ABSTRACT

Acquiring high-resolution hyperspectral (HS) images is a very challenging task. To this end, hyperspectral pansharpening techniques have been widely studied, which estimate an HS image of high spatial and spectral resolution (high HS image) from a pair of an HS image of high spectral resolution but low spatial resolution (low HS image) and a high spatial resolution panchromatic (PAN) image. However, since these methods do not fully utilize the piecewise-smoothness of spectral information on HS images in estimation, they tend to produce spectral distortion when the low HS image contains noise. To tackle this issue, we propose a new hyperspectral pansharpening method using a spatio-spectral regularization. Our method not only effectively exploits observed information but also properly promotes the spatio-spectral piecewise-smoothness of the resulting high HS image, leading to high quality and robust estimation. The proposed method is reduced to a nonsmooth convex optimization problem, which is efficiently solved by a primal-dual splitting method. Our experiments demonstrate the advantages of our method over existing hyperspectral pansharpening methods.

Index Terms— hyperspectral pansharpening, spatio-spectral total variation, primal-dual splitting

1. INTRODUCTION

A hyperspectral (HS) image is a spatio-spectral datacube that contains rich information on invisible light and narrow wavelength intervals. Since HS images reveal the intrinsic characteristics of scene objects and environmental lighting, hyperspectral imaging is a promising research topic and offers many applications in a wide range of fields, spanning from remote sensing, geoscience and astronomy to biomedical imaging and signal processing [1, 2].

Essentially, it is very difficult to capture an HS image of high spatial and spectral resolution (we call high HS image). This is because the amount of incident energy is limited, and there are critical tradeoffs between the spatial resolution and the spectral resolution of HS imaging systems. On the other hand, such a high HS image can be a key item in many applications. To resolve this dilemma, hyperspectral pansharpening techniques have been studied (see comprehensive review papers [3, 4] and references therein).

In hyperspectral pansharpening, a high HS image is estimated from a pair of an HS image of high spectral resolution but low spatial resolution (we call low HS image) and a high spatial resolution panspectral image (we call PAN image). Hyperspectral pansharpening methods can be roughly classified into two groups. The first group contains the methods of using Principal Component Analysis (PCA) [5, 6], Gram-Schmidt (GS) [7, 8] and multiresolution analysis (MRA) [9–11]. The merit of these methods are low computational cost and relatively easy implementation. However, since these methods do not utilize a-priori knowledge, they are very sensitive to noise and are difficult to achieve high-quality estimation. Meanwhile, the methods in the second group [12–15] are based on variational approaches, i.e., estimating a high HS image by solving optimization problems. In general, these methods yield better results than the methods in the first group because they use a-priori knowledge on HS images, such as spatial smoothness and low dimensionality. However, they do not give due consideration to the piecewise-smoothness of the spectral information of the resulting high HS image, so that they tend to produce spectral distortion when the low HS image contains noise.

To overcome this difficulty, we propose a novel hyperspectral pansharpening method built upon a newly-formulated convex optimization problem. The objective function of the problem consists of a spatio-spectral regularization term and an edge similarity term with a PAN image. In addition, two hard constraints are imposed on the problem: data-fidelity to a low HS image and a dynamic range constraint. This problem formulation not only fully exploits the spectral information on the low HS image and the spatial information on the PAN image but also properly promotes the spatio-spectral piecewise-smoothness of the resulting high HS image. After reformulation, the problem can be efficiently solved by a primal-dual splitting method [16], which is a proximal splitting algorithm and has been successfully applied to image restoration [17–20]. Our experiments illustrate the advantages of the proposed method over existing hyperspectral pansharpening methods.

2. PROPOSED METHOD

2.1. Observation Model

Let \( \mathbf{u} \in \mathbb{R}^{NB} \) be a true high HS image with \( N \) pixels and \( B \) spectral bands. In hyperspectral pansharpening, a low HS image \( \mathbf{v} \) and a PAN image \( \mathbf{p} \) are assumed to be given with the observation model:

\[
\mathbf{v} = \mathbf{S} \mathbf{u} + \mathbf{n} \in \mathbb{R}^{NB},
\]

\[
\mathbf{p} = \mathbf{R} \mathbf{u} \in \mathbb{R}^N,
\]

where \( \mathbf{S} \in \mathbb{R}^{NB \times NB} \) is a downsampling matrix with a downsampling rate of \( r \) (\( N \) is divisible by \( r \)), \( \mathbf{B} \) is a blur matrix, \( \mathbf{n} \) is an additive white Gaussian noise with standard deviation \( \sigma \), and \( \mathbf{R} \in \mathbb{R}^{N \times NB} \) is a matrix that represents the spectral response of the PAN image (\( \mathbf{R} \) calculates weighted average along the spectral direction). This model assumes that the low HS image contains considerable noise, which is a natural situation in hyperspectral imagery. Existing hyperspectral pansharpening methods are also based on the same or very similar model.

