INTER-USER INTERFERENCE IN MOLECULAR COMMUNICATION NETWORKS

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ABSTRACT

As the nanotechnology becomes more and more mature, the concept of nano communication emerged and attracted lots of researchers’ attention. To implement the nano communication system, the diffusion-based molecular communication is considered as a promising bio-inspired approach, where the nano transmitter emits molecules into the medium to transmit data and the nano receiver absorbs molecules from the medium to receive data. Since all the nano machines share the same propagation medium and the molecules are identical, the interference among nano transmitters are unavoidable. In this paper, we analyze the inter-symbol and inter-user interference in the diffusion-based molecular communication systems. Based on the interference analysis, we further study the bit error rate performance and derive the optimal decision threshold for the nano receiver. Simulation results are shown to verify our analysis.

Index Terms— Nano communication, molecular communication, inter-symbol interference, inter-user interference.

1. INTRODUCTION

Recently, nano communication emerges as a new communication paradigm, which refers to the information exchange between nano machines through nano mechanical, acoustic, electromagnetic and chemical or molecular communication means [1]. It is expected that nano communication networks can be applied in many different scenarios including human health monitoring, food and water quality control, air pollution control, as well as aggressive chemical agent detection [2]. Since molecules migration naturally occurs within both living organisms and abiotic components, diffusion-based molecular communication is considered as the most promising approach for nano communication networks [3]-[4]. In the molecular communication model, the nano transmitter release molecules, that are modulated and coded to carry information, into the fluid medium. The molecules propagate to the receiver through the medium, which are demodulated and decoded to restore the information.

In the literature, various topics about the diffusion-based molecular communications have been studied, including channel model [5]-[10], modulation and coding [11]-[15], receiver design [16]-[17]. The earlier works regarding diffusion-based molecular communications were mainly focused on channel capacity analysis [5]-[10], where a common accepted channel model is based on Brownian motion [18]. In [5], Eckford analyzed the achievable bound on information rate for the diffusion-based channel with one-dimensional Brownian motion. As an extension, Brownian motion of molecules in fluid medium with drift velocity instead of static environment, was analyzed in [10]. In addition to the channel analysis, a comprehensive physical end-to-end model, including molecules emission, diffusion and reception, was proposed in [19]. Meanwhile, simulation-based approaches for exploring the diffusive molecules were conducted in [20] and [21]. As for channel noise and interference analysis, Pierobon and Akyildiz studied the molecules sampling and counting noise in [13], as well as the inter-symbol and co-channel interference in [14]; Kadloor et al. presented an additive inverse Gaussian noise channel model in [11]. To enhance system performance, various coding schemes were also introduced, including a forward error correction coding scheme in [12] and a rate-delay tradeoff network coding scheme in [15]. In terms of receiver design, an optimal receiver design based on weighted sum detectors was proposed in [16], and a ligand-binding reception model was studied in [17]. Moreover, the consensus problem and relaying probe under diffusion-based molecular communication were recently studied in [22] and [23], respectively.

However, the inter-user interference in molecular communication networks has not been well investigated in the existing works. Since all the nano transmitters share the same propagation medium and the emitted molecules by different transmitters are identical and indistinguishable, the inter-user interference would appear when multiple transmitters emit molecules simultaneously. Considering this problem, in this paper, we study the inter-user interference in molecular communication systems by using a two-transmitter and one-receiver system as an example. Based on the interference analysis, we find the optimal decision threshold using the maximum-a-posterior (MAP) detection method and derive the corresponding bit error rate (BER) performances of different transmitters. In the rest of this paper, we first introduce the system model of molecular communication in Section 2. Then, we study the inter-symbol and inter-user interference in Section 3. The BER performance is analyzed in Section 4, as well as the simulation results. Finally, the conclusion is drawn in Section 5.

