EXPLORING THE NON-LOCAL SIMILARITY PRESENT IN VARIATIONAL MODE FUNCTIONS FOR EFFECTIVE ECG DENOISING

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ABSTRACT

ECG is corrupted by various noises and denoising of ECG is central to proper diagnosis of cardiac diseases. The main objective of this work is to exploit the efficacy of non-local means (NLM) for ECG denoising. In the presented work, noisy ECG signal is decomposed into variational mode functions (VMFs) using variational mode decomposition (VMD) technique. The decomposed VMFs represent the different frequency band of the noisy ECG signal. The non-local similarity present in each VMFs was exploited using NLM estimation for effective ECG denoising. The two-stage VMD decomposition and NLM estimation process is performed on different set of VMFs at both stages. The proposed method is tested upon MIT-BIH Arrhythmia database. The denoising performance of existing techniques is compared to proposed method. The computed results shows that the proposed method gives superior denoising performance.

Index Terms— Electrocardiogram, denoising, variational mode decomposition, variational mode function, Non-local means

1. INTRODUCTION

Electrocardiogram (ECG) signal represents the electrical picture of human heart. It consists of important clinical parameters and aids in diagnosis of cardiac diseases [1]. The primary problem associated with ECG recording process is noise contamination. During ECG acquisition process, various noises such as baseline wander (BLW), power-line interference (PLN), motion artifacts (MA), disturbance in recording device, etc affects ECG signal [1, 2]. During ambulatory ECG recording white Gaussian noise (WGN) abducts ECG signal [3]. Denoising becomes key for any further signal processing on ECG signal. These noises affect different frequency band of original ECG signal. BLW noise corrupts the lower frequency band, PLN adds 50/60 Hz line frequency where as MA, device noises, WGN and other burst noises affects the entire frequency range of the ECG signal.

Many ECG denoising techniques can be found in literature [4, 5, 6, 7, 8, 9, 10, 11, 12]. In [13, 12, 5], PLN and BLW affected signal were denoised using empirical mode decomposition (EMD) and discrete wavelet transform (DWT) methods. In [14, 7], denoising of ECG signals affected by MA is performed by wavelet based method. In [4, 6, 9, 10, 3], WGN noise based denoising frameworks were proposed. In [8], nonlinear Bayesian filtering framework is proposed.

Most of the these methods are based on EMD-domain thresholding [5, 9, 13] and wavelet domain filtering [7, 15, 11]. In [4], patch-based non-local means (NLM) technique applied on raw noisy ECG for its denoising is proposed. It’s performance is quite competitive with existing state-of-art methods in terms of distortion and mean-square error measurement of denoised output. The major challenge with NLM method is the “rare-patch” effect [4]. The abrupt amplitude and frequency variation around QRS-complex region leads to rare-patch effect, resulting in ineffective QRS-region denoising.

Recently, variational mode decomposition (VMD) [16] has been proposed to decompose the signal into predefined number of narrow-band variational mode functions (VMFs). Each VMFs represents smaller portion of the overall frequency band of the signal. Unlike ECG, these VMFs does not have abrupt amplitude or frequency variation, thus suitable for NLM technique. Motivated by this, a novel two-stage VMD-NLM framework for is proposed effective ECG denoising. The set of VMFs in desired frequency-band are computed from noisy ECG signal for both stages and NLM estimation is performed on selective VMFs. The effective implementation of NLM in VMF denoising results in overcoming the “rare-patch” effect in NLM technique. The simulations are performed on MIT-BIH Arrhythmia database [17]. The proposed method outperforms the existing approaches based on the qualitative and quantitative approaches.

The rest of paper is organized as follow: Section 2 describes the Proposed ECG denoising methodology. Section 3 presents the experimental results and a discussion on the same. Finally, the paper is concluded in Section 4.

2. PROPOSED METHODOLOGY

The overall proposed ECG denoising process is depicted in Figure 1. Firstly, the noisy ECG signal is decomposed into
Stage 1 and stage-2 processes are marked by vertical arrow.

