INDIVIDUAL DIFFERENCE OF ULTRASONIC TRANSDUCERS FOR PARAMETRIC ARRAY LOUDSPEAKER

Shota Minami, Jun Kuroda, and Yasuhiro Oikawa

Department of Intermedia Art and Science, Waseda University, Tokyo, Japan

ABSTRACT
A parametric array loudspeaker (PAL) consists of a lot of ultrasonic transducers in most cases and is driven by an ultrasonic which is modulated by audible sound. Because each ultrasonic transducer has different frequency resonant frequency, there is the individual difference in ultrasonic transducers of a PAL in a manufacturing process. In this paper, two PALs are made of each set of transducers with large and small variance of resonant frequencies. Quality factor of PAL with the large variance of resonant frequencies is smaller than that of PAL with small variance, and the demodulated audible sound pressure level (SPL) is large and almost flat to 3 kHz in PAL with the large variance of resonant frequencies.

Index Terms— SSB Modulation, Resonant Frequency, Ultrasonic Transducer

1. INTRODUCTION
Two large amplitude sound waves in the air which have neighboring frequency interact with each other, and then generate each harmonic sound, the sum tones and the difference tones during propagation by the nonlinearity of air. This phenomenon was discovered by Westervelt in 1963 [1]. Yoneyama proposed an application of this phenomenon to a new type of loudspeaker design, which is now called a parametric array loudspeaker (PAL), in 1983 [2]. PAL is constructed of a lot of ultrasonic transducers in most cases and driven by ultrasonic sound which is modulated by audible sound. PAL can make a sharp directivity of audible sound beam and also realize the audio spotlight in the field. It has a wide range of the application [3–7].

PAL, however, has several problems, low effectivity of demodulation, sound quality in low frequency, etc. For resolving these problems, a number of studies were conducted. For example, the method of modulation was considered for the improvement of sound quality [8–12], the methods for simulating the propagation were conducted [13, 14] and various transducers were designed by many researchers [15–19]. In addition, there was a research which includes the method of arrangement of transducers as an array [20].

They assume the same characteristics of all transducers in a PAL and do not consider the individual difference in ultrasonic transducers. It is important to research the influence of the individual difference in ultrasonic transducers for driving PAL to ultrasonic and audible sound in industrial applications. This paper focuses on the individual difference of resonant frequency of transducers and describes the influences to PAL from the transducers’ individual differences.

2. THEORY

2.1. Theory of finite-amplitude sound
A large amplitude sound wave is called a finite-amplitude sound wave. The interaction between two finite-amplitude sound waves (which are called primary sound) with nearby frequency makes harmonics of each sound, and sum / difference frequency sound waves (which are called secondary sound). For example, two finite-amplitude sound of 40 and 41 kHz make a difference tone of 1 kHz which is audible sound. PAL is a loudspeaker based on this phenomenon.

The discussion about this phenomenon is usually started from the Khokhlov-Zabolotskaya-Kuznetsov(KZK)-equation [21]:

\[ \frac{\partial^2 p}{\partial t^2} - \frac{c_0}{2} \nabla^2 p - \frac{\delta}{2c_0} \frac{\partial^3 p}{\partial t^3} = \frac{\beta}{2 \rho_0 c_0^3} \frac{\partial^2 p^2}{\partial t^2}, \]

where \( z \) is the direction of propagation, \( p \) is sound pressure, \( c_0 \) is sound speed, \( \rho_0 \) is density, \( \beta \) is nonlinearity coefficient, \( t' = t - z/c_0 \) is a time delay caused by the sound traveling and \( \nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2 \) is the Laplacian.

From this theory, secondary sound \( p_s (t) \) is radiated, and

\[ p_s (t) = \frac{\beta S}{16\pi \rho_0 c_0^3 \alpha z} \frac{\partial^2}{\partial t^2} p_i^2 (t), \]

where \( p_i \) is sound pressure of primary sound, \( \alpha \) is absorption coefficient of primary sound, \( S \) is the area of loudspeaker array, and \( z \) is distance from the loudspeaker array to a measurement point. In this equation, \( p_i^2 (t) \) is the square of primary sound, which indicates that secondary sound depends on the sound pressure of the primary sound, and \( \partial^2 / \partial t^2 \) means the frequency of primary sound influences the sound pressure of secondary sound.