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2.2. Problem Formulation

Based on the model in Sec. 2.1, we formulate hyperspectral pan-
sharpening as the following convex optimization problem:

\[
\begin{align*}
\min_u & \quad \lambda \| D u - D M p \|_{1,2} + \operatorname{SSTV}(u) \\
\text{s.t.} & \quad S B u \in B_{2, \varepsilon}^2 := \{ x \in \mathbb{R}^{N_B} : \| x - v \| \leq \varepsilon \}, \\
& \quad u \in [\mu_{\min}, \mu_{\max}]^{N_B},
\end{align*}
\]

where $D = (D_h^T, D_h^T)^T \in \mathbb{R}^{2N_B \times N_B}$ is a spatial difference operator with $D_h$ and $D_h$ being vertical and horizontal difference opera-
tors respectively. $\| \cdot \|_{1,2}$ is the mixed $\ell_{1,2}$ norm, and $M \in \mathbb{R}^{N_B \times N}$
is a linear operator that replicates the PAN image $B$ times along the spectral
direction.

The first term in (3) is originally proposed in [21], which evaluates edge similarity between the high HS image of interest and the PAN image. Specifically, we can assume that the non-zero differences of the high HS image are sparse and correspond to edges, and that their positions should be the same as those of the PAN image. Hence, evaluating their errors by the mixed $\ell_{1,2}$ norm is a reasonable approach for exploiting the spatial information on the PAN image.

The second term in (3) is the spatio-spectral total variation (SSTV) proposed in [22] for HS image denoising, defined by

\[
\operatorname{SSTV}(u) := \| D D u \|_1,
\]

where $D_h$ is a spectral difference operator, and $\| \cdot \|_1$ is the $\ell_1$ norm. This regularization evaluates the spatio-tmetric smoothness of an HS image, and it has been shown to be very effective in HS image denoising, so that the use of SSTV would robustly hyperspectral pansharpening against spectral distortion when the low HS image contains noise. The first constraint in (3) serves as data-fidelity to the low HS image $v$ and is defined as the $v$-centered $\ell_2$-norm ball with the radius $\varepsilon > 0$. As mentioned in [23-27], such a hard constraint facilitates the parameter setting because $\varepsilon$ has a clear meaning. The second constraint in (3) represents the dynamic range of HS images with $\mu_{\min} < \mu_{\max}$. 

2.3. Optimization

Since Prob. (3) is a convex but highly nonsmooth optimization problem, a suitable iterative algorithm is required to solve it. In this paper, we adopt a primal-dual splitting method [16]. It can solve (possibly nonsmooth) convex optimization problems of the form:

\[
\min_u g(u) + h(L u),
\]

where $g$ and $h$ are proper lower semicontinuous convex functions, and $L$ is a linear operator. Here, we assume that $g$ and $h$ are prox-
imals, i.e., the proximity operators\(^1\) [28] of $g$ and $h$ are computable.

For any $y_1^{(0)}$ and $\gamma_1, \gamma_2 > 0$ satisfying $\gamma_1 \gamma_2 \| L \|_{op}^2 \leq 1$ ($\| \cdot \|_{op}$ is the operator norm), the algorithm is given by

\[
\begin{align*}
\begin{aligned}
u^{(n+1)} &= \text{prox}_{\gamma_1 g}(u^{(n)} - \gamma_1 L^T y^{(n)}), \\
v^{(n+1)} &= \text{prox}_{\gamma_2 h}(y^{(n)} + \gamma_2 L(2u^{(n+1)} - u^{(n)}) - u^{(n)}),
\end{aligned}
\end{align*}
\]

The function $h^*$ is the convex conjugate of $h$, and the proximity operator of $h^*$ is available via that of $h$ [29, Theorem 14.3(ii)] as follows:

\[
\text{prox}_{\gamma h^*}(x) = x - \gamma \text{prox}_{\frac{1}{\gamma} h}(\frac{x}{\gamma}).
\]

Algorithm 1: Primal-dual splitting algorithm for Prob. (3)

\[
\text{input : } u^{(0)}, y_1^{(0)}, y_2^{(0)}, y_3^{(0)}
\]