Fig. 1. Proposed method for ECG denoising. Stage-1 and stage-2 processes are explained as follows:

1. **Variational Mode Decomposition**
   - Stage 1 variational mode functions (VMFs). The first VMFs (vmf1) to vmf3) at stage-1. These VMFs are arranged from low-frequency to high-frequency band region of the ECG signal. The low-frequency band VMFs (i.e from vmf1 to vmf3) are processed using NLM estimation. The rest of stage-1 high-frequency VMFs (vmf4 to vmf8) are summed together and further decomposed into n VMFs at stage-2. The value of n is kept greater than m to get the fine information residing in high-frequency ECG signal for effective denoising. The selection of VMFs subjected to NLM estimation at both stage is explained in section 2.3.
   - The first i VMFs (vmf1) to vmf2) in stage-2 having non-local similarity and containing fine clinical information are then estimated by NLM technique. The rest of very-high frequency VMFs (vhf - vmfs) from vmf5 to vmf7) are mostly high frequency noises. The vhf - vmfs contains very little significant information as they mostly lie outside the typical ECG signal frequency range. The denoised output is achieved by summing the stage-1 and stage-2 NLM estimated VMFs.

2. **NLM estimation process**
   - NLM technique finds the non-local similarity present in the noisy input signal y for denoising purpose. It is a patch based technique which compares two patches to find non-local similarity. For a given sample p, the denoised estimation \( \hat{x}(p) \) is weighted sum of sample values at another point q in the search-neighborhood Z(p)

\[
\hat{x}(p) = \frac{1}{Y(p)} \sum_{q \in Z(p)} w(p, q)y(q)
\]

where \( Y(p) = \sum_{q} w(p, q) \). The weight value (w(p, q)) is computed by weighted squared-difference of patches centered

\[
\min_{\{vk, \{f_k\}\}} \left\{ \sum_k \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) * v_k(t) \right] e^{-jf_k t} \right\|_2^2 \right\}
\]

such that \( \sum_k v_k = x \). k is total number of decomposed modes. The reconstruction constraint is solved with help of Lagrangian multipliers, \( \lambda(t) \) and quadratic penalty term. The resultant VMFs computed in frequency domain, \( \hat{v}_k \), is as follows:

\[
\hat{v}_{k+1}(\omega) = \frac{\hat{x}(\omega) - \sum_{i \neq k} \hat{v}_i(\omega) + \lambda(\omega)}{1 + 2\alpha(\omega - \omega_k)^2}
\]

The inverse Fourier transform is used to calculate the corresponding time domain VMFs, \( v_k \). Figure 2 and 3 represents the decomposed VMFs and their corresponding magnitude spectrum for a noisy ECG signal at k = 5. The detailed discussion on working of VMD is given in [16].

2.2. **NLM estimation process**

The NLM technique finds the non-local similarity present in the noisy input signal y for denoising purpose [4]. It is a patch based technique which compares two patches to find non-local similarity. For a given sample p, the denoised estimation \( \hat{x}(p) \) is weighted sum of sample values at another point q in the search-neighborhood Z(p).
amplitude and frequency variation around the QRS-complex region. This effect results in ineffective denoising of QRS-region. On other hand if properly decomposed, the VMFs can have lesser amplitude and frequency variation as suitable for NLM. Also to compliment NLM efficacy, these VMFs have non-local similarity. Keeping in mind the fact that the very large and small VMD decomposition levels results in under-binning (loss of ECG information) or over-binning (mode duplication) of modes [16], a 5 mode decomposition is performed at stage-1. The stage-1 VMFs \((v mf_{1}^{m} = (v mf_{1}^{1}, v mf_{1}^{2}, \ldots, v mf_{1}^{5}))\) and its magnitude spectrum \((ms_{m}^{1} = ms_{1}^{1}, ms_{2}^{1}, \ldots, ms_{5}^{1})\) are shown in Figure 2 and 3 respectively. It can be observed that, the initial two modes \((v mf_{1}^{1} \) and \(v mf_{1}^{2} \)) contains lesser amplitude-frequency variation and significant non-local similarity, thus ideal for NLM estimation. The remaining VMFs \((v mf_{1}^{3} \) to \(v mf_{1}^{5} \)) lie in the high-frequency range and have abrupt amplitude variation. But they contain some significant ECG information and needs further inspection. To do so, these remaining stage-1 VMFs are summed together and subjected further to 7-mode VMD decomposition at stage-2. These VMFs are represented as \(v mf_{2}^{n} = (v mf_{2}^{1}, v mf_{2}^{2}, \ldots, v mf_{2}^{7})\) as shown in Figure 4. The initial 3 VMFs \((v mf_{2}^{1} \) to \(v mf_{2}^{3} \)) are subjected to NLM estimation as they contain some significant high-frequency-ECG information. The remaining stage-2 VMFs mostly contains high-frequency noises are discarded. The final reconstruction is done by adding the processed VMFs at both stages.