2. SYSTEM MODEL

2.1. Network Entity

As shown in Fig. 1, we consider a molecular communication system with two transmitters and one receiver. We would like to point out that since the interference from different transmitters are additive due to the independence among transmitters, all the analysis and results in this paper can be easily extended to the multi-transmitter and multi-receiver case. As we can see from Fig. 1, there are four components in a diffusion-based molecular communication system: transmitter, molecule, propagation medium and receiver. The transmitters can modulate a series of “0-1” symbols by controlling the speed (frequency modulation) [19], time (timing modulation) [11] or number (amplitude modulation) [12] of the emitted molecules. In...
this paper, we consider the amplitude modulation. Specifically, at
the beginning of each time slot, the transmitter releases \( n \) molecules
to represent transmitting bit “1”, while releases 0 molecule to rep¬
sent transmitting bit “0”, i.e., only one bit transmission per time
slot. The length of each time slot \( t_s \) is considered as identical for
all transmitters. For the reception side, the receiver counts and ac¬
culates the number of absorbed molecule during each time slot,
comparing the number with some predefined threshold to make a de¬
cision on receiving bit “1” or “0”. Note that \( n \) can be regarded as the
molecules emission power.

The molecules are considered as identical and undistinguishable
between each other, and the interactions among them are not taken
into account in general [11]. The Brownian motion is a generally
accepted model for the propagation of molecules in the medium,
which can be characterized by two parameters: drift velocity and
diffusion constant determined by the physical properties of the fluid
medium [24]. In this paper, we only consider the diffusion effect of
molecules. Note that [10] considered both diffusion effects, and
all our analysis and results can be easily extended to that case.
When the molecules arrive at the receiver, they are absorbed by the
receiver and disappear from the medium. The receiver can measure
the number of absorbed molecule in one time slot and use such in¬
formation to determine the information sent by the transmitter.

2.2. Channel Model

In the molecular communication systems, the communication chan¬
el is the fluid medium and the propagation model is based on the
Brownian motion [18]. Due to the random behavior of the molecules
with Brownian motion, after escaping from the transmitter, the time
they arrive at the receiver is probabilistic. Suppose the transmitter is
located at the origin and releases a molecular at time \( t = 0 \). Let \( X(t) \)
denote the position of the released molecule at some specific time \( t \).
According to [14], under the Brownian motion in three dimensional
space, the probability density of \( X(t) \) is

\[
P_X(x, t) = \frac{1}{\sqrt{4\pi D t}} \exp \left( -\frac{x^2}{4Dt} \right), \tag{1}
\]

where \( x \) represents the distance from the original emission point and \( D \)
is the aforementioned diffusion constant. The (1) illustrates the
prorogation model of the molecules in the medium, which can be reg¬
sarded as the impulse response of the Brownie motion based chan¬
el. According to the aforementioned amplitude modulation model,
only the molecules absorbed by the receiver within time \( t_s \) are con¬
sidered as useful ones. Suppose the distance between the transmitter
and receiver is \( d \) and the radius of the receiver is \( R \). Then, according
to [12], the probability that a molecule is absorbed by the receiver
within \( t_s \) can be calculated by

\[
P_a(d, t_s) = \frac{d}{R} \text{erfc} \left( \frac{d - R}{\sqrt{4Dt_s}} \right). \tag{2}
\]

3. INTERFERENCE ANALYSIS

In this section, we analyze the interference issues in the molecular
communication system. Similar to the traditional wireless commu¬
nication systems, there are inter-symbol interference and inter-user in¬
terference in the molecular communication systems. For each trans¬
mitter, the molecules released in the previous time slots may leak
into the current time slot, i.e., the current symbol, which causes the
inter-symbol interference (ISI), as shown in the left part of Fig. 2.
Moreover, the molecules released by one transmitter may also leak
into the time slot of other transmitters, which causes the inter-user in¬
terference (IUI), as shown in the right part of Fig. 2. Based on
the interference analysis, we can further derive the optimal deci¬
sion threshold based on maximum-a-posterior (MAP) method and
the corresponding bit error rate (BER) performance.

