COORDINATED BEAMFORMING IN MIMO FBMC/OQAM SYSTEMS

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ABSTRACT

In this contribution, we propose a coordinated transmit beamforming technique for point-to-point multiple-input-multiple-output (MIMO) filter bank based multi-carrier with offset quadrature amplitude modulation (FBMC/OQAM) systems. To enable reliable transmissions when the number of transmit antennas does not exceed the number of receive antennas and the channel