Channel gain cartography relies on sensor measurements to construct maps providing the attenuation profile between arbitrary transmitter-receiver locations. Existing approaches capitalize on tomographic models, where shadowing is the weighted integral of a spatial loss field (SLF) depending on the propagation environment. Currently, the SLF is learned via regularization methods tailored to the propagation environment. However, the effectiveness of existing approaches remains unclear especially when the propagation environment involves heterogeneous characteristics. To cope with this, the present work considers a piecewise homogeneous SLF with a hidden Markov random field (MRF) model under the Bayesian framework. Efficient field estimators are obtained by using samples from Markov chain Monte Carlo (MCMC). Furthermore, an uncertainty sampling algorithm is developed to adaptively collect measurements. Real data tests demonstrate the capabilities of the novel approach.

Index Terms—channel gain cartography, radio tomography, Markov chain Monte Carlo, active learning

1. INTRODUCTION

Based on the measurements collected by a network of spatially distributed sensors, channel gain cartography constructs maps providing channel-state information for links between locations where no sensors are present [1]. Such maps can be employed in cognitive radio setups to control the interference that the secondary network inflicts to primary users that do not transmit – setup encountered with television broadcasting systems [2, 3, 4, 5]. The non-collaborative nature of these primary users precludes any direct form of channel estimation between secondary transmitters and primary receivers.

Existing methods for channel gain cartography build upon the intuitive principle that spatially close radio links exhibit similar shadowing [6]. Most of these methods adopt a tomographic approach [7], where shadowing attenuation is modeled as the weighted integral of an unknown spatial loss field (SLF) capturing the absorption induced by objects located in the propagation medium [7, 8, 9, 10, 11]. The weights in the integral are determined by a function depending on the transmitter-receiver locations that is either selected based on heuristic criteria [7, 12], or blindly learned via the non-parametric kernel regression method [13]. A channel map can thus be obtained once the SLF has been estimated.

Conventionally, the SLF is learned via regularized least-squares (LS) methods tailored to the propagation environment [11, 12, 14]. However, those approaches are less effective when the propagation environment exhibits heterogeneous characteristics. Different from past works, the present one leverages the Bayesian framework to learn the piecewise homogeneous SLF through a hidden Markov random field (MRF) model [15], which captures spatial correlations of neighboring regions exhibiting related statistical behavior. Efficient field estimators will be derived by using Markov chain Monte Carlo (MCMC) sampling [16], which is a powerful tool for Bayesian inference when the analytical solutions of the minimum mean-square error (MMSE) or the maximum a posteriori (MAP) estimators are not available. Furthermore, an adaptive data acquisition method will be developed, with the goal of reducing uncertainty of the SLF.

Notation: $I_n$ is the $n \times n$ identity matrix. Superscript $\top$ represents transposition. $\cdot_{\mathsf{c}}$ stands for a cardinality of the set.

2. MODEL AND PROBLEM STATEMENT

Consider a set of sensors deployed over a two-dimensional geographical area indexed by a set $\mathcal{A} \subset \mathbb{R}^2$. After averaging out small-scale fading effects, the channel gain measurement over a link between a transmitter located at $x \in \mathcal{A}$ and a receiver located at $x' \in \mathcal{A}$ can be represented (in dB) as

$$g(x, x') = g_0 - \gamma 10 \log_{10} d(x, x') - s(x, x')$$

where $g_0$ is the path gain at unit distance; $d(x, x') := ||x - x'||_2$ is the distance between the transceivers at $x$ and $x'$; $\gamma$ is the pathloss exponent; and $s(x, x')$ is the attenuation due to shadow fading. In CG cartography, a tomographic model for the shadow fading is adopted [7, 12, 11], namely

$$s(x, x') \simeq \sum_{i=1}^{N} w(x, x', \hat{x}_i) f(\hat{x}_i).$$

where \{\hat{x}_i\}_{i=1}^{N} is a grid of points over $\mathcal{A}$, $f : \mathcal{A} \to \mathbb{R}$ denotes the spatial loss field (SLF) capturing the attenuation at each location, and $w(x, x', \hat{x})$ is the weight function modeling the
influence of the SLF at \( \tilde{x} \) to the shadowing experienced by link \( x-x' \). Examples of the weight function include the normalized ellipse model taking the form [14]

\[
w(x, x', \tilde{x}) = \begin{cases} 1/\sqrt{d(x, x')}, & \text{if } d(x, \tilde{x}) + d(x', \tilde{x}) < d(x, x') + \lambda \\ 0, & \text{otherwise} \end{cases}
\]  

(3)

where \( \lambda > 0 \) is a tunable parameter. The value of \( \lambda \) is commonly set to half the wavelength to assign non-zero weights only within the first Fresnel zone. Overall, the model in (2) shows how the nature and spatial distribution of obstructions in the propagation medium influence the attenuation between a pair of locations.

