1 \\
\overrightarrow{v}[1-d], \ldots, \overrightarrow{v}[L] , 0 , \ldots, 0 & \text{if } -d \leq d \leq -1 \\
\emptyset & \text{if } |d| > L-1 
\end{cases}
$$

(1)

This is easily illustrated via an example. For instance, if $\overrightarrow{v} = (a, b, c, d, e)^T$ then $\overrightarrow{v}^{(-3)} = (d, c, b, a, 0)^T$, $\overrightarrow{v}^{(-3)} = (0, 0, 0, 0, 0)^T$. $\overrightarrow{v}^{(-6)}$ is an empty vector of dimension zero.

Following the discussion above on the signal contribution to collected packets $\overrightarrow{y}_n$ and using the notation in (1) we have

- $\overrightarrow{y}_1 = \overrightarrow{y}_1^{(0)} + \overrightarrow{y}_1^{(2)} + \overrightarrow{y}_1^{(P-3)} + N_{P,1}$
- $\overrightarrow{y}_2 = r_1 \overrightarrow{y}_1^{(0)} + r_2 \overrightarrow{y}_2^{(0)} + r_3 \overrightarrow{y}_3^{(2)} + r_4 \overrightarrow{y}_4^{(P-3)} + \overrightarrow{y}_4^{(-3)} + N_{P,1}$
- $\overrightarrow{y}_3 = r_1^2 \overrightarrow{y}_1^{(0)} + r_2^2 \overrightarrow{y}_2^{(0)} + r_3^2 \overrightarrow{y}_3^{(2)} + r_4^2 \overrightarrow{y}_4^{(P-3)} + \overrightarrow{y}_4^{(-3)} + N_{P,1}$

and so on. Suppose the receiver collects $N = 6$ packets $\{\overrightarrow{y}_n\}_{n=1}^6$. Stacking them in a matrix and using their expressions above we obtain

$$
\begin{pmatrix}
\overrightarrow{y}_1 \\
\overrightarrow{y}_2 \\
\vdots \\
\overrightarrow{y}_6
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & 1 & 1 & 0 \\
r_1 & 1 & r_3 & r_4 & 1 \\
r_1^2 & r_2 & r_3^2 & r_4^2 & r_4 \\
r_1^3 & r_2^2 & r_3^3 & r_4^3 & r_4^2 \\
r_1^4 & r_2^3 & r_3^4 & r_4^4 & r_4^3 \\
r_1^5 & r_2^4 & r_3^5 & r_4^5 & r_4^4
\end{pmatrix}
\times
\begin{pmatrix}
\overrightarrow{y}_1^{(0)} \\
\overrightarrow{y}_1^{(2)} \\
\overrightarrow{y}_1^{(P-3)} \\
\overrightarrow{y}_1^{(-3)} \\
\overrightarrow{y}_1^{(-3)} \\
\overrightarrow{y}_1^{(-3)}
\end{pmatrix}
+ N_{6,P}
$$

(2)

\[ Y_6 = W_6 \times S + N_{6,P} \]

C. General expression of received matrix of packets $Y_n$

Recall that packet $\overrightarrow{s}_k$ of active transmitter $k$ arrives at the receiver at $t_k = ((n_k - 1)P + p_k)\tau$. In the example of Figure 1a, transmitters 1, 2, 3 and 4 form the active set of transmitters, where $p_1 = p_2 = 0$, $p_3 = 2$ and $p_4 = 3$. The expression of the received matrix of packets in (2) suggests an equivalent collision scenario illustrated in Figure 1b. Transmitters 1a, 2a, 3a, 4a and 4b want to send packets $\overrightarrow{s}_1^{(0)}$, $\overrightarrow{s}_2^{(0)}$, $\overrightarrow{s}_3^{(0)}$, $\overrightarrow{s}_4^{(0)}$, respectively to the receiver. All five transmitters are synchronized to the start of a time slot: $p_{1a} = p_{2a} = p_{3a} = p_{4a} = p_{4b} = 0$.

We point out the following pattern: each of transmitters 1, 2 and 3 in Figure 1a has $p_k \leq G$. These are replaced by single transmitters in the equivalent set of Figure 1b and each occupies one column of matrix $W_6$ and one row of matrix $S$ in (2). Transmitter 4 in Figure 1a has $p_k > G$ and is replaced by two transmitters in Figure 1b. Thus it occupies two columns of $W_6$ and two rows of $S$ in (2). Given this observation, we derive a general expression for the received matrix of packets $Y_n$. Define the $n$-extension of the coding vector of transmitter $k$ as

$$
\overrightarrow{w}_{k,n} = \begin{pmatrix} 1 & r_k & r_k^2 & \ldots & r_k^{n-1} \end{pmatrix}^T
$$

(3)

The arrival time of packet $\overrightarrow{w}_{k,n}$ of transmitter $k$ is $t_k = ((n_k - 1)P + p_k)\tau$. Using (3) and the notation in (1), transmitter $k$’s contribution to the coding matrix $W_n$ in the expression of $Y_n$ is given by

$$
\overrightarrow{w}_{k,n} = \begin{pmatrix} w_{k,n}^{(n_k-1)} \\
w_{k,n}^{(n_k-1)} \\
w_{k,n}^{(n_k-1)} \\
w_{k,n}^{(n_k-1)} \\
w_{k,n}^{(n_k-1)} \\
w_{k,n}^{(n_k-1)} \end{pmatrix}
$$

(4)