While a stopping criterion is not satisfied do do

\[
\begin{align*}
u^{(n+1)} &= \text{prox}_{\gamma_1 g}(u^{(n)} - \gamma_1 L^T y^{(n)} + \gamma_2 D^T y^{(n)} + \Phi^T y^{(n)})), \\
y^{(n+1)} &= \text{prox}_{\gamma_2 h}(y^{(n)} + \gamma_2 D D u^{(n+1)} - u^{(n)}), \\
\end{align*}
\]

\[
\begin{align*}
\begin{aligned}
u^{(n+1)} &= \text{prox}_{\gamma_1 g}(u^{(n)} - \gamma_1 L^T y^{(n)} + \gamma_2 D^T y^{(n)} + \Phi^T y^{(n)}), \\
y^{(n+1)} &= \text{prox}_{\gamma_2 h}(y^{(n)} + \gamma_2 D D u^{(n+1)} - u^{(n)}), \\
\end{aligned}
\end{align*}
\]

In what follows, we reformulate Prob. (3) into Prob. (4) to solve it by the primal-dual splitting method.

First, we introduce the indicator functions of the two constraints in (3) to put them into the objective function. The indicator function of a nonempty closed convex set $C$ is defined by

\[
\iota_C(x) := \begin{cases} 
0, & \text{if } x \in C, \\
\infty, & \text{otherwise.}
\end{cases}
\]

Then, Prob. (3) can be rewritten as

\[
\begin{align*}
\min_u & \quad \lambda \| D u - D M p \|_{1,2} + \| D D u \|_1 \\
& \quad + t \gamma_2 y_2 (S B u) + t \iota_{[\mu_{\min}, \mu_{\max}]^N} (u).
\end{align*}
\]

Note that Prob. (3) and Prob. (8) are equivalent from the definition of the indicator function.

Then, by letting

\[
\begin{align*}
g : \mathbb{R}^{N_B} &\to \mathbb{R} : u \mapsto t \iota_{[\mu_{\min}, \mu_{\max}]^N} (u), \\
h : \mathbb{R}^{(4+\frac{1}{2})N_B} &\to \mathbb{R} \cup \{\infty\} : \\
(y_1, y_2, y_3) &\mapsto \lambda \| y_1 - D M p \|_1 + \| y_2 \|_1 + t \gamma_2 y_2 (y_3), \\
L : \mathbb{R}^{N_B} &\to \mathbb{R}^{(4+\frac{1}{2})N_B} : u \mapsto (D u, D D u, S B u),
\end{align*}
\]

Prob. (8) is reduced to Prob. (4). Using (6), the resulting algorithm for solving (3) is summarized in Algorithm 1.

Let us explain how to compute each step of Alg. 1. Since the proximity operator of the indicator function of $C$ is equivalent to the metric projection\(^2\) onto $C$, the proximity operators in steps 2 and 8 can be computed as follows: for $i = 1, \ldots, N B$

\[
\begin{align*}
\text{prox}_{\gamma_2 h}(x) &= P_{\gamma_2 h}(x) \left( x, v, \frac{(x-v)^{\top}(x-v)}{\| x - v \|^2} \right), \\
\text{prox}_{\gamma_2 \gamma_2 g}(x) &= P_{\gamma_2 \gamma_2 g}(x) \left( x, v, \frac{(x-v)^{\top}(x-v)}{\| x - v \|^2} \right), \\
\end{align*}
\]

Meanwhile, the proximity operators of the $\ell_1$ norm and the mixed $\ell_{1,2}$ norm in steps 6 and 7 are reduced to simple soft-thresholding

\[
\begin{align*}
\text{prox}_{\gamma_1 g}(x) &= \arg \min_y \| y - x \|_1 + \frac{1}{2} \| y - x \|^2, \\
\text{prox}_{\gamma_2 h}(x) &= \arg \min_y \| y - x \|_2 + \frac{1}{2} \| y - x \|^2.
\end{align*}
\]

\[^1\]The proximity operator of index $\gamma_2 > 0$ of a proper lower semicontinuous convex function $f$ is defined by $\text{prox}_{\gamma f}(x) := \arg \min_y (f(y) + \frac{1}{2\gamma} \| y - x \|^2)$.

\[^2\]Given a vector $x$ and a nonempty closed convex set $C$, the metric projection onto $C$ is characterized by $P_C(x) := \inf \{ \| x - \tilde{x} \| \text{ s.t. } \tilde{x} \in C \}$. 

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Table 1. Quality measures for $\sigma = 0.05$ (left) and $\sigma = 0.1$ (right).