To estimate the channel gain map, \( N \) sensors located at \( \{x_1, \ldots, x_N\} \in A \) collaboratively obtain channel gain measurements. At time slot \( t \), the radios indexed by \( n(t) \) and \( n'(t) \) measure the channel gain \( g_t := g(x_{n(t)}, x_{n'(t)}) \) by exchanging pilot sequences, where \( n(t), n'(t) \in \{1, \ldots, N\} \). It is supposed that \( g_0 \) and \( \gamma \) have been estimated during a calibration phase. After subtracting these \( g_t \), the shadowing estimate \( \hat{s}_t := \hat{s}(x_{n(t)}, x_{n'(t)}) \) of \( s(x_{n(t)}, x_{n'(t)}) + \nu_t \) is obtained, where \( \nu_t \) denotes measurement noise. Given these measurements \( \hat{s}_t := [\hat{s}_1, \ldots, \hat{s}_T] \in \mathbb{R}^T \) along with the known set of links \( \{(x_{n_{(r)}}, x_{n'(r)})\}_{r=1}^{N_{s}} \) and the weight function \( w \), the problem is to estimate \( g(x, x') \) between any pair of locations \( (x, x') \in A \). To this end, it suffices to estimate \( f \), or equivalently \( \hat{f} := [\hat{f}(\tilde{x}_1), \ldots, \hat{f}(\tilde{x}_N)] \in \mathbb{R}^N \). Afterwards, the arbitrary channel gain \( g(x, x') \) can be obtained by substituting (2) into (1) and replacing \( f \) with its estimate.

### 3. ADAPTIVE BAYESIAN CG CARTOGRAPHY

In this section, we propose a two-layer Bayesian model for the SLF, as well as, an MCMC-based approach for inference. Furthermore, an adaptive data acquisition strategy to select informative measurements is introduced.

#### 3.1. Field estimation via Markov chain Monte Carlo

Let \( A := \{x \mid \mathbb{E}[f(x)] = \mu_f, \text{Var}[f(x)] = \sigma_f^2, x \in A\} \) and \( A_1 := \{x \mid \mathbb{E}[f(x)] = \mu_f, \text{Var}[f(x)] = \sigma_f^2, x \in A\} \), giving rise to a hidden label field \( z := [z(\tilde{x}_1), \ldots, z(\tilde{x}_N)] \in \{0, 1\}^N \) of binary labels with \( z(\tilde{x}_i) = k \) if \( \tilde{x}_i \in A_k \). We then model the conditional distribution of \( f(x) \) as

\[
f(\tilde{x}_i) | z(\tilde{x}_i) = k \sim \mathcal{N}(\mu_{f, k}, \sigma_{f, k}^2),
\]

(4)

while the Ising prior [17], which is a binary version of the discrete MRF Potts prior [15], is assigned to \( z \) to capture the dependency among spatially correlated labels. By the Hammersley-Clifford theorem [18], the Ising prior of \( z \) follows a Gibbs distribution

\[
p(z|\beta) = \frac{1}{C(\beta)} \exp \left[ \beta \sum_{i=1}^{N_s} \sum_{j \in \mathcal{N}(\tilde{x}_i)} \delta(z(\tilde{x}_j) = z(\tilde{x}_i)) \right]
\]

(5)

where \( \mathcal{N}(\tilde{x}_i) \) is a set of indices associated with 1-hop neighbors of \( \tilde{x}_i \) on the rectangular grid, \( \beta \) is the granularity coefficient to control the degree of homogeneity in \( z \), \( \delta(\cdot) \) is the Kronecker delta function, and \( C(\beta) := \sum_{z \in \mathcal{Z}} \exp \left[ \beta \sum_{i=1}^{N_s} \sum_{j \in \mathcal{N}(\tilde{x}_i)} \delta(z(\tilde{x}_j) = z(\tilde{x}_i)) \right] \) is the partition function with \( \mathcal{Z} := \{0, 1\}^{N_s} \). By assuming conditional independence of \( \{f(\tilde{x}_i)\}_{i=1}^{N_s} \) given \( z \), the resulting model is referred to as the Gauss-Potts model [19] with two labels.

Let \( \nu_t \) be independent and identically distributed (i.i.d) Gaussian with zero mean and variance \( \sigma^2_\nu \), and \( \theta \) denote the known parameter vector including \( \sigma^2_\nu, \beta \), and \( \theta_f := [\mu_f, \tau_f, \sigma^2_{f, 0}, \sigma^2_{f, 1}]^\top \). The weight matrix \( \mathbf{W}_t \in \mathbb{R}^{N_s \times T} \) is constructed with columns equal to \( w(\mathbf{x}_{n(t)}, \mathbf{x}_{n'(t)}, \tilde{x}_i) \), \( w(x_{n(r)}, x_{n'(r)}, \tilde{x}_i) \}_{r=1}^{N_s} \) of the link \( x_{n(r)} \sim x_{n'(r)} \) for \( r = 1, 2, \ldots, T \). Then, one can cast Bayesian CG cartography by writing the joint posterior as

\[
p(f, z, \theta | \tilde{s}_t) \propto p(\tilde{s}_t | f) p(z | \beta) p(\theta),
\]

(6)

where \( p(\tilde{s}_t | f, \sigma^2_\nu) \sim \mathcal{N}(\mathbf{W}_t \mathbf{f} + \mathbf{f}_t \mathbf{I}_T) \) is the likelihood, and \( p(f, z, \theta) \) are the priors of \( \{f, z, \theta\} \), respectively. By utilizing the posterior in (6), the MMSE estimator of \( f \) is found as \( \hat{f}_{MMSE} := \mathbb{E}[f | z = z_{MAP}, \tilde{s}_t] \), where \( z \) is fixed to the marginal MAP estimate of \( z \), i.e., \( z_{MAP} = \arg \max_z p(z | \tilde{s}_t) \).

Although the suggested estimators have been advocated [20, 21], analytical solutions are not available due to the complex form of the posterior in (6) for marginalization or maximization. To bypass this challenge, one can use samples generated from the posterior in (6) as its proxy and then numerically obtain the desired estimators from those samples. MCMC is a class of algorithms to generate samples from a complex distribution [16]. Among MCMC methods, Gibbs sampling is particularly suitable for this work. It draws samples following the target distribution (e.g., the posterior in (6)) by sweeping through each variable to sample from its conditional distribution while fixing the others to their up-to-date values. Although the samples at early iterations of Gibbs sampling with random initialization are not representative of the desired distribution (such duration is called the burn-in period \( N_{\text{burn}} \)), the theory of MCMC guarantees that the stationary distribution of those samples is matched with the target distribution [16].

Gibbs sampling requires only the proportionality of the conditional distribution, as described in Alg. 1. Particularly for the posterior conditional of \( f \), it is easy to show

\[
p(f | \tilde{s}_t, z, \theta) \propto p(\tilde{s}_t | f, \sigma^2_\nu) p(f | z, \theta) \sim \mathcal{N}(\mu_{f \mid z}, \Sigma_{f \mid z}),
\]

(7)

where

\[
\Sigma_{f | z} := \left(\sigma^2_{f, 0}^{-1} W_t W_t^\top + \Delta_{f | z}^{-1}\right)^{-1}
\]

(8)

\[
\hat{\mu}_{f | z} := \Sigma_{f | z} \left(\sigma^2_{f, 0}^{-1} W_t \tilde{s}_t + \Delta_{f | z}^{-1} \mu_f \right)
\]

(9)

since \( p(f | z, \theta_f) \) follows \( \mathcal{N}(\hat{\mu}_{f | z}, \Delta_{f | z}) \) by (4), with \( \mu_f := \mathbb{E}[f | z] \) and \( \Delta_{f | z} := \text{diag}(\{\text{Var}[f_i | z_i] \}_{i=1}^{N_s}) w_i \).

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Algorithm 1 Generic Gibbs sampler for $f$ and $z$

**Input:** $z^{(0)}$, $\theta$, $\hat{s}_i$, $N_{\text{Burn}}$, and $N_{\text{iter}}$.

1: for $l = 1$ to $N_{\text{Burn}}$ do
2: Generate $f^{(l)} \sim p(f|\hat{s}_i, z^{(l-1)}, \theta)$ in (7)
3: Generate $z^{(l)} \sim p(z|\hat{s}_i, f^{(l)}, \theta)$ via Alg. 2
4: end for
5: return $S := \{f^{(l)}, z^{(l)}\}_{l=N_{\text{Burn}}+1}^{N_{\text{iter}}}$

