PHYSICAL OBJECT AUTHENTICATION:
DETECTION-THEORETIC COMPARISON OF NATURAL AND ARTIFICIAL RANDOMNESS

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ABSTRACT

In this paper, we compare two methods that can be used by the anti-counterfeiting industry to protect physical objects, which are either based on an object’s natural randomness or on artificial randomness embedded on the object. We show that the considered verification architectures rely either on a comparison between an enrolled fingerprint and an extracted one or between a tag and a fingerprint. We compare these setups from detection-theoretic perspectives for both types of architectures. Authentication performance using false and miss error probabilities of the two systems are analysed and then compared using two practical setups. We highlight the advantages and limitations of each architecture. These theoretical results derived for binary fingerprints are useful to construct and optimise practical methods and to help select the appropriate architecture.

1. INTRODUCTION

Anti-counterfeiting of physical objects based on digital solutions has attracted a lot of attention in the last years. This interest is caused by the urgency for technologies to deal with modern counterfeiting, which implies having access to easy, fast, reliable and user-friendly verification of objects authenticity. Moreover, objects authentication on consumer mobile phones de-facto becomes a standard solution for brand protection. In this respect, there is significant interest in cheap and simple secure methods, which are suitable for protection of various objects such as pharmaceutical products (including both drugs and its packaging), cosmetics, food, luxury goods, spare parts, as well as medical equipment, components and implants.

Recent technologies that address the above demands are based on physical uncloneable properties, a.k.a. physical uncloneable features (PUF), that are easy to verify but difficult to clone, generally referred to as a randomness. As a matter of fact, optically visible microscopic features which exhibit random features are of special interest for mobile verification since the advent of high definition of modern imaging sensors.

Depending on the origin of randomness the antounterfeiting technologies can be divided into natural randomness, when the system exploits the randomness inherently created by nature, and artificial randomness, when the randomness is created on purpose. Natural randomness systems typically use object surface images such as shown in Figure 1a whereas artificial randomness systems are based on various uncontrolled effects occurring during marking (printing, embossing, laser engraving, etc.) of random-like structures such as graphical codes (GC) as shown in Figures 1b-1c.

Several papers address the practical and theoretical performance and security of natural randomness [1, 2] and artificial randomness [3–7] systems. However, to our best knowledge, there is little if no work that compares the theoretical performance of these systems using the same assumptions behind the statistical models of randomness and acquisition/clonning processes. Therefore, the goal of this paper is to compare the two systems and to analyse the potential advantages of each system.

The paper is organised as follows. The problem formulation as well as the statistical models behind the natural randomness and artificial randomness systems are introduced in Section 2, and section 3 summarises the main detection-theoretic results. The comparison between the two practical setups is given in Section 4.

Notation. We use capital letters to denote scalar random variables $X$, bold capital letters to denote vector random variables $\mathbf{X}$, corresponding small letters $x$ and $\mathbf{x}$ to denote the realizations of scalar and vector random variables, respectively, i.e., $\mathbf{x} = (x_1, x_2, \ldots, x_N)^T$. We use $X \sim p(x)$ to indicate that a random variable $X$ follows $p_X(x)$. The sign $\ast$ denotes the convolution of probabilities $p \ast q = p(1 - q) + q(1 - p)$.

2. PROBLEM FORMULATION: STATISTICAL MODELS

We present here the models associated with authentication using either natural or artificial randomness.

The block diagram for systems based on natural randomness is shown in Figure 2. At the enrollment stage, the microstructure of object $w$ corresponding to object index $w \in \{1, 2, \ldots, M\}$ is acquired by a device (e.g. a camera) resulting in the image $\mathbf{x}(w) \in \mathbb{R}^N$ and the extracted fingerprint $f(w) \in \{0, 1\}^n$ which is stored in the database for future authentication. The helper data are given in the form of a tag $w$ pointing to the fingerprint $f(w)$ stored in the database. In this paper, we consider only helper data in a form of a
pointer to the concerned fingerprint. However, other forms of helper data encoding developed in biometric applications are possible and the reader is referred to [8,9] for more details.

The authentication consists in: (i) acquisition of an object microstructure by the acquisition device resulting in image $y$, (ii) computation of binary $f_e \in \{0,1\}^n$ or soft $f_e \in \mathbb{R}^n$ fingerprint and (iii) making a binary decision by comparing $f_e$ to $f_e(w)$. Because of this last step, we will refer to these systems as fingerprint-to-fingerprint (F2F) architectures.

We will assume that the F2F setup can be modeled as memoryless source and the acquisitions are modeled by memoryless channels. The authentication model can be represented by a chain in the form of a probabilistic graphical model $F_e \leftarrow X \leftarrow O \rightarrow Y \rightarrow F_o$ or by the joint distribution $p(f_e, x, o, y, f_o) = p(o)p(x|o)p(f_e|o)p(f_o|y)g with p(x, y|o) = p(x|o)p(y|o).

