Quantifying the Transmit Diversity Order of Euclidean Distance Based Antenna Selection in Spatial Modulation

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Abstract—In this letter, we quantify the transmit diversity order of the SM system operating in a closed-loop scenario. Specifically, the SM system relying on Euclidean distance based antenna subset selection (EDAS) is considered and the achievable diversity gain is evaluated. Furthermore, the resultant trade-off between the achievable diversity gain and switching gain is studied. Simulation results confirm our theoretical results. Specifically, at a symbol error rate of about $10^{-4}$, the signal-to-noise ratio gain achieved by EDAS is about $7$ dB in case of $16$-QAM and about $5$ dB in case of $64$-QAM.

Index Terms—Antenna selection, diversity, limited feedback, spatial modulation, switching gain.

I. INTRODUCTION

C onsider a spatial modulation (SM) based system [1]–[9] having $N_t$ transmit antennas (TA) and $N_r$ receive antennas (RA), whose system model is given by

$$y = \sqrt{\rho} h_i s + n,$$

where $\rho$ is the average signal-to-noise ratio (SNR) at each receive antenna, $y \in \mathbb{C}^{N_r}$ is the received signal vector, $h_i$ is the channel vector corresponding to the $i$th TA, $s$ is a random symbol selected from a unit-energy $M$-QAM or $M$-PSK signal set represented by $S$ and $n \in \mathbb{C}^{N_r}$ is the noise vector. The entries of both channel matrix $H$ and the noise vector $n$ are from a circularly symmetric complex-valued Gaussian distribution $\mathcal{CN}(0,1)$. The input bitstream in SM is divided into blocks of $\log_2(N_t)M$ bits and in each such block, $\log_2 M$ bits select a symbol $s$ from an $M$-QAM or $M$-PSK signal set, while $\log_2 N_t$ bits per channel use (bpcu) select an antenna $i$ out of $N_t$ transmit antennas for the transmission of the selected symbol $s$. Thus, the SM system achieves $\log_2 N_t$ bits higher throughput than

the single-input multiple-output (SIMO) system using $M$-ary modulation.

Definition 1: In the SM system, the number of bits transmitted per channel use through the TA indices is defined as the spatial switching gain (SSG) of the SM system.

Although SM has the benefit of having a single RF chain, the related disadvantage is that it has no transmit diversity gain due to the activation of single TA in every channel use. Some recently proposed schemes to increase the diversity order of SM beyond one are time-orthogonal signal design assisted spatial modulation [3], [10], coherent space-time shifting [11], time-orthogonal signal design assisted SM relying on STBC [12] and space-time block coded spatial modulation [13]. These schemes constitute some of the existing open-loop techniques conceived for SM. Recently, some closed-loop techniques were conceived for SM [14]–[16], where the SM transmitter relies on the feedback information sent back from the receiver. In this paper, we focus our attention on the Euclidean distance (ED) based antenna subset selection (EDAS) technique [14]–[16], which is briefly described as follows.

EDAS: Let $N_{SM}$ out of $N_t$ TAs be selected for achieving a SSG of $\log_2 N_{SM}$ bpcu. Let $\mathcal{I} = \{i\}_{i=1}^n$ represent the set of enumerations of all possible $n - \left(\frac{N_t}{N_{SM}}\right)$ combinations of selecting $N_{SM}$ out of $N_t$ TAs. Among the $\left(\frac{N_t}{N_{SM}}\right)$ possibilities, the specific TA set that maximizes the minimum ED among all possible transmit vectors (TV) [15] is obtained as

$$I_{ED} = \arg\max_{\mathcal{I} \in \mathcal{Z}} \left\{ \min_{s \in \mathcal{S}} \frac{\|H_i(s - x)\|_2}{\|x\|_2} \right\},$$

where $H_i \in \mathbb{C}^{N_r \times N_{SM}}$ has $N_{SM}$ columns given by $\{e_s\}_{s=1}^{N_{SM}}$, where $s \in S$ and $e_s$ is a $N_{SM} \times 1$ vector having 1 as the only non-zero element at the $s$th location, with $|\mathcal{S}| = N_{SM}$. The chosen $I_{ED}$ is encoded into bits, which are then sent to the transmitter once every coherence interval. Upon receiving this information, the transmitter starts data transmission through the TA indexed by the set $I_{ED}$. Note that the conventional TA selection scheme [17]–[19] is a special case of EDAS corresponding to $N_{SM} = 1$.

Prior Work: A pair of TA selection schemes were proposed in [16], namely capacity optimized antenna selection (COAS) and EDAS. Furthermore, the components of the SM symbol error at the receiver were studied. Explicitly, it was shown that if $P_e(S_M)$ represents the SM symbol error with components
Pr(A) and Pr(S, A'), where A' represents the event of an antenna index not being in error and S represents the event of a transmitted symbol error under ML detection [16], then the component Pr(S, A') in COAS achieves a diversity order of \( N_t - N_{SM} + 1, \log_2 N_{SM} \). However, the effective diversity order of COAS was shown to be only \( N_e \) owing to A.

**New Contributions:** The EDAS of SM systems was shown to achieve significant symbol-error rate (SER) improvements over both COAS and the conventional SM system. However, to the best of our knowledge, the diversity order of this system has not been quantified in the open literature. Hence in this paper, we show that SM employing EDAS will achieve a transmit diversity order of \( d_t = (N_t - N_{SM} + 1) \), and strikes an attractive trade-off between the transmit diversity and SSG, given by \( (d_t, SSG) = (N_t - N_{SM} + 1, \log_2 N_{SM}) \). A further substantial benefit of antenna selection is that the number of high-cost radio-frequency chains is reduced.

**Notations:** The lowercase boldface letters represent vectors and uppercase boldface letters represent matrices. The notations of \( | \cdot | \) and \( \| \cdot \|_F \) represent the two-norm of a vector and the Frobenious norm of a matrix, respectively. Trace of a matrix is represented by \( \text{Tr}(\cdot) \). The notations of \( (\cdot)^T \) and \( (\cdot)^H \) indicate the transpose and Hermitian transpose of a vector/matrix, respectively, while \( \cdot \) represents the cardinality of a given set, or the magnitude of a complex quantity. Expected value of a random variable \( X \) is denoted by \( \mathbb{E}(X) \), while the smallest non-zero Eigenvalue of matrix \( \mathbf{H} \) is denoted by \( \lambda_1(\mathbf{H}) \). Furthermore, \( Q(x) \) is the tail probability of standard normal distribution given by \( \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{u^2}{2}) \, du \).

**II. MAIN RESULTS**

Let \( L = \{i\}_{i=1}^{N_e} \) represent the set of TA indices and \( \mathcal{C}_i = \{e_{ij} \in S, j \in L_i\} \) for \( 1 \leq i \leq n \) represent the \( n \) possible subsets of TVs, where \( e_{ij} \) is the \( j \)-th column of the \( N_t \times N_t \)-element identity matrix. Let \( \Delta \mathcal{C}_i = \{x_1, x_2, x_3, x_4 \in \mathcal{C}_i, x_1 \neq x_2 \} \) be the set of difference vectors corresponding to the codebook \( \mathcal{C}_i \). Consider the set of matrices given by

\[
\Delta \mathcal{D} = \{[x_1, x_2, \ldots, x_n] \mid x_1 \in \Delta \mathcal{C}_1, x_2 \in \Delta \mathcal{C}_2, \ldots, x_n \in \Delta \mathcal{C}_n\},
\]

which is obtained by concatenating all possible difference vectors, one taken from each of the \( n \) subsets. Note that each element in \( \Delta \mathcal{D} \) is of size \( (N_t \times n) \). Let \( p = \min\{\text{rank}(X) | X \in \Delta \mathcal{D}\} \).

**Proposition 1:** The SM system employing EDAS achieves a diversity order of \( N_e p \).

**Proof:** We adopt the steps in the proof of Theorem 2 in [20], showing that the rate of decay for the pairwise error probability (PEP) between any two TVs is at least \( N_e p \).

Let the TVs in each codebook be indexed as \( \mathcal{C}_i = \{x_{i1}, x_{i2}, \ldots, x_{ip}\} \). Let the optimal set of TAs for a given channel realization \( \mathbf{H} \) be \( I_k \), as in (2). Then the PEP between any two distinct TVs indexed by \( l_1, l_2 \) in the codebook \( \mathcal{C}_{k*} \) is given by

\[
\text{PEP}(x_{i1} \rightarrow x_{i2}; \mathbf{H}) = Q \left( \sqrt{\frac{p}{2}} \left\| \mathbf{H} (x_{i1} (k*) - x_{i2} (k*)) \right\| \right),
\]

\[
\leq \frac{1}{2} \exp \left( -\frac{p}{4} \left\| \mathbf{H} (x_{i1} (k*) - x_{i2} (k*)) \right\|^2 \right).
\]