According to the system model, if the transmitter intends to send
bit “1”, it would release \( n \) molecules to the medium. Let \( N_a \) denote
the number of molecules absorbed by the receiver located \( d \) away
from the transmitter within time period \( t_s \). Since all the molecules
are independent of each other and each reaches the receiver with
probability \( P_a(d, t_s) \), \( N_a \) follows a binomial distribution as follows [25]–[26]:

\[
N_a \sim \mathcal{B}(n, P_a(d, t_s)). \tag{3}
\]

When \( n \) is sufficiently large (e.g. \( n \geq 20 \)), which is generally sat¬
sified by the molecular communication systems, the binomial distri¬
bution can be approximated by a normal distribution [12] [25]. In
such a case, the distribution of \( N_a \) can approximated by

\[
N_a \sim \mathcal{N} \left( nP_a(d, t_s), nP_a(d, t_s)(1 - P_a(d, t_s)) \right). \tag{4}
\]

3.1. Inter-Symbol Interference

The inter-symbol interference (ISI) defined in traditional wireless
communication systems represents the power leakage of one trans¬
mitted symbol into the following symbols, which is usually caused
by multi-path propagation. Similarly, in the molecular communica¬
tion system, due to the random nature of Brownian motion based dif¬
fusion process, molecules emitted in previous time slots may arrive
at the receiver in current time slot, which leads to the interference
to the current bit reception. Such interference is defined as ISI in
molecular communications, as shown in the left part of Fig. 2. Sup¬
pose that the current reception time slot is \( m \), let us denote \( N_{a, m-t_s} \)
as the number of molecules that were sent at the beginning of i time
slots before, i.e., at \( (m - i)t_s \), and leak into the current time slot \( m \).
In such a case, \( N_{a, m-t_s} \) should follow the distribution as follows:

\[
N_{a, m-t_s} \sim \frac{1}{2} \mathcal{N} \left( nP_a(d, (i + 1)t_s), nP_a(d, (i + 1)t_s)(1 - P_a(d, (i + 1)t_s)) \right) - \frac{1}{2} \mathcal{N} \left( nP_a(d, it_s), nP_a(d, it_s)(1 - P_a(d, it_s)) \right). \tag{5}
\]
where $\frac{1}{4}$ is based on the assumption that the bit “1” and “0” are equally generated, the first term represents the total number of molecules that were sent at $n_{ds} - n_{ds}$ and absorbed by the receiver within all subsequent $i + 1$ time slots, while the second term represents that within all subsequent $i$ time slots, both of which follow the normal distribution as defined in (4). Thus, the total interference caused by all previous time slots, $N_{ISI}$, can be calculated by

$$N_{ISI} = \sum_{i=0}^{\infty} N_{a, i+t_s}.$$  \hspace{1cm} (6)

According to [25], the experiment results have shown that only the interference from one previous time slot, i.e., the ISI from the last symbol, needs to be considered. In such a case, the distribution of ISI $N_{ISI}$, can be approximated by

$$N_{ISI} \sim \frac{1}{2} N \left(nP_a(d, 2t_s), nP_a(d, 2t_s)(1 - P_a(d, 2t_s))\right) - \frac{1}{2} N \left(nP_a(d, t_s), nP_a(d, t_s)(1 - P_a(d, t_s))\right).$$  \hspace{1cm} (7)

### 3.2. Inter-User Interference

The inter-user interference (IUI) defined in traditional wireless communication systems represents the interference power from one transmitter to the other. Similarly, in the molecular communication systems, when there are two or more transmitters releasing molecules into the medium, they would be interfered by each other at the receiver. In the amplitude modulation based system, the receiver compares the number of molecules absorbed in one time slot with predefined threshold $\tau$ to decode the bit. In such a case, the BER performance is heavily related with the threshold $\tau$. Therefore, we need to first derive the optimal $\tau$ that minimizes the BER of each transmitter.