D. Decoding of matrix of packets $S$

(6) shows that the received matrix of packets $Y_n$ can be expressed as a coding matrix $W_n$ times a matrix of packets $S$ plus a noise matrix, where the columns of $W_n$ are shifted versions of the coding vectors $\overrightarrow{w}_{k,n}$ in (3). This is the same format as the expression of $Y_n$ in [12] for the case of synchronized transmissions $(n_k \leq N, p_k = 0)_k$. The receiver thus may apply the same decoding algorithm as in [12] to resolve asynchronous collisions except for two main differences:

- In [12], the receiver solves a system of $N - 1$ equations to find roots $\{r_k\}$ that identify the active transmitters. For the case of asynchronous transmissions, characteristic complex exponential $r_k$ of active transmitter $k$ will be a duplicate root of two consecutive equations in the system solved by the receiver whenever $p_k > G$. This is because in this case transmitter $k$ occupies two columns of $W_n$ as in (4). Upon identifying duplicate roots the receiver modifies the reconstruction of $W_n$ accordingly during decoding.

- Upon successful decoding, the receiver recovers matrix of packets $S$ in (6). As opposed to synchronous collisions
in [12], $S$ does not necessarily hold original packets \( \{s_k\}_k \) but possibly shifted versions as in (5). However, for a particular packet \( s_k \) there are two cases: a) if \( p_k \leq G \) then there is a single row of $S$ that holds all the elements of \( s_k \) possibly shifted to the right. b) if \( p_k > G \) there is one row of $S$ that holds the first $P - p_k$ elements of \( s_k \) and starts with $p_k$ zeros, while another row of $S$ holds the last $p_k$ elements of \( s_k \) in addition to $P - p_k$ trailing zeros. In both cases original packet \( s_k \) can still be recovered.

There are few additional technical details for the blind signal separation in (6) that arise, either due to the fact that more than one column of the $W_n$ correspond to the same duplicate root \( r_k \), or due to the loss of the full rank property of $S$ for particular sets of shifts \( \{p_k\}_k \). We do not deal with such complexities for the sake of brevity.

IV. THROUGHPUT ANALYSIS

Consider a network of $K \geq 8$ transmitters and one receiver. Suppose data arrives at each transmitter according to a Poisson distribution of mean $\lambda$. At a given instant of time the number of active transmitters is random. In case of collisions, the receiver builds matrix $Y_N$ for $N$ large enough in order to separate the colliding signals. Assume the SNR is high enough so that one extra packet stacked in $Y_N$ after noiseless rank saturation suffices to identify the active transmitters and successfully resolve the collisions. We consider three scenarios:

1) All transmitters are synchronized to the receiver.
2) None is synchronized to the receiver.
3) Only the closest four transmitters are synchronized to the receiver.

In Figure 2 we inspect the network throughput (defined as the ratio of the total number of successfully transmitted packets within the network to the total simulated time) versus $\lambda$ for the three scenarios. In Figure 3, we inspect the average delay experienced by a packet inside a single transmitter’s buffer versus $\lambda$. In the third scenario we consider a far transmitter (unsynchronized). We vary $\lambda$ per transmitter up to $1/(K+1)$. The queues have infinite memory and are simulated till steady state. We assume the guard interval $G$ is $10\%$ of $P$. In the case of asynchronous transmissions and upon resolving a collision, some transmitters might still be amid a retransmission. We assume $G$ is large enough to acknowledge the transmitters and clear the medium for new transmissions.

![Fig. 2: Network throughput versus data arrival rate $\lambda$ for a population of $K = 8$ transmitters.](image)

![Fig. 3: Average queue delay versus data arrival rate $\lambda$ for a population of $K = 8$ transmitters.](image)

Figure 2 shows that the network throughput increases with the data arrival rate $\lambda$ at the transmitters. Notice that the network throughput for all three scenarios is the same. This is counterintuitive since the decoding time for asynchronous transmissions in case of collisions is in general longer than that of synchronous transmissions. However, this can be explained by the fact that the network throughput is function of the expected number of attempted transmissions. This does not only depend on the data arrival rate at the buffers of the transmitters but also the rate of accumulation of packets inside these buffers. For a given rate $\lambda$, synchronous collision resolution intervals are shorter, so the accumulation of packets inside the buffers of unsynchronized transmitters is faster. Thus it becomes more frequent in the asynchronous case to have a large number of transmitters with non-empty buffers directly after a collision resolution interval, in which case the packet alignments \( \{p_k\}_k \) will be in general less than $G$ in the next resolution interval while $K$ is large. This boosts the throughput of the asynchronous scheme and balances the overall number of successful transmissions over the simulation time among the three scenarios.

That said, Figure 3 shows that the average delay experienced by a packet inside a transmitter’s buffer before transmission is longer in the asynchronous case compared to synchronized transmissions. The queueing delay increases with $\lambda$. For a maximum decoding time of 20 slots, $\lambda(K+1) \leq 0.8$ for the asynchronous case as opposed to $\lambda(K+1) \leq 0.95$ if the transmitters are synchronized to the receiver. Since synchronization is difficult to implement in large networks, an intermediate approach is to synchronize only those transmitters close to the receiver. This reduces the queueing delay even for a far transmitter without sacrificing the overall throughput.

V. CONCLUSION

We presented a blind collision resolution algorithm based on temporal diversity for slow fading channels. The method supports immediate transmissions but perfectly applies to synchronous networks similar to slot-synchronized NDMA protocols. The decoding complexity solely depends on the number of colliding packets. Simulation results show high throughputs but an increased queueing delay of data at the transmitters. The network is still stable for relatively high data rates. In future work we carry out analytical throughput analysis of the proposed scheme and refine the algorithm to reduce the decoding time.
REFERENCES