The block diagram of systems based on artificial randomness is shown in Figure 3. At the enrollment stage, the object tag $m(w) \in \{0,1\}^n$ is in a form of any random-like modality is generated based on the index $w$ and reproduced on the object surface resulting in object $o(w)$. In the general case, the tag $w$ is known at the verification stage too, it can be added to the object at the enrollment either as a standalone index encoded in any machine readable form or integrated directly into the object tag $m(w)$. Additionally, the object tag $m(w)$ can be generated pseudo-randomly from $w$ or coded from $w$ using error correction codes.

The authentication process consists in (i) the acquisition of the object's microstructure by the capturing device resulting in image $y$, (ii) extraction of the tag estimate $\hat{f}_e \in \{0,1\}^n$ or soft $\hat{f}_e \in \mathbb{R}^n$ and (iii) making a binary decision by comparing $\hat{f}_e$ to $m(w)$, or decoding $\hat{w}$ based on $\hat{f}_e$ and comparing the obtained estimate $\hat{w}$ to the claimed object tag $w$. We will refer to such artificial randomness systems as tag-to-fingerprint (T2F) architectures.

We assume that tag statistics are governed by a Bernoulli distribution with parameter $P_r\{M_i = 1\} = \theta_o, 1 \leq i \leq n$. The above T2F setup can be represented by a Markov chain in the form of a probabilistic graphical model $M \rightarrow O \rightarrow Y \rightarrow F_o$ or by the joint distribution $p(m, o, y, f_o) = p(m)p(o|m)p(y|o)p(f_o|y)$.

Note that the physical protection based on the T2F architecture is based on a fact that any attempt of counterfeiter to clone the original object tag will lead to the additional distortions between the digital tag $m(w)$ and its cloned version, that can be detected using an appropriate test (see section 3.2).

3. DETECTION-THEORETIC ANALYSIS

3.1. Statistical models under consideration

This analysis relies on a set of assumptions that can be summarised as follows: (a) the pmf of sources and channels are known and the sources are assumed to be memoryless and binary and the channels to be binary symmetric channel (BSC) models; (b) the synchronization between all enrolled sequences and probe is perfect.

F2F setup (Figure 4a): We consider a hypothetical model of natural randomness generated according to a Bernoulli distribution

\[ p(f_x, f_y) = \theta_o f_x \theta_y f_y \]

with parameter $\theta_o = \Pr\{F_o = 1\}, 1 \leq i \leq n$. Therefore, the statistical model of the source governing all samples is $F_o \sim \text{Bern}(\theta_o)$.

Accordingly, we assume that all corresponding components of the model representing enrolled data $F_e$, probe $F_e$ for the authentic object and $F_e'$ for the fake object, are binary, i.e., with the alphabet $\{0,1\}$.

The enrollment channel is assumed to be represented by an additive modulo-2 channel, i.e., $F_o$ is XORed with independent enrollment noise $F_{z_o}$ resulting into enrolled data $F_o$,

\[ F_o = F_e \oplus F_{z_o} \]

where the noise $F_{z_o} \sim \text{Bern}(P_{z_o})$ and the enrolled fingerprint $F_o \sim \text{Bern}(\theta_o \ast P_{z_o})$. This model also represents the BSC with the cross-over probability $P_{z_o}$.

The authentic verification channel is also assumed to be an additive modulo-2 channel with:

\[ F_o = F_o \oplus F_{z_o} \]

where $F_{z_o} \sim \text{Bern}(P_{z_o})$ and $F_o \sim \text{Bern}(\theta_o \ast P_{z_o})$.

The opponent verification channel corresponds to the case when the opponent tries to clone the original $F_o$ and since the process is noisy:

\[ F_o = F_e \oplus F_{z_e} \oplus F_{z_o} \]

3The statistics of extracted binary fingerprint are determined by the parameter $\theta_o$. In some cases, some data independent or data-dependent (learned) transform can be applied to the image $x(w)$ with a following binarization to ensure the desirable value $\theta_o$ [12,13], for example $\theta_o = 0.5$.  

\[ \begin{array}{c}
\text{Object tag } w \\
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\text{Object tag } w \\
\text{Object tag } w \\
\end{array} \]
where \( F_{Z_v} \sim \text{Bern}(P_{b_v}) \) cloning to object and \( F_y' \sim \text{Bern}(\theta_m * P_{b_p} * P_{b_v}) \).