For case “00”, there is no IUI since no molecule is released. For case “01”, only the current symbol “1” of TX I causes interference to TX II. In this case, the number of molecules that leaks into the current symbol of TX II from TX I, denoted by $N_{01}^I$, follows normal distribution as

$$N_{01}^I \sim N\left(n_1 P_{a}(d_1, t_s), n_1 P_{a}(d_1, t_s)(1 - P_{a}(d_1, t_s))\right) = N\left(n_1 P_{ae}, n_1 P_{ae}(1 - P_{ae})\right).$$  \hspace{1cm} (8)

where $P_{ae} = P_a(d_1, t_s)$ defined in (2) represents the absorbing probability of molecules of TX I’s current symbol at current time slot, which is interference to TX II. For case “10”, only the previous symbol “1” of TX I causes interference to TX II. In this case, the number of molecules that leaks into the current symbol of TX II from TX I, denoted by $N_{10}^I$, also follows normal distribution as

$$N_{10}^I \sim N\left(n_1 P_a(d_1, 2t_s), n_1 P_a(d_1, 2t_s)(1 - P_a(d_1, 2t_s))\right) - N\left(n_1 P_a(d_1, t_s), n_1 P_a(d_1, t_s)(1 - P_a(d_1, t_s))\right) = N\left(n_1 P_{ae}, n_1 P_{ae}(1 - P_{ae})\right) - N\left(n_1 P_{ae}, n_1 P_{ae}(1 - P_{ae})\right).$$  \hspace{1cm} (9)

where $P_{ae} = P_a(d_1, 2t_s)$ represents the absorbing probability of molecules of TX I’s previous symbol at both the previous and current time slots. Note that the IUI in case “10” is quite similar to the ISI in (7), while the difference is that ISI means the interference from the transmitter’s own previous symbol, while IUI in case “10” means the interference from the other transmitter’s previous symbol. For case “11”, both the previous and current symbols of TX I can cause interference to TX II. Thus, the number of molecules that leaks into the current symbol of TX II from TX I, denoted by $N_{11}^I$, follows another normal distribution as

$$N_{11}^I \sim N\left(n_1 P_{ae}, n_1 P_{ae}(1 - P_{ae})\right) + N\left(n_1 P_{ae}, n_1 P_{ae}(1 - P_{ae})\right).$$  \hspace{1cm} (10)

Overall, based on the assumption that the probabilities of all the four cases are identical, i.e., $P_00 = P_{01} = P_{10} = P_{11} = \frac{1}{4}$, the IUI from TX I to TX II, denoted by $N_{II}^I$, follows the distribution as follows:

$$N_{II}^I \sim \frac{1}{4} N\left(n_1 P_{ae}, n_1 P_{ae}(1 - P_{ae})\right) + 2n_1 P_{ae}(1 - P_{ae})$$

Similarly, the IUI from TX II to TX I, denoted by $N_{II}^H$, follows the distribution as follows:

$$N_{II}^H \sim \frac{1}{4} N\left(n_2 P_{ae}, n_2 P_{ae}(1 - P_{ae})\right) + 2n_2 P_{ae}(1 - P_{ae}).$$

where $P_{ae} = P_a(d_2, t_s)$ and $P_{ae} = P_a(d_2, 2t_s)$ can be calculated by (2).

### 4. BIT ERROR RATE PERFORMANCE

#### 4.1. Bit Error Rate Analysis

The bit error rate (BER) is defined as the number of bit errors divided by the total number of transmitted bits during an evaluated time interval, which is usually approximated by the bit error probability, i.e., the probability that bit “0” or “1” is wrongly decoded by the receiver. In the amplitude modulation based system, the receiver compares the number of molecules absorbed in one time slot with some predefined threshold $\tau$ to decode the bit. In such a case, the BER is heavy related with the threshold $\tau$. Therefore, we need to first derive the optimal $\tau$ that minimizes the BER of each transmitter.