$f(\hat{x}_i)$ and $z_i := z(\hat{x}_i)$. Hence, $f$ can be easily simulated by a standard sampling method. On the other hand, another Gibbs sampler is required to simulate $p(z|\hat{s}_i, f, \theta) \propto p(f|z, \theta_f)p(z|\beta)$ to avoid the intractable computation of $C(\beta)$ in (5). Let $z_{-i}$ and $z_{N(\hat{x}_i)}$ represent replicas of $z$ without its $i$-th entry, and only with the entries of $N(\hat{x}_i)$, respectively. By the Markovianity of $z$ and conditional independence between $f_i$ and $f_j$ $\forall i \neq j$ given $z$, the conditional distribution of $z_i$ is given by

$$p(z_i|z_{-i}, \hat{s}_i, f, \theta) \propto \exp \left[ \ell(z_i) + \beta \sum_{j \in N(\hat{x}_i)} \delta(z_j = z_i) \right]$$

where $\ell(z_i) := \ln p(f_i|z_i, \theta_f)$. After evaluating (10) for $z_i = 0, 1$ and normalizing, one can obtain $p(z_i = 1|z_{-i}, \hat{s}_i, f, \theta) = (1 + h_i)^{-1}$, where

$$h_i := \exp \left[ \ell(z_i = 0) - \ell(z_i = 1) + \sum_{j \in N(\hat{x}_i)} \beta (1 - 2z_j) \right]$$

with $\delta(z_j = 0) - \delta(z_j = 1) = 1 - 2z_j$. Then, the sample of $z$ can be obtained via the single-site Gibbs sampler by using (11), as summarized in Alg. 2.

Building on [20], the elementwise MAP estimator of $z$ and its sample-based approximation are

$$\hat{z}_{i,\text{MAP}} = \arg \max_{z_i \in \{0, 1\}} p(z_i|\hat{s}_i)$$

$$\hat{z}_{i,\text{MAP}} \simeq \arg \max_{z_i \in \{0, 1\}} \frac{1}{|S|} \sum_{l=N_{\text{Burn}}+1}^{N_{\text{iter}}} \delta(z_i^{(l)} = z_i)$$

for $i = 1, \ldots, N_g$. After obtaining $\hat{z}_{\text{MAP}}$, the sample-based elementwise MMSE estimator of $f$ similarly follows as

$$\hat{f}_{i,\text{MMSE}} \simeq \frac{1}{|S|} \sum_{l=N_{\text{Burn}}+1}^{N_{\text{iter}}} f_i^{(l)} \delta(z_i^{(l)} = \hat{z}_{i,\text{MAP}}), \forall i,$$

where $S_l \subset S$ is a subset of samples such that $z_i^{(l)} = \hat{z}_{i,\text{MAP}}$ for $l = N_{\text{Burn}} + 1, \ldots, N_{\text{iter}}$.

### 3.2. Adaptive data acquisition via uncertainty sampling

The proposed Bayesian CG cartography accounts for the uncertainty of $f$, through the variance in (8). Therefore, one can adaptively collect a measurement (or a mini-batch of measurements) when a set of available sensing radio pairs are revealed, with the goal of reducing the uncertainty of $f$. Note that the resulting sampling algorithm has been studied under the name of active learning [22] in the machine learning community. To this end, the conditional entropy [23] is considered as an uncertainty measure of $f$ at time slot $t$, namely,

$$H_t := H(f|\hat{s}_t, z, \theta) = \sum_{z \in Z} \int p(s_t, z', \theta)$$

$$\times H(f|s_t = s_t', z = z', \theta = \theta')d\theta'ds_t'$$

where $H(f|s_t = s_t', z = z', \theta = \theta') := -\int p(f|s_t', z', \theta')d\theta'$. Once expressing $H_{t+1} = H_t - \sum_{z' \in Z} \int p(s_{t+1}', \theta')p(s_t'|s_t, \theta')q(z', \theta', w_{t+1})d\theta'ds_t'$ with $q(z, \theta, w) := \ln (1 + (\sigma_0^2)^{-1}w^T Z f|z, w)/2$ by use of the matrix determinant identity lemma [24, Chapter 18], given $\hat{s}_t = \hat{s}_t'$, it yields $w_{t+1}$ as the solution of

$$\max_{w \in \mathcal{W}_{t+1}} \mathbb{E}_{z, \theta|h_t = \hat{s}_t}[q(z, \theta, w)]$$

$$= \sum_{z' \in Z} \int p(z', \theta'|s_t = \hat{s}_t')q(z', \theta', w)d\theta'$$

where $\mathcal{W}_{t+1}$ is a set of weight vectors found from locations of available sensing radio pairs at time slot $t + 1$.