**T2F setup** (Figure 4b): The statistical model of the source (the mark here) is also assumed to be generated as \( M(w) \in \{0, 1\}^n \) with \( \Pr[M_i = 1] = \theta_m, \) i.e., it is generated from the Bernoulli pmf as \( M_i \sim \text{Bern}(\theta_m) \). The marking process is modeled as a BSC with transition probability \( \Pr[O_i \neq M_i] = P_{b_m} \). Thus the marked object is modeled as: \( F_{o} \sim \text{Bern}(\theta_m * P_{b_p}) \). The reproduction process can be equivalently represented by the additive modulo-2 channel:

\[
F_{o} = M \oplus F_{Z_p},
\]

where \( F_{Z_p} \sim \text{Bern}(P_{b_p}) \) stands for the marking noise.

The authentic verification channel is also memoryless where for each element:

\[
F_{i} = M \oplus F_{Z_p} \oplus F_{Z_v},
\]

where \( F_{Z_v} \sim \text{Bern}(P_{b_v}) \) and \( F_{i} \sim \text{Bern}(\theta_m * P_{b_p} * P_{b_v}) \).

The verification channel for the opponent is represented by:

\[
F_{y} = M \oplus F_{Z_p} \oplus F_{Z_v} \oplus F_{Z_e},
\]

where \( F_{Z_e} \sim \text{Bern}(P_{b_e}) \) model the copying process and \( F_{y} \sim \text{Bern}(\theta_m * P_{b_p} * P_{b_e} * P_{b_v}) \).

### 3.2. Hypothesis testing

We measure the system performance using probability of miss \( P_{M} \) and probability of false acceptance \( P_{F} \). Probability of miss \( P_{M} \) corresponds to the error event when authentic object with tag \( w \) is rejected by the authentication system. Probability of false acceptance \( P_{F} \) reflects a probability of falsely accepting any fake item as an authentic one with index \( w \).

It should be pointed out that a fake object presented for authentication might be chosen blindly without any reference to the original object with tag \( w \). However, a fake might also be designed more meticulously using any available information about object \( o(w) \) or its features \( f_i(w) \). We refer to this case as an **informed attack**. The corresponding probability of false acceptance under the informed attack is denoted probability of **successful attack** \( P_{F_A} \). An informed attack is more dangerous than a blind attack since it might generate a fake object whose features are considerably closer to the features of an authentic object. For this reason, we focus on informed attacks here.

We consider this authentication problem as a binary hypothesis testing with hypothesis \( H_0^c \), representing the hypothesis that the presented object is not authentic (fake), and \( H_w \) its authentic counterpart. Moreover, we assume the worst case attack, i.e., an informed attack where the object presented for authentication under hypothesis \( H_0^c \) is reproduced from an authentic object. The distributions under the corresponding hypothesis are [2]:

\[
\begin{align*}
H_0^c : p(f_i|f_i(w), H_0^c) &= P_{b}^{d_H(w)} (1 - P_b)^{n-d_H(w)}, \\
H_w : p(f_i|f_i(w), H_w) &= P_{b}^{d_H(w)} (1 - P_b)^{n-d_H(w)},
\end{align*}
\]

where \( d_H(w) = d_H(f_i, f_i(w)) \). Probability of bit error for the hypothesis \( H_0^c \) characterises the opponent channel \( P_{b}^c = P_{b_m} * P_{b_p} * P_{b_v} \) for F2F, and \( P_{b}^c = P_{b_m} * P_{b_e} * P_{b_v} \) for T2F systems. Hypothesis \( H_w \) corresponds to the case of legitimate channel and is equal to \( P_{b} = P_{b_m} * P_{b_p} * P_{b_v} \) for F2F and \( P_{b} = P_{b_m} * P_{b_e} * P_{b_v} \) for T2F systems. Note that for blind attacks, \( P_{b}^c = 0.5 \) in both cases. We also assume that the fingerprinting scheme in the case of F2F systems is designed to maximise the entropy of the source, i.e., such that \( \theta_0 = 0.5 \) for F2F\(^4\) and \( \theta_m = 0.5 \) for the T2F.

The authentication test is performed based on rule \( d_H(f_i, f_i(w)) \leq \gamma n \), where \( \gamma \) is a threshold relying on \( P_{M} \) and \( P_{F} \).