Let us consider the two-transmitter case in Fig. 1. When TX I transmits bit “0”, i.e., no molecule is released, the number of molecules absorbed by the receiver within current time slot, denoted by $N_{0}^I$, includes both ISI from its previous symbol and IUI from TX II, which follows normal distribution as follows:

$$N_{0}^I = N_{ISI}^I + N_{II}^I \sim N(\mu_{01}, \sigma_{01}^2).$$  \hspace{1cm} (13)

where $N_{ISI}^I$ can be calculated by (7) and $\mu_{01}, \sigma_{01}^2$ are

$$\mu_{01} = \frac{1}{2}(n_1 P_{ae} + n_2 P_{2p}),$$

$$\sigma_{01}^2 = \frac{1}{4}n_1 P_{ae}(1 - P_{ae}) + \frac{1}{4}n_1 P_{ae}(1 - P_{ae}) + \frac{1}{8}n_2 P_{2p}(1 - P_{2p}).$$  \hspace{1cm} (14)
On the other hand, when TX I transmits bit “1”, i.e., $n_1$ molecules are released, the number of molecules absorbed by the receiver within current time slot, denoted by $N_1^I$, follows another normal distribution as follows:

$$N_1^I = \mathcal{N}(n_1 P_{Ic}, n_1 P_{Ic}(1 - P_{Ic})) + N_0^I \sim \mathcal{N}(\mu_{1,1}, \sigma_{1,1}^2), \quad (16)$$

where the first term represents the number of molecules absorbed by the receiver among the $n_1$ molecules from TX I, and $\mu_{1,1}, \sigma_{1,1}^2$ are

$$\mu_{1,1} = \frac{1}{2} (n_1 P_{Ic} + n_1 P_{Ic} + n_2 P_{2p}), \quad \sigma_{1,1}^2 = \frac{1}{4} n_2 P_{2p} (1 - P_{2p}) + \frac{1}{2} n_2 P_{2p} (1 - P_{2p}). \quad (17)$$

At the receiver side, to demodulate TX I’s information, it is confronted with the following hypothesis test based on the absorbed molecules $Y$:

$$\mathcal{H}_0 : \quad Y = N_0^I \sim \mathcal{N}(\mu_{0,1}, \sigma_{0,1}^2)$$
$$\mathcal{H}_1 : \quad Y = N_1^I \sim \mathcal{N}(\mu_{1,1}, \sigma_{1,1}^2). \quad (19)$$

With the hypothesis test model above, we can derive the optimal decision that minimizes the BER using the maximum-a-posterior (MAP) detection method. Since we assume the bit “0” and “1” are uniformly distributed, i.e., $p(\mathcal{H}_0) = p(\mathcal{H}_1)$, the MAP test is equivalent to the likelihood-ratio test (LRT) as follows:

$$p(Y|\mathcal{H}_1) = \frac{\sigma_{0,1}}{\sigma_{1,1}} \exp \left\{ \frac{(Y - \mu_{1,1})^2}{2\sigma_{1,1}^2} \right\} \quad (20)$$

$$p(Y|\mathcal{H}_0) = \frac{\sigma_{0,1}}{\sigma_{1,1}} \exp \left\{ \frac{(Y - \mu_{0,1})^2}{2\sigma_{0,1}^2} \right\} \quad (21)$$