We now have to show that the PEP decays as \( 1/p^{N_e p} \). Let \( x_{min}(k) = \arg \min_{x \in \Delta \mathcal{D}_k} \left\| \mathbf{H} x \right\|^2 \) for \( 1 \leq k \leq n \) represent the difference vector from \( \Delta \mathcal{D}_k \) corresponding to the minimum ED and \( X_{min} = [x_{min}[1], x_{min}[2], \ldots, x_{min}[n]] \). Then we have

\[
\left\| \mathbf{H} (x_{i1} (k*) - x_{i2} (k*)) \right\|^2 \geq \left\| X_{min} (k*) \right\|^2,
\]

\[
\geq \frac{1}{n} \left\| \mathbf{H} X_{min} \right\|_F^2.
\]

The inequality (7) is due to the fact that \( x_{min}(k*) \) corresponds to the maximum ED among the elements in \( X_{min} \). Furthermore, we have

\[
\left\| \mathbf{H} X_{min} \right\|_F^2 = \text{Tr}(\mathbf{H}^H \mathbf{H} X_{min} \mathbf{X}_{min}^H),
\]

\[
= \text{Tr}(\mathbf{H}^H \mathbf{H} X_{min} \mathbf{H}^H),
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\[
= \text{Tr}(\mathbf{H} \mathbf{H}^H),
\]

where \( X_{min} \mathbf{H}^H = \mathbf{U} \) is the Eigen decomposition of \( X_{min} \mathbf{H}^H \) and \( \mathbf{H} \) is the Eigenvalue of matrix \( \mathbf{H} \). Since \( \left\| \mathbf{H} X_{min} \right\|_F^2 \geq \frac{1}{n} \sum_{i=1}^{N_e} \lambda_i \), we have

\[
\left\| \mathbf{H} X_{min} \right\|_F^2 \geq \sum_{i=1}^{N_e} \sum_{j=1}^{p} \lambda_i \left\| h_{i,j} \right\|^2 > \lambda^* \sum_{i=1}^{N_e} \sum_{j=1}^{p} \left\| h_{i,j} \right\|^2.
\]

Therefore averaging over \( \mathbf{H} \), we have

\[
\text{PEP}(x_{i1} \rightarrow x_{i2}) \leq \frac{1}{2} \prod_{i=1}^{N_e} \prod_{j=1}^{p} \left( 1 + \frac{\rho \lambda^*}{4n} \right)^{-1}.
\]

Thus, the probability of symbol errors in EDAS at high SNRs can be bounded as

\[
P_e \leq \frac{1}{\left| C_{k*} \right|} \sum_{x_{i1} \in C_{k*}} \sum_{x_{i2} \neq x_{i1} \in C_{k*}} \text{PEP}(x_{i1} \rightarrow x_{i2}),
\]

\[
\leq \left( \frac{C_{k*} - 1}{2} \right) \left( \frac{\rho \lambda^*}{4n} \right)^{-N_e p},
\]

\[
= \left( \frac{(N_t - N_{SM}) - 1}{2} \right) \left( \frac{\rho \lambda^*}{4n} \right)^{-N_e p}.
\]

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Note that the exponent of $\rho$ is $N_r p$. This concludes the proof.

**Proposition 2:** The SM system employing EDAS achieves a diversity order of $N_r(N_t - N_{SM} + 1)$.

**Proof:** Owing to Proposition 1, it is sufficient to prove that we have $p \geq N_t - N_{SM} + 1$ in order to prove the proposition. Let us now consider any matrix $X$ from the set $\Delta D$. We can then show that this matrix has at least $(N_t - N_{SM} + 1)$ linearly independent columns and hence has a rank of at least $(N_t - N_{SM} + 1)$.

Let $\Delta S$ denote the set of non-zero difference constellation points given by $\{s_1 - s_2, s_1, s_2 \in S, s_1 \neq s_2\}$. Every element in $\Delta C_i$ is either from the set $E_k = \{c_{e_k} \in \Delta S\}$ for $1 \leq k \leq N_t$, or from the set $E_{pq} = \{c_1 e_p + c_2 e_q | c_1, c_2 \in S, 1 \leq p \neq q < N_t\}$. We show that it is impossible to construct a matrix $X$ with rank less than $N_t - N_{SM} + 1$ by picking columns from the set $E_k \cup E_{pq}$.

First, we show that by restricting the columns of $X$ to be from the collection of sets $\{E_k\}$ the least possible rank of $X$ can be $N_t - N_{SM} + 1$. Then, we show that by picking the columns from the collection of sets $\{E_k \cup E_{pq}\}$ the minimum achievable rank of $X$ does not reduce than that achieved with $E_k$ alone.