Although (P1) can be solved in a greedy fashion when $\theta$ is fixed as in this work, evaluating $\mathbb{E}_{z, \theta|h_t = \hat{s}_t}[q(z, \theta, w)]$ is still intractable for large $N_g$ since $|Z| = 2^{N_g}$. Fortunately, the samples from Alg. 1 help to approximate

Algorithm 2 Single-site Gibbs sampler for $z$

**Input:** $f^{(l-1)}$ and $z^{(l-1)}$.

1: Initialize $\zeta^{(l-1)} = z^{(l-1)}$

2: for $i = 1$ to $N_g$ do
3: Obtain $h_i$ in (11) with $z = \zeta^{(l-1)}$ and $f = f^{(l-1)}$
4: if $u \sim U(0, 1) < (1 + h_i)^{-1}$ then
5: Set $\zeta_i^{(l)} = 1$
6: else
7: Set $\zeta_i^{(l)} = 0$
8: end if
9: end for
10: return $z^{(l)} = \zeta^{(l)}$

Algorithm 3 Adaptive Bayesian CG cartography

**Input:** $z^{(0)}$, $s_0$, $\theta$, $N_{\text{Burn}}$, $N_{\text{iter}}$, $\gamma_0$ and $\gamma$.

1: for $\tau = 0, 1, \ldots$ do
2: Obtain $S^{(\tau)}$ via Alg. 1 ($z^{(0)}, \theta, \hat{s}_i, N_{\text{Burn}}, N_{\text{iter}}$)
3: Obtain $\hat{z}_{\text{MAP}}^{(\tau)}$ from (12) by using $S^{(\tau)}$
4: Obtain $f_{\text{MMSE}}^{(\tau)}$ from (13) by using $\hat{z}_{\text{MAP}}^{(\tau)}$ and $S^{(\tau)}$
5: Calculate $\hat{u}(w)$ in (16) for $w \in W_{t+1}$ by using $S^{(\tau)}$
6: Collect $\hat{s}_{t+1}$ from sensors associated with max $\hat{u}(w)$
7: Set $\hat{s}_{t+1} = [\hat{s}^T_t, \hat{s}_{t+1}]^T$ and $z^{(\tau)} = \hat{z}_{\text{MAP}}^{(\tau)}$
8: end for
9: Consider arbitrary locations $\{x, x'\} \in A$
10: Estimate $\hat{s}(x, x')$ via (2) by using $f_{\text{MMSE}}^{(\tau)}$
11: Estimate $\hat{g}(x, x')$ via (1) by using $\gamma_0$, $\gamma$, and $\hat{s}(x, x')$

$$H_t := H(f|\hat{s}_t, z, \theta) = \sum_{z \in Z} \int p(s_t, z', \theta)$$

$$\times H(f|s_t = s_t', z = z', \theta = \theta')d\theta'ds_t'$$

where $H(f|s_t = s_t', z = z', \theta = \theta') := -\int p(f|s_t', z', \theta')d\theta'$. Once expressing $H_{t+1} = H_t - \sum_{z' \in Z} \int p(s_{t+1}', \theta')p(s_t'|s_t, \theta')q(z', \theta', w_{t+1})d\theta'ds_t'$ with $q(z, \theta, w) := \ln (1 + (\sigma_0^2)^{-1}w^T Z f|z, w)/2$ by use of the matrix determinant identity lemma [24, Chapter 18], given $\hat{s}_t = \hat{s}_t'$, it yields $w_{t+1}$ as the solution of

$$\max_{w \in \mathcal{W}_{t+1}} \mathbb{E}_{z, \theta|h_t = \hat{s}_t}[q(z, \theta, w)]$$

$$= \sum_{z' \in Z} \int p(z', \theta'|s_t = \hat{s}_t')q(z', \theta', w)d\theta'$$

where $\mathcal{W}_{t+1}$ is a set of weight vectors found from locations of available sensing radio pairs at time slot $t + 1$.
This paper developed a novel adaptive Bayesian channel gain cartography algorithm capable of constructing maps that provide the channel gain between arbitrary locations in a region of interest while revealing the structure of the propagation medium through a spatial loss field, equipped with adaptive data acquisition capability. Efficacy of the novel algorithm was validated through real data tests.
6. REFERENCES


