ON THE DESIGN OF ROBUST STEERABLE FREQUENCY-IN Variant BEAMPATTERNS WITH CONCENTRIC CIRCULAR MICROPHONE ARRAYS

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ABSTRACT

This paper studies the problem of frequency-invariant beamforming with concentric circular microphone arrays (CCMAs). We develop a beamforming algorithm based on an optimal approximation of the beampattern with the Jacobi-Anger expansion. In comparison with the existing frequency-invariant beamformers with CCMAs or CDMAs, the developed algorithm offers the following advantages: 1) it can mitigate the deep-null problem encountered in CMMAs and therefore has a consistent directivity factor over the frequency range of speech signals; 2) it is more flexible in terms of steering flexibility and the resulting beampattern can be steered to any direction; and 3) it does not require the microphones in different rings of the CCMAs to be aligned, which is very useful in practice, particularly when microphone arrays with small and compact apertures have to be used.

Index Terms— Microphone arrays, concentric circular microphone arrays, fixed beamforming, frequency-invariant beampattern, white noise gain, directivity factor.

1. INTRODUCTION

Microphone array beamforming, an important problem in acoustic signal processing for voice communications and human-machine interfaces, has attracted a considerable amount of attention over the past few decades. Many beamforming algorithms have been developed. In real applications, circular microphone arrays (CMAs) are often used due to their steering ability. One type of CMAs, i.e., circular differential microphone arrays (CDMAs), have been shown to be particularly useful in speech and audio applications since they can form frequency-invariant beampatterns and attain high directivity factors. However, CDMAs may suffer from the so-called deep-null problem, which is more serious at high frequencies. This has become a limiting factor that restricts the application of CDMAs in practical systems. Recently, a robust beamforming algorithm was developed with the use of concentric CDMAs (CCDMAs). It can deal with the deep-null problem while achieving good performance over the frequency range of interest. But the resulting beampattern can only be steered to a limited number of directions. Moreover, with the existing methods, the geometry of the CCDMA must satisfy perfect symmetry, i.e., 1) the number of microphones in outer rings must be integral multiples of the number of microphones in inner rings; and 2) microphones in different rings need to be aligned. To deal with the aforementioned limitations, we develop a frequency-invariant beamforming algorithm with CCMAs based on the Jacobi-Anger series expansions. This method offers the following advantages in comparison with existing methods: 1) it has full steering flexibility, i.e., the directivity pattern can be steered to any directions in the plane in which the sensors are located; 2) it can mitigate the deep-null problem; and 3) it does not require alignment of microphones in different rings, which is very useful in practical applications, especially when CCMAs with small and compact apertures are used.

2. SIGNAL MODEL, PROBLEM FORMULATION, AND PERFORMANCE MEASURES

Considering a farfield source signal (plane wave), that propagates in an anechoic acoustic environment at the speed of sound, i.e., \( c = 340 \text{ m/s} \), and impinges on a CMA composed of \( n \) microphones. Without loss of generality, we assume that all the sensors are in the horizontal plane, centered inside the circular microphone array, which coincides with the origin of the Cartesian coordinate system. Azimuthal angles are measured anticlockwise from the positive direction of axis, and sensor 1 of the array is placed on the positive side of the \( x \) axis. The direction of the source signal to the array is parameterized by the azimuth angle, \( \theta \). In this scenario, the steering vector of length \( M \), where \( M = \sum_{p=1}^{P} M_p \) is the total number of microphones, is defined as [1,26]

\[
\mathbf{d}(\omega, \theta) = \left[ d_{1}^T(\omega, \theta) \ d_{2}^T(\omega, \theta) \ \cdots \ d_{P}^T(\omega, \theta) \right]^T,
\]

where the superscript \( ^T \) is the transpose operator,

\[
\mathbf{d}_p(\omega, \theta) = \left[ e^{j2\pi \frac{\omega \tau_p}{c} \cos (\theta - \psi_{p,1})} e^{j2\pi \frac{\omega \tau_p}{c} \cos (\theta - \psi_{p,2})} \ \cdots \ e^{j2\pi \frac{\omega \tau_p}{c} \cos (\theta - \psi_{p,M_p})} \right]^T
\]

is the \( p \)th ring's steering vector, \( j \) is the imaginary unit with \( j^2 = -1 \), \( \tau_p = \omega r_p/c \), with \( \omega = 2\pi f \) being the angular frequency and \( f > 0 \) being the temporal frequency, and

\[
\psi_{p,m} = \frac{2\pi (m-1)}{M_p}
\]

is the angular position of the \( m \)th \((m = 1, 2, \ldots, M_p) \) element on the \( p \)th \((p = 1, 2, \ldots, P) \) ring. In order to avoid spatial aliasing, it is necessary that the interelement spacing is less than half of the acoustic wavelength. In this paper, we consider fixed beamformers with small values of the interelement spacing, so that this condition easily holds [3,5].