Pick $g_1 \in E_k$ for some legitimate $k$. Since $g_1$ belongs to $(N_t - 1, N_{SM} - 1)$ of the $n$ sets $\{\Delta C_i\}_{i=1}^n$, we pick $g_1$ from these $\Delta C_i$’s that constitute say first $(N_t - 1, N_{SM} - 1)$ columns of $X$. Similarly, we pick $g_2 \in E_i$, $l \neq k$. Since $g_2$ belongs to $(N_t - 2, N_{SM} - 1)$ of the remaining $(n - N_t + 1)$ sets in the collection $\{\Delta C_i\}$, we pick $g_2$ from these sets that constitute the next $(N_t - 2, N_{SM} - 1)$ columns of $X$. Now we have picked $(N_t - 1, N_{SM} - 1) + (N_t - 2, N_{SM} - 1)$ columns of $X$, which has a rank equal to two. Proceeding in the same lines, at the $m^{th}$ step, we would have chosen $\sum_{i=1}^m (N_t - i, N_{SM} - 1)$ columns of $X$, which has a rank equal to $m$.

Since $n = \sum_{i=1}^{N_t - 1} (N_{SM} - 1)$, we would have picked all the columns of $X$ when $m = N_t - N_{SM} + 1$. Thus the minimum rank of $X$ when the columns are picked from the collection $\{E_k\}$ is $N_t - N_{SM} + 1$. Now we show that by considering the set $\{E_k \cup E_{pq}\}$, we will not have any advantage in reducing the rank of $X$ any further.

Suppose, assume that we pick $g_1 \in span(e_l, e_m)$ from the set $E_{pq}$ instead of $g_1 \in E_k$. Since $g_1$ belongs to $(N_t - 2, N_{SM} - 2)$ of the $n$ sets, which is strictly lesser than $(N_t - 1, N_{SM} - 1)$, any choice of the vector from the collection $\{E_k \cup E_{pq}\}$ to fill in the remaining $(N_t - 1, N_{SM} - 1) - (N_{SM} - 2)$ columns will increase the rank of $X$ to two, whereas $g_1$ gives a rank of only one for $(N_t - 1, N_{SM} - 1)$ columns. However, we can pick vectors whose basis element is either $e_l$ or $e_m$ while increasing the rank of $X$ only by one. There are totally $(N_{SM} - 1)$ vectors for each of the basis. Since we have already chosen those columns which have both $e_l$ and $e_m$, we have effectively $(N_{SM} - 1) + (N_{SM} - 1) - (N_{SM} - 2)$ columns, which is equal to $(N_t - 1, N_{SM} - 1) + (N_t - 1, N_{SM} - 1)$. Now, $X$ has a rank of two with both $(N_{SM} - 1)$ columns filled. Note that by restricting the columns of $X$ to be from $E_k$, a rank of two was achieved with the same number of columns filled. Thus, picking elements from $E_{pq}$ will not give any advantage in reducing the rank further. This concludes the proof.

Thus, it becomes clear from Proposition 2 that the SM system cannot achieve full transmit diversity, while achieving a non-zero SSG. Fig. 1 compares the trade-off between the achievable transmit diversity and SSG for various number of TAs with different choices of $N_{SM}$. Considering for example $N_t = 12$, we can have $N_{SM} = [2, 4, 8] \leq N_t$. The achievable transmit diversity gains in these cases are 11, 9, and 5 respectively. Note that $N_{SM} = 1$ corresponds to the conventional TA selection scheme of [17].

### III. Simulation Results and Discussion

**Simulation Scenario:** In all our simulations, we assume block Rayleigh fading channels and perfect channel estimation at the receiver. Furthermore, the receiver is assumed to perform maximum-likelihood detection in all the transmission schemes considered. For each channel realization, the antenna subset $I_{EDAS}$ of (2) is obtained by employing the low-complexity EDAS algorithm given in [16].

We demonstrate that the SM system employing EDAS indeed achieves a transmit diversity of order $d_t = (N_t - N_{SM} + 1)$. Consider an SM system operating with $N_t = 5$, $N_r = 1$, 16-QAM and 32-QAM signal sets, where $N_{SM} \in \{1, 2, 3, 4\}$. The choice of $N_r = 1$ ensures that the achieved diversity is only due to TA subset selection. Fig. 2 gives the SER performance of

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In this paper, we have quantified the achievable transmit diversity order of SM employing EDAS. Furthermore, the resultant trade-off between the diversity order and the switching gain was quantified. Our simulation results confirmed the theoretical formulae.

**REFERENCES**