2.2. Influences by individual difference of transducers
Piezoelectric transducers are used for PAL to generate ultrasonic. For generating large ultrasonic sound pressure, many of these transducers are connected in parallel and placed as an array. For driving PAL, the amplitude-modulation (AM) is
used and a modulated signal is radiated as primary sound from the array. The frequency of carrier signal is selected as same as a resonant frequency of the piezoelectric transducer. Each transducer, however, has individual differences due to several factors in the manufacturing process. This means each transducer used in PAL has a different resonant frequency, therefore the driving of PAL may not follow the theories above. Let $Y_n(f)$ be the admittance of the $n$-th transducer in an array, and $f_n = \arg \max(Y_n(f))$. Assuming that all $f_n$ are same, each $Y_n(f)$ has the same maximum value $Y_{max}$ and the same mechanical quality factor. The quality factor is calculated by

$$Q = \frac{f_0}{\nu}$$

where $f_0$ is the resonant frequency, $\nu$ and $f_i$ are the upper / lower frequencies at which the conductance is half of that at $f_0$. The admittance of the array $Y(f)$ is

$$Y(f) = \sum_{n=1}^{N} Y_n(f) = NY_n(f),$$

where $N$ is the number of transducers. Assuming that $f_a$ follows the normal distribution, since each $Y_n(f)$ has different value at the same frequency $f$, $Y(f)$ does not have same characteristic as that of each transducer $Y_n(f)$. The radiated sound has the same frequency characteristic as $Y(f)$.

Figure 1 shows the example in which the resonant frequency of each transducer influences that of an array in the case $N = 2$. The blue line demonstrates if $f_1 = f_2$, and the red line demonstrates if $f_1 \neq f_2$. The left and right figure show the resonant characteristics of each transducer and the array, respectively. Both arrays have the different amplitude at resonant frequency $f_0$. If $f_1 = f_2$, $\max(Y(f))$ is $2Y_{max}$, however if $f_1 \neq f_2$, $\max(Y(f))$ is $Y_{max}$. In manufacturing PAL, a lot of transducers are used as an array. The resonance of the array of PAL is not always sharp.

From Eq. (2), the demodulated sound in the frequency domain is

$$P_s(f) = \frac{\beta S}{16\pi \rho_0 c_0^3 \alpha z} (2\pi f)^2 \times P_i(f_0) \times P_i(f_0 \pm f)$$

$$\propto f^2 \times P_i(f_0) \times P_i(f_0 \pm f),$$

where $f$ is the frequency of self-demodulated audible sound, and $f_0$ is the frequency of carrier. The sound pressure of self-demodulated audible sound is calculated by the multiplication of two sound pressure, which are the sound pressure of carrier wave and that of sideband one. At the resonant frequency of the array $f_0$, the amplitude of blue line is larger than that of red line, while at $f_{side}$ in the sideband, the amplitude of blue line is smaller than that of red line as shown in Fig. 1. In this situation, it is considered the array that consists of transducers which have individual differences can radiate the larger sound in the sideband, which includes the information of audible sound.

### Table 1. Information about arrays.

<table>
<thead>
<tr>
<th>Transducer</th>
<th>Array1</th>
<th>Array2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Center Frequency [kHz]</td>
<td>41.0</td>
<td>40.7</td>
</tr>
<tr>
<td>Variance Value</td>
<td>$2.32 \times 10^4$</td>
<td>$1.34 \times 10^6$</td>
</tr>
</tbody>
</table>

### Table 2. Admittance measurement conditions.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>HIOKI IM3570 Impedance Analyzer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Range [kHz]</td>
<td>From 35 to 45</td>
</tr>
<tr>
<td>Input Voltage [V]</td>
<td>1</td>
</tr>
</tbody>
</table>