Beamforming aims at recovering a signal of interest (also called the desired signal) from the noisy observation vector. In this paper, we consider the general case where the desired signal comes based on the Jacobi-Anger series expansions [17]. This method offers the following advantages in comparison with existing methods: 1) it has full steering flexibility, i.e., the directivity pattern can be steered to any directions in the plane in which the sensors are located; 2) it can mitigate the deep-null problem; and 3) it does not require alignment of microphones in different rings, which is very useful in practical applications, especially when CCMAs with small and compact apertures are used.

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from the direction $\theta_s$ and the corresponding propagation vector is $\mathbf{d}(\omega, \theta_s)$. Then, our objective is to design a desired, frequency-invariant beampattern with its main beam pointing to the direction $\theta_s$. To do that, a complex weight, $H_{p,m}(\omega)$, is applied to the output of the $m$th sensor on the $p$th ring, where the superscript $*$ denotes complex conjugation. The weighted outputs are then summed together to form the beamformer’s output. The weights can be put together into a vector of length $M$ as

$$\mathbf{h}(\omega) = \left[ h_{1}^{T}(\omega) \quad h_{2}^{T}(\omega) \quad \cdots \quad h_{M}^{T}(\omega) \right]^T,$$

(4)

where

$$h_{p}(\omega) = \left[ H_{p,1}(\omega) \quad H_{p,2}(\omega) \quad \cdots \quad H_{p,M}(\omega) \right]^T$$

(5)

is the weighting vector on the $p$th ring.

To let the source signal pass through the beamformer without distortion, the distortionless constraint in the desired direction is needed, i.e.,

$$\mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta_s) = 1,$$

(6)

where the superscript $H$ is the conjugate-transpose operator.

Now, we give some useful measures, i.e., the beampattern (or directivity pattern), the directivity factor (DF), and the white noise gain (WNG), to evaluate the performance of the proposed beamformer.

The beampattern describes the sensitivity of the fixed beamformer to a plane wave impinging on the CCMA from the direction $\theta$ [5]. Mathematically, it is defined as

$$\mathcal{B}(\mathbf{h}(\omega), \theta) = \mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta)$$

(7)

$$= \sum_{p=1}^{P} \sum_{m=1}^{M_p} H_{p,m}^{*}(\omega) e^{j \omega \psi_{p,m}} \cos \left( \theta - \psi_{p,m} \right).$$

The DF quantifies the ability of the beamformer in suppressing spatial noise from directions other than the look direction. It is written as [3, 5]

$$\mathcal{D}[\mathbf{h}(\omega)] = \left| \frac{\mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta_s)}{\mathbf{h}^H(\omega) \mathbf{\Gamma}_d(\omega) \mathbf{h}(\omega)} \right|^2,$$

(8)

where $\mathbf{\Gamma}_d(\omega)$ is the pseudo-coherence matrix of the noise signal in a diffuse (spherically isotropic) noise field, and the $(i,j)$th element of $\mathbf{\Gamma}_d(\omega)$ is

$$[\mathbf{\Gamma}_d(\omega)]_{ij} = \sin \left( \frac{\omega \delta_{ij}}{c} \right),$$

(9)

with $\delta_{ij} = \| \mathbf{r}_i - \mathbf{r}_j \|_2$ being the distance between microphone $i$ and $j$, $\| \cdot \|_2$ being the Euclidean norm, $\mathbf{r}_i, \mathbf{r}_j \in \{ \mathbf{r}_{1,1}, \mathbf{r}_{1,2}, \ldots, \mathbf{r}_{p,M}, \ldots, \mathbf{r}_{p,M_p} \}$, and $\mathbf{r}_{p,m}$ is the coordinates of the $m$th microphone on the $p$th ring.

The WNG evaluates the sensitivity of a beamformer to some of its imperfections. It can be written as [3]

$$\mathcal{W}[\mathbf{h}(\omega)] = \left| \frac{\mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta_s)}{\mathbf{h}^H(\omega) \mathbf{h}(\omega)} \right|^2.$$  

(10)

3. DESIRED BEAMPATTERNS

In our context, the objective of beamforming is to find a proper beamforming filter, $\mathbf{h}(\omega)$, so that its beampattern is as close as possible to a desired (target) frequency-invariant beampattern. In acoustic and speech applications, we generally use the frequency-invariant beampatterns of different orders that were developed in the context of differential microphone arrays (DMAs) as the desired beampatterns [2, 4, 5]. As shown in [5], an $N$th-order frequency-invariant beampattern with its main beam pointing to the direction of $0^\circ$ is given by

$$\mathcal{B}(\mathbf{a}_N, \theta) = \sum_{n=0}^{N} a_{N,n} \cos (n \theta) = \mathbf{a}_N^T \mathbf{p}_c(\theta),$$