By setting the LRT test $\frac{p(Y|\mathcal{H}_1)}{p(Y|\mathcal{H}_0)}$ as 1, we can find the MAP decision threshold for demodulating information from TX I as

$$\tau_1 = \text{round} \left( \frac{\sqrt{2A \ln C + B^2 - B}}{A} \right), \quad (22)$$

where $\text{round}$ represents the rounding operation since the decision threshold should be an integer, the parameters $A, B$ and $C$ are

$$A = \left( \frac{1}{\sigma_{1,1}^2} - \frac{1}{\sigma_{0,1}^2} \right), \quad B = \frac{\mu_{1,1}}{\sigma_{1,1}} - \frac{\mu_{0,1}}{\sigma_{0,1}}, \quad (23)$$

$$C = \frac{\sigma_{1,1}^2}{\sigma_{0,1}^2} \exp \left\{ \frac{1}{2} \left( \frac{\mu_{1,1}^2}{\sigma_{1,1}^2} - \frac{\mu_{0,1}^2}{\sigma_{0,1}^2} \right) \right\}. \quad (24)$$

Based on the optimal decision threshold, we can calculate the BER of information from TX I as follows:

$$P_e^I = \frac{1}{2} \left( P(N_0^I \geq \tau_1) + P(N_1^I \leq \tau_1) \right)$$

$$= \frac{1}{2} \left( 1 - Q \left( \frac{\tau_1 - \mu_{1,1}}{\sigma_{1,1}} \right) + Q \left( \frac{\tau_1 - \mu_{0,1}}{\sigma_{0,1}} \right) \right), \quad (25)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$. Similarly, we can derive the BER of information from TX II as

$$P_e^II = \frac{1}{2} \left( 1 - Q \left( \frac{\tau_2 - \mu_{0,II}}{\sigma_{0,II}} \right) + Q \left( \frac{\tau_2 - \mu_{1,II}}{\sigma_{1,II}} \right) \right). \quad (26)$$

where $\tau_0, \mu_{0,II}, \mu_{1,II}, \sigma_{0,II}$ and $\sigma_{1,II}$ are dual with $\tau_1, \mu_{0,1}, \mu_{1,1}, \sigma_{0,1}$ and $\sigma_{1,1}$, and hence the detailed expressions are omitted here.

### 4.2 Simulation Results

In this subsection, we conduct simulation to show the IUI and BER performance in the molecular communication system. In the simulation, the length of each time slot is set as $t_s = 10s$ (second), the radius of the receiver is configured as $R = 10\mu m$ and the diffusion coefficient is a empirical parameter set as $D = 79.4\mu m^2/s$ [24]. As for the location settings, the distance between TX I and the receiver is fixed at $d_1 = 30\mu m$, while the distance between TX II and the receiver $d_2 = 40\mu m$. Since TX I is located closer to the receiver, it is supposed to have better BER performance than TX II.

In Fig.3, we plot the TX I’s BER performance $P_e^I$ and TX II’s BER performance $P_e^{II}$ versus different molecule emission power of TX I and II, $n_1$ and $n_2$. From Fig.3-(a), we can see that TX I’s BER $P_e^I$ is a decreasing function in terms of $n_1$ and an increasing function in terms of $n_2$. This is consistent with traditional wireless communication systems that the higher emission power TX I adopts, i.e., the more molecules, $n_1$, are transmitted to represent bit “1”, the lower BER performance can be achieved. On the other hand, when TX II enhances its emission power $n_2$, the interference to TX I increases, due to which the BER performance of TX I is degraded. For TX II’s BER performance shown in Fig.3-(b), an opposite phenomenon can be found, where $P_e^{II}$ is a decreasing function in terms of its own emission power $n_2$, while an increasing function in terms of TX I’s emission power $n_1$.

### 5. CONCLUSION

In this paper, we analyzed the inter-symbol and inter-user interference in diffusion-based molecular communication systems. Based on the interference analysis, we derived the optimal decision threshold and the BER performance of different transmitters. The simulation results showed that when the molecules emission power of one transmitter is increased, the BER performance of it can be enhanced, while the interference to the other transmitter is increased and the BER performance is impaired. Thus, an important problem in the molecular communication systems is how to control the transmitters’ molecules emission to achieve not only efficiency but also fairness, which is one of our works in the near future.
6. REFERENCES