<table>
<thead>
<tr>
<th>method</th>
<th>CC</th>
<th>SAM</th>
<th>RMSE</th>
<th>ERGAS</th>
<th>CC</th>
<th>SAM</th>
<th>RMSE</th>
<th>ERGAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFIM [11]</td>
<td>0.8758</td>
<td>23.08</td>
<td>511.4</td>
<td>9.310</td>
<td>0.6711</td>
<td>35.72</td>
<td>996.8</td>
<td>17.25</td>
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<tr>
<td>MTF-GLP [9]</td>
<td>0.8814</td>
<td>23.47</td>
<td>512.7</td>
<td>9.114</td>
<td>0.7309</td>
<td>34.22</td>
<td>888.9</td>
<td>14.95</td>
</tr>
<tr>
<td>MTF-GLP-HPM [10]</td>
<td>0.8742</td>
<td>23.23</td>
<td>524.8</td>
<td>9.533</td>
<td>0.6653</td>
<td>56.00</td>
<td>1031</td>
<td>17.74</td>
</tr>
<tr>
<td>GS [7]</td>
<td>0.8042</td>
<td>25.40</td>
<td>658.4</td>
<td>11.97</td>
<td>0.6353</td>
<td>38.97</td>
<td>1042</td>
<td>19.57</td>
</tr>
<tr>
<td>GSA [8]</td>
<td>0.8706</td>
<td>27.18</td>
<td>558.8</td>
<td>10.25</td>
<td>0.7105</td>
<td>40.37</td>
<td>998.4</td>
<td>18.71</td>
</tr>
<tr>
<td>PCA [5]</td>
<td>0.7968</td>
<td>25.56</td>
<td>677.0</td>
<td>12.63</td>
<td>0.6314</td>
<td>39.12</td>
<td>1054</td>
<td>19.77</td>
</tr>
<tr>
<td>GPCA [6]</td>
<td>0.9264</td>
<td>9.097</td>
<td>409.6</td>
<td>7.087</td>
<td>0.9219</td>
<td>10.62</td>
<td>417.7</td>
<td>7.255</td>
</tr>
<tr>
<td>CNMF [15]</td>
<td>0.9587</td>
<td>9.977</td>
<td>522.5</td>
<td>5.351</td>
<td>0.9343</td>
<td>15.03</td>
<td>407.4</td>
<td>6.599</td>
</tr>
<tr>
<td>Bayesian Naive [12]</td>
<td>0.9517</td>
<td>15.23</td>
<td>314.3</td>
<td>5.686</td>
<td>0.8789</td>
<td>25.15</td>
<td>519.2</td>
<td>9.772</td>
</tr>
<tr>
<td>Bayesian Sparse [13]</td>
<td>0.9546</td>
<td>15.11</td>
<td>300.7</td>
<td>5.484</td>
<td>0.8807</td>
<td>24.98</td>
<td>514.0</td>
<td>9.708</td>
</tr>
<tr>
<td>HySure [14]</td>
<td>0.9645</td>
<td>9.306</td>
<td>292.2</td>
<td>4.990</td>
<td>0.9452</td>
<td>13.88</td>
<td>349.6</td>
<td>6.088</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.9757</td>
<td>7.460</td>
<td>235.2</td>
<td>4.081</td>
<td>0.9691</td>
<td>8.739</td>
<td>263.7</td>
<td>4.604</td>
</tr>
</tbody>
</table>

Fig. 1. Quality measures versus $\lambda$ in (3).

3. EXPERIMENTS

We demonstrate the advantages of the proposed method over existing hyperspectral pansharpening methods. Specifically, we examine the proposed and existing methods as follows: (i) generate a pair low HS and PAN images from a true high HS image; (ii) add white Gaussian noise with standard deviation $\sigma$ to the low HS image; (iii) estimate the true high HS image from the pair by each method, and (iv) evaluate the estimated high HS images based on standard quality measures (introduced later). For the true high HS image, we used a Moffett field dataset, where we cropped a region of size 256 $\times$ 128 $\times$ 176 and normalized its dynamic range into $[0, 1]$. The low HS and PAN images were generated based on the model in Sec. 2.1, where the downsampling rate was set as $r = 4$. $B$ was set to a 9 $\times$ 9 Gaussian blur matrix, and $R$ was set to an weighted-average matrix with its weights $w_i$ ($i = 1, \ldots, B$) were defined as $w_i = 1$, for $1 \leq i \leq 41$; and 0 otherwise. The above procedures follow Wald's protocol [31], a standard quality assessment methodology of hyperspectral pansharpening.