(11)

where $a_{N,n}$, $n = 0, 1, \ldots, N$, are real coefficients, and

$$\mathbf{a}_N = \left[ a_{N,0} a_{N,1} \cdots a_{N,N} \right]^T,$$

$$\mathbf{p}_c(\theta) = \left[ \cos \theta \cdots \cos (N \theta) \right]^T.$$

It can be checked that $\mathcal{B}(\mathbf{a}_N, \theta)$ is symmetric about the axis $0 \leftrightarrow \pi$. The values of the coefficients $a_{N,n}$, $n = 0, 1, \ldots, N$, in (11) determine the shape of the directivity pattern as well as the corresponding DF. In the direction of the main beam, i.e., $\theta = 0^\circ$, the directivity pattern should be equal to 1, i.e., $\mathcal{B}(\mathbf{a}_N, 0^\circ) = 1$. Therefore, we have

$$\sum_{n=0}^{N} a_{N,n} = 1.$$  

(12)

We write the directivity pattern corresponding to a steering angle $\theta_s$ as [17]:

$$\mathcal{B}(\mathbf{b}_{2N}, \theta - \theta_s) = \sum_{n=-N}^{N} b_{2N,n} e^{j \omega (\theta - \theta_s)}$$

(13)

$$= \left[ \mathbf{Y}(\theta_s) \mathbf{b}_{2N} \right]^T \mathbf{p}_c(\theta)$$

$$= \mathbf{c}_{2N}^T(\theta_s) \mathbf{p}_c(\theta)$$

$$= \mathcal{B}(\mathbf{c}_{2N}(\theta_s), \theta),$$

where

$$\begin{cases}
    b_{2N,0} = a_{N,0}, \\
    b_{2N,i} = b_{2N,-i} = \frac{1}{2}a_{N,i}, \quad i = 1, 2, \ldots, N
\end{cases}$$

and

$$\mathbf{Y}(\theta_s) = \text{diag} \left( e^{j N \theta_s}, \ldots, 1, \ldots, e^{-j N \theta_s} \right),$$

$$\mathbf{b}_{2N} = \left[ b_{2N,-N} \cdots b_{2N,0} \cdots b_{2N,N} \right]^T,$$

$$\mathbf{p}_c(\theta) = \left[ e^{-j N \theta} \cdots 1 \cdots e^{j N \theta} \right]^T,$$

$$\mathbf{c}_{2N}(\theta_s) = \mathbf{Y}(\theta_s) \mathbf{b}_{2N}$$

$$= \left[ c_{2N,-N}(\theta_s) \cdots c_{2N,0}(\theta_s) \cdots c_{2N,N}(\theta_s) \right]^T.$$

Clearly, the main beam of (13) points in the direction $\theta_s$ and $\mathcal{B}(\mathbf{b}_{2N}, \theta - \theta_s)$ is symmetric about the axis $\theta_s \leftrightarrow \theta_s + \pi$. From (13), it is clearly seen that a rotation of the directivity pattern corresponds to a simple modification of its coefficients.
4. BEAMPATTERN DESIGN

In this section, we show how to derive the beamforming filter, \( h(\omega) \), such that the designed beampattern, \( \mathcal{B}_1(\omega) \), approaches the desired symmetric directivity pattern, \( \mathcal{B}_0(\theta) \).

It has been shown that the optimal approximation of the beamformer’s beampattern with a CMA from a least-squares error (LSE) perspective is the Jacobi-Anger expansion [17]. Following this principle, the Jacobi-Anger expansions of the exponential function in the beamformer’s beampattern with a CCMA is [27]

\[
e^{j\omega p \cos(\theta - \psi_{p,m})} = \sum_{n=-\infty}^{\infty} j^n J_n(\pi \psi_{p,m}) e^{jn(\theta - \psi_{p,m})},
\]

where \( J_n(\pi \psi_{p,m}) \) is the \( n \)th-order Bessel function of the first kind with \( J_{-n}(\pi \psi_{p,m}) = (-1)^n J_n(\pi \psi_{p,m}) \). Substituting (14) into the definition of the beamformer’s beampattern with a CMA in (7) and limiting the Jacobi-Anger series to the order \( N \), we obtain

\[
\mathcal{B}_N[h(\omega), \theta] \approx \sum_{p=1}^{P} \sum_{m=1}^{M_p} H_{p,m}^*(\omega) \sum_{n=-N}^{N} j^n J_n(\pi \psi_{p,m}) e^{jn(\theta - \psi_{p,m})}
\]

\[
= \sum_{n=-N}^{N} e^{jn\theta} \sum_{p=1}^{P} \sum_{m=1}^{M_p} J_n(\pi \psi_{p,m}) e^{-jn\psi_{p,m}} H_{p,m}^*(\omega)
\]

\[
= \sum_{n=-N}^{N} e^{jn\theta} c_{2N,n}(\theta_a),
\]