### 3. EXPERIMENTS AND RESULTS

#### 3.1. Array construction

Two arrays which consist of 50 transducers are compared. One is a collection of transducers with small variance (Array1), and the other is an array of transducers with large variance (Array2). One hundred transducers used in these two arrays were chosen from the same kind of 450 transducers collected in order to measure the variation of same manufacturing. Figure 2 shows the histogram of the resonant frequency of all transducers, Array1 and Array2. In addition, Fig.3 shows the admittance of each array. Variance influences the electrical frequency response of the arrays. The information of arrays is shown in Table 1 and measurement conditions are shown in Table 2.

#### 3.2. Frequency response of PAL

Figure 4 shows the frequency characteristics of sound pressure level (SPL) of each array in ultrasonic, and experimental conditions are shown in Table 3. Table 4 shows the mechanical quality factors of Arrays 1 and 2. The quality factor is calculated as same as Eq. (3).

Arrays 1 and 2 have different frequency response, different frequency of peak, and different quality factor. Array1 can make the large SPL of ultrasonic carrier for PAL efficiently.

#### 3.3. Radiated ultrasonic

Figure 5 shows the SPL of radiated ultrasonic from PAL. $f_0$ is the resonant frequency. In the lower frequency, the SPL of Array1 is larger than that of Array2, and frequency response of Array2 is flat. In the middle frequency, the SPL of Array2 is larger than that of Array1. In the higher frequency band above

### Table 3. Radiated ultrasonic measurement conditions.

<table>
<thead>
<tr>
<th>Measuring Frequency [kHz]</th>
<th>From 20 to 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage [V rms]</td>
<td>10</td>
</tr>
<tr>
<td>Microphone</td>
<td>B&amp;K 4939-A-011</td>
</tr>
<tr>
<td>Distance [cm]</td>
<td>30</td>
</tr>
</tbody>
</table>
Fig. 1. Resonant frequencies of two transducers. When the resonant frequencies of the two transducers are exactly same (blue lines), the pair of the two has sharp resonance, while the resonance frequencies are a little different (red lines), the pair does not have sharp resonance. The amplitude of the array with same resonance is larger than that of the array with a little different resonance at the resonant frequency. On the other hand, the amplitude of the array with same resonance is smaller than that of the array with a little different resonance at the sideband frequency.

Fig. 2. Histogram of resonant frequency. Gray one shows all transducers. Blue and red ones correspond to the transducers chosen as Arrays 1 and 2, respectively.

6 kHz, the magnitudes of two arrays are almost equal. This is due to the individual transducers’ frequency responses. From the point of view of the flattening the frequency response, Array2 is more suitable than Array1.

3.4. Demodulated sound
The ultrasonic signal is modulated by lower sideband (LSB) modulation. LSB modulation is one kind of single sideband (SSB) modulation. Using Hilbert transform $\mathcal{H}$, the LSB modulated signal $S_{LSB}$ is expressed as,

$$S_{LSB} = \text{Re} \left[ (s(t) + i\mathcal{H}s(t)) \exp(-i\omega_0 t) \right],$$

where $s(t)$ is audible signal, $\omega_0 = 2\pi f_0$ is angular frequency of carrier, $i = \sqrt{-1}$, and $\text{Re} \{Z\}$ is real part of $Z$. SSB modulation method is useful, because the method can reduce the frequency band. As mentioned in Section 2.2, the array used in PAL does not usually have wide frequency band. Therefore, SSB modulation is useful to drive PAL [22, 23]. The experiment conditions are shown in Table 5. The carrier frequency is 41.0 kHz in Array1 and 40.7 kHz in Array2.

Figure 3 shows that the peak of conductance of Array2 is approximately half of Array1, and Fig. 4 shows that SPL of Array2 is almost 6 dB lower than that of Array1. These figures show that the linearity between the input voltage and SPL of the radiated sound holds around the resonant frequency.