For comparison, we adopted 11 existing methods: SFIM [11], MTF-GLP [9], MTF-GLP-HPM [10], GS [7], GSA [8], PCA [5], GPCA [6], CNMF [15], Bayesian Naive [12], Bayesian Sparse [13] and HySure [14]. All parameters of these methods except HySure were set to the recommended values in a MATLAB toolbox of hyperspectral pansharpening distributed by the authors of [3]. For HySure, we set its hyperparameter as $\lambda_{\text{opt}} = 0.1\sigma$ to enhance its performance, and other parameters were set to the recommended values. For our method, the parameters in (3) were set to $\epsilon = \|v - SBu\|_2$ and $\lambda = 0.3$. We set the max iteration number and the stopping criterion of the primal-dual splitting method to 5000 and $\|u^{(n+1)} - u^{(n)}\|_2^2/\|u^{(n)}\|_2 < 1 \times 10^{-4}$, respectively.

For quality measures, we use Cross Correlation (CC), the Spectral Angle Mapper (SAM) [32], the Root Mean Squared Error (RMSE) and Erreur Relative Globale Adimensionnelle de synthèse (ERGAS) [33], which are defined, respectively, by

$$
\text{CC}(u, \hat{u}) = \frac{1}{B} \sum_{j=1}^{B} \frac{\sum_{i=1}^{N} (u_{i+j-1,N} - \alpha_{u,j})(\hat{u}_{i+j-1,N} - \alpha_{u,j})}{\sqrt{\sum_{i=1}^{N} (u_{i+j-1,N} - \alpha_{u,j})^2 \sum_{i=1}^{N} (\hat{u}_{i+j-1,N} - \alpha_{u,j})^2}}
$$

$$
\text{SAM}(u, \hat{u}) = \frac{1}{N} \sum_{i=1}^{N} \arccos \left( \frac{u_{i} \hat{u}_{i}}{\|u_{i}\|_2 \|\hat{u}_{i}\|_2} \right)
$$

$$
\text{RMSE}(u, \hat{u}) = \frac{\|u - \hat{u}\|_2}{\sqrt{NB}}
$$

$$
\text{ERGAS}(u, \hat{u}) = \frac{100}{\alpha} \sqrt{\frac{1}{B} \sum_{j=1}^{B} \frac{|u_{i+j-1,N} - \alpha_{u,j}|^2}{(p^1 u_{i+j-1,N})^2}}
$$
where \( \mathbf{u}_i = [u_{i+1}, \ldots, u_{i+(B-1)N}] \in \mathbb{R}^B \) (\( i = 1, \ldots, N \)) and \( \mathbf{u}_j = [u_{N(j-1)+1}, u_{N(j-1)+2}, \ldots, u_{N(j-1)+N}] \in \mathbb{R}^N \) (\( j = 1, \ldots, B \)) are the spectral and spatial vectors of \( \mathbf{u} \), respectively, \( \alpha_{\mathbf{u},j} = \sum_{i=1}^{N} u_{i+(j-1)N} \), \( \alpha_{\mathbf{u},j} = \sum_{i=1}^{N} \bar{u}_{i+(j-1)N} \) and \( 1 = [1, \ldots, 1] \in \mathbb{R}^N \).

Table 1 shows CC, SAM, RMSE and ERGAS of the high HS images estimated by the existing and proposed methods for \( \sigma = 0.05 \) and \( 0.1 \). One can see that for all the quality measures and for both \( \sigma \), the proposed method outperforms all the existing methods (note that only for CC, higher is better). Fig. 1 plots CC, SAM, RMSE and ERGAS of the high HS images estimated by the proposed method versus \( \lambda \) in (3) (\( \sigma = 0.05 \) and \( 0.1 \)). We found that \( \lambda \in [0.3, 0.5] \) is a good choice in this experimental setting.

Fig. 2 depicts the estimated high HS images (\( \sigma = 0.1 \)) as RGB images, where R, G and B bands were set to the 16th, 32nd and 64th bands of them. One can see that (i) most of the existing methods produce artifacts, (ii) GFPCA, CNMF and HySure achieve relatively good estimation but spectral information is distorted in their results. and (iii) the proposed method has a strong ability of spatial and spectral detail-preserving estimation, and the result is most similar to the true high HS image.

4. CONCLUSION

We have proposed a new hyperspectral pansharpening method using a spatio-spectral regularization. This method not only fully exploits observed information but also properly promotes spatio-spectral piecewise-smoothness, an intrinsic property of HS images, leading to robust and effective estimation. The proposed method is reduced to a nonsmooth convex optimization problem, and the optimization is efficiently solved by a primal-dual splitting method. Our experiments revealed the advantages of the proposed method over existing methods.

Finally, we remark that our method can be naturally extended to methods for fusing a pair of HS and multispectral images, which is an interesting direction of future work.
5. REFERENCES