(15)

This gives the relation between the beamformer’s beampattern with a CCMA and the \( N \)th order desired directivity pattern. Based upon this relationship, the beamforming filter, \( h(\omega) \), can be obtained through solving a linear equation constructed from the optimal approximation of the beampattern with the Jacobi-Anger series expansions. By writing (16) in a vector form, we have

\[
j^n \psi_n^T(\omega) h^T(\omega) = c_{2N,n}(\theta_a),
\]

(17)

where

\[
\psi_n(\omega) = \begin{bmatrix} J_n(\pi \psi_{1,1}) \psi_{n,1}^T \ J_n(\pi \psi_{1,2}) \psi_{n,2}^T \\ \vdots \ J_n(\pi \psi_{p,m}) \psi_{n,m}^T \end{bmatrix}^T
\]

(18)

is a vector of length \( M_p \) with \( n = \pm 1, \ldots, \pm N \),

\[
\psi_{n,p} = \begin{bmatrix} e^{-jn\psi_{p,1}} & e^{-jn\psi_{p,2}} & \cdots & e^{-jn\psi_{p,M_p}} \end{bmatrix}^T,
\]

(19)

and \( p = 1, 2, \ldots, P \).

From (17), we obtain the following equation:

\[
\Psi(\omega) h(\omega) = J^* \mathbf{Y}^T(\theta_a) \mathbf{b}_{2N},
\]

(20)

where

\[
\mathbf{J} = \text{diag} \left[ \frac{1}{j^{-N}}, \ldots, \frac{1}{j^N} \right]
\]

(21)

5. SIMULATIONS

In this section, we study the performance of the developed beamforming algorithm for the design of the first-order hypercardioid [5].
Microphone array beamforming has long been a very important problem in acoustic, speech, and audio signal processing and many beamforming algorithms have been developed over the last few decades, such as the delay-and-sum, filter-and-sum, superdirective, and differential beamformers [10, 12, 31–33]. Among those, beamformers with CDMAs have attracted much R&D interest for their frequency-invariant response and steering flexibility [21, 23–25]. However, conventional beamformers with CDMAs, e.g., algorithms in [21], suffers from two problems: 1) the deep-null problem in both DF and WNG (so the beamformer cannot perform consistently and robustly at different frequencies), and 2) limited steering ability (so the beampattern, without any changes, can only be steered to a limited number of directions). To deal with the first problem, we studied in [17] beamforming with CCMAs. The resulting beamformers are free from deep nulls in either DF or WNG but they lack steering flexibility. To deal with the second problem, we recently developed a new approach with CDMAs in [26]. The resulting beamformer has full steering flexibility, i.e., the beampattern can be perfectly steered to any wanted direction in the plane where the sensors are located. Moreover, the developed method does not require to align the microphones in different rings, which is convenient and useful in real applications, particularly when arrays with small and compact apertures have to be used. Simulation results demonstrated the advantage of this proposed algorithm over conventional frequency-invariant beamformers with either CMAs or CCMAs.

7. RELATION TO PRIOR WORK

In this paper, we studied the problem of designing frequency-invariant beamformers with CCMAs. We proposed an FIB-CCMA algorithm based on an optimal approximation of the beampattern with the Jacobi-Anger series expansions. The developed beamformer can mitigate the deep nulls problem as compared to existing methods with CDMAs and the deduced beampattern can be steered to any wanted direction in the plane where the sensors are located. Moreover, the developed method does not require to align the microphones in different rings, which is convenient and useful in real applications, particularly when arrays with small and compact apertures have to be used. Simulation results demonstrated the advantage of this proposed algorithm over conventional frequency-invariant beamformers with either CMAs or CCMAs.

6. CONCLUSIONS

In this paper, we studied the problem of designing frequency-invariant beamformers with CCMAs. We proposed an FIB-CCMA algorithm based on an optimal approximation of the beampattern with the Jacobi-Anger series expansions. The developed beamformer can mitigate the deep nulls problem as compared to existing methods with CDMAs and the deduced beampattern can be steered to any wanted direction in the plane where the sensors are located. Moreover, the developed method does not require alignment of microphones in different rings of the CCMA.
8. REFERENCES