Figure 6 shows the frequency response of demodulated sound of LSB modulation signal, of which modulation coefficient $m$ is 0.5 and input voltage is 10 Vrms. Array2 has flatter frequency response than Array1. The SPLs of demodulated sound for Array1 and Array2 are 72 dB and 77 dB at 1 kHz, and 70 dB and 77 dB at 2.5 kHz, respectively. This result shows that the audible SPL of Array2 is 5–7 dB larger than that of Array1. This is caused by the flat response of Array2, whose frequency band is wider than Array1, and then the radiated ultrasonic used as sideband sound in Array2 is

<table>
<thead>
<tr>
<th>Table 4. Values regarding arrays’ resonance.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-Width [kHz]</td>
</tr>
<tr>
<td>Frequency of Peak [kHz]</td>
</tr>
<tr>
<td>Quality Factor</td>
</tr>
</tbody>
</table>

Fig. 3. Admittance of each array. Array1 has larger quality factor than Array2.
Table 5. Measurement conditions of sound pressure level of demodulate sound.

<table>
<thead>
<tr>
<th>Audible Signal</th>
<th>0 – 10 kHz chirp signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage [Vrms]</td>
<td>10</td>
</tr>
<tr>
<td>Modulation Coefficient</td>
<td>0.1, 0.2, ..., 1.0</td>
</tr>
<tr>
<td>Modulation Signal</td>
<td>LSB</td>
</tr>
<tr>
<td>Microphone</td>
<td>RION NL-32 Sound Level Meter</td>
</tr>
<tr>
<td>Distance [cm]</td>
<td>30</td>
</tr>
</tbody>
</table>

Fig. 4. Sound pressure level of each array.

Fig. 5. Sound pressure level of ultrasonic when radiating modulated sound. The modulation coefficient \( m \) and input voltage were set to 0.5 and 10 Vrms, respectively.

more than that of Array1. As is expressed in Eq. (5), the audible sound pressure is determined by the multiplication of the carrier and sideband sound pressure. It is considered that Array2 can generate demodulated sound from ultrasonic with better efficiency.

However, it should be considered more carefully that SPL of audible sound in all frequency measured in Array2 is larger than that of Array1. If PAL’s audible sound pressure follows the theory expressed in Section 2.2, it is easily expected that the SPL at the lower frequency band in Array1 is larger than that of Array2 because the SPL at resonant frequency in Array1 is larger than that of Array2, and SPL of the sideband ultrasonic close to the resonant frequency in Array1 is also larger than that in Array2 as shown in Fig. 5. One cause of this result is the difference of demodulated sound at the position close to each array. From the theory by Westervelt [1, 2], the carrier and sideband sound decay while the demodulated sound is generated and accumulated in phase from the array to far-field. The theory says that a demodulated sound accumulates according to a transmission beam of ultrasonic. In addition, the longer the distance is, the smaller the accumulation becomes because of ultrasonic decay by propagation. Therefore, it is considered that the SPL of initial demodulated sound, which is generated close to an array, could be more important, and Array2 has larger demodulated SPL at the neighborhood of the array than Array1 because of these individual differences. For analyzing this phenomenon, it is necessary to measure the modulated ultrasonic and the demodulated sound at the neighborhood of the array.

4. CONCLUSION

In this paper, we studied the effect of individual characteristics of the transducer to the driving of PAL. We fabricated two kinds of array for PAL with small and large variance of transducers and evaluated the demodulated audible sound and radiated ultrasonic. An array with small variance resonant frequency has large SPL of ultrasonic carrier, but extremely small SPL of sideband. On the other hand, an array with large variance has a small carrier and sideband which is an almost same level as carrier. This leads to less ultrasonic in driving PAL. In order to decrease ultrasonic and obtain flat frequency responses of audible sound, an array which has a large variance of the resonant frequency is better suited. Future works include calculating an array’s characteristic considering the placement of transducers, considering other parameters when choosing transducers used in arrays. In addition, future works include simulating and measuring the influences to the initial demodulated sound by the individual differences of transducers close to the array.
5. REFERENCES