### 3. EXPERIMENTAL EVALUATION

#### 3.1. Experimental setup

The experimental evaluation is performed on MIT-BIH Arrhythmia database [17]. The records in database are sampled at 360 Hz and at 11 bits resolution. Each record contains 2 leads ECG data, MLII lead and V5 lead. This work uses MLII lead and the record used for comparison are kept similar to ones used in existing discussed techniques [4]. This work uses 10 second raw ECG (3600 samples) data for all simulation purpose. To simulate various noises, white Gaussian noise (WGN) is added to raw signal at different signal-to-noise ratio (SNR). Also random noise at standard deviation of 0.20 is added to simulate Motion artifact (MA). The performance metrics used for all quantitative result comparison are kept similar to the ones used in [4]: signal-to-noise ra-

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**Fig. 3.** Magnitude Spectrum of stage 1 VMFs. They are arranged from low- to high-frequency band region (left to right).

**Fig. 4.** Stage 2 variational mode functions (VMFs). The modes are arranged from low- to high-frequency band (top to bottom).

\[
w(p, q) = \exp \left( -\frac{\sum_{\Delta \delta} (y(p + \Delta) - y(q + \Delta))^{2}}{2L_{\delta} \tau^{2}} \right)
\]

where \(\tau\) is bandwidth parameter, \(\delta\) represents the patch of samples with \(L_{\delta}\) sample points. The NLM novelty is that the weight’s depends on similarity between patches and not on physical distance between patches.

2.3. VMFs selection, denoising and signal reconstruction

The selection of VMFs at two-stages in the proposed methods is done based on the efficacy of NLM estimation. As pointed out in section 1 that, the NLM denoising technique works effectively by finding the non-local similarity but suffers from the rare-patch problem. The resulting rare-patch effect is due to the fact that, the ECG signal contains abrupt amplitude and frequency variation.
3.2. Results and Discussion

To establish the efficacy of proposed approach, it has been compared with existing DWT-thresholding [15] and NLM-estimation [4] techniques. Figure 5 shows the qualitative comparison of the proposed method with existing methods for both WGN and MA noises. The morphological structure is effectively retained by proposed method compared to existing one for both noises.

The quantitative comparison results are shown through Figure 6 and Table 1. The results are compared based on performance metrics discussed in subsection 3.1. In Figure 6(a) and (b), the $SNR_{imp}$ for discussed methods are compared for different set of input $SNR$ and test signals respectively. Similar comparison is made based on PRD values in Figure 6(c) and (d). The MSE values for the compared methods are shown in Table 1 for various test signals. The resultant $SNR_{imp}$ value for proposed method is significantly higher than the existing methods. Also the MSE and PRD values are lower for the proposed work as desired.

These results show that the proposed denoising method performs better compared to discussed methods both qualitatively and quantitatively. The NLM ill-effect of rare-patch effect has been effectively overcome in this work. It is to note that, NLM algorithm will work efficiently if applied on narrow-band signals with lesser amplitude variation compared to raw ECG with abrupt amplitude variation around QRS-complex region.

4. CONCLUSION

In this paper, a two-stage VMD-NLM based ECG denoising techniques has been proposed. The noisy ECG is decomposed into VMFs with exclusive decomposition capability of VMD. The resultant narrow-band VMFs selected at both stages are efficiently processed through NLM technique for effective denoising. The proposed work overcomes the rare-patch effect of NLM technique by proper VMFs selection. The proposed method works effectively on both white Gaussian noise (WGN) and motion artifact (MA), and retains the significant morphological information of ECG. In Future, efforts will be made to incorporate denoising of power-line interference and baseline wander. The results are found to be superior compared to the existing method both qualitatively and quantitatively.
5. REFERENCES