In this paper, we follow the approach proposed in [2] that considered the performance of authentication systems under the informed attacks. The probability of successful attack is defined as [2]:

\[
P_{F_A}(\gamma) = \Pr\{D_H(w) \leq \gamma n | H_0^c\} \leq \sum_{d_H=0}^{\lfloor \frac{\gamma n}{d_H} \rfloor} \binom{n}{d_H} P_{b}^{d_H}(1 - P_b)^{n-d_H},
\]

and the probability of miss is [2]:

\[
P_M(\gamma) = \Pr\{D_H(w) > \gamma n | H_w\} = \sum_{d_H=\lceil \frac{\gamma n}{d_H} \rceil+1}^{n} \binom{n}{d_H} P_{b}^{d_H}(1 - P_b)^{n-d_H}.
\]

### 4. COMPARISON BETWEEN T2F AND F2F SCHEMES

#### 4.1. Setups and assumptions

The considered models are generic and allow considering different relationships between the parameters of enrollment and verification for both the legitimate user and the opponent. We consider here a conservative scenario assuming that the counterfeiter uses the same marking and acquisition equipment as a legitimate user, as well as

\[^4\text{This can be achieved even for correlated input images by randomly projecting the input image and binary quantizing the resulting projections as shown in [14].}\]
the same fingerprinting extraction algorithms. Other possible scenarios could include: (a) High-Quality clones, when the opponent has the high quality scanning-reproduction equipment w.r.t. the genuine enrollment, and (b) Low-Quality clones, when the situation is an inverse one. More particularly, we assume that: (a) \( P_{th} = P_{th} = P_E \) to be a generic extraction probability of error, (b) \( P_{th} = P_E \) with \( P_C \) to be a probability of reproduction in F2F systems and (c) \( P_{th} = P_E \) with \( P_P \) to be a probability of reproduction/printing in T2F systems. We also set \( \theta_0 = \theta_m = 0.5 \) for the above discussed reasons. The setups for the F2F and T2F authentication methods are depicted in Figures 5a and 5b, respectively.

4.2. Performance of authentication systems

We compute \( P_{SA} \) and \( P_M \) according to (2) and (3) with: (a) F2F parameters \( P_b = P_E \) and \( P'_b = P_E + P_C + P_E \) and (b) T2F parameters \( P_b = P_P + P_E \) and \( P'_b = P_P + P_E \).

The probabilities \( P_M(\gamma) \) and \( P_{SA}(\gamma) \) depend on the selection of threshold \( \gamma \). We simulated the equal-error-rate strategy popular in biometrics, i.e., \( P_M = P_{SA} \equiv P_{EER} \) shown in Figure 6a assuming \( P_P = P_E = 0.1 \) and \( n = 500 \). We have observed the same behavior of plots with the threshold \( \gamma \) selection under the Neyman-Pearson (NP) strategy for the bounded \( P_{SA}(\gamma) \). For both strategies of threshold \( \gamma \) selection, \( P_{EER} \) for the F2F systems is lower of those of T2F systems whenever \( P_E \leq P_P \). For both schemes there may exist an optimal extraction value \( P_E \) that minimizes \( P_{EER} \). Note that the possible existence of an optimal value is due to the fact that: for low \( P_E \), it is easier for the opponent to reproduce an accurate copy; for large \( P_E \), the original and fake fingerprint tend to be equally noisy and are not distinct.

Finally, Figure 6b summarises the behavior of \( P_{EER} \) as a function of \( P_P = P_C \) for \( P_E = 0.1 \). In this scenario we assume that counterfeiting devices are comparable. Accordingly, we set the extraction error to a given value \( P_E = 0.1 \). It is interesting to note that the behaviour of these two schemes w.r.t. the duplication error is completely different. The authentication performance decreases w.r.t. \( P_P \) (after reaching a maximum for \( P_E \)) for T2F setup but increases w.r.t. \( P_C \) for F2F setup. The first phenomenon can be explained by the fact that if a printing device is highly noisy, it is difficult to distinguish between two equally noisy fingerprints. The second phenomenon can be explained by the fact that only the opponent has to use a duplication device for the F2F scheme which makes the discrimination between the fake object and the original one growing w.r.t. the duplication noise. It is interesting to point out that in this case the F2F has an advantage over the T2F systems due to the independence of the legitimate channel from these parameters which only determine the opponent channel.

\[ \text{These results are not shown due to the paper length restrictions.} \]

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5. CONCLUSIONS AND PERSPECTIVES

In this paper, we have presented and compared the F2F and T2F systems from the detection-theoretic perspectives. These systems can model a large variety of practical scenarios used for object identification and authentication. For conservative scenarios, we demonstrate that F2F authentication systems have lower equal error rate than T2F systems for large reproduction errors. At the same time, we have found that T2F systems may reach a minimum for a given parameters of extraction and printing. In our future research, we intend to consider the impact of security and information theoretic oriented constraints such as the unclonability of the fingerprint, the information leakage of the secret parameter in the T2F setup or the identification capacity associated to the authentication system.

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Fig. 5. Simplified setups under analysis: (a) F2F and (b) T2F.

Fig. 6. Comparison of performance of F2F and T2F systems.
6. REFERENCES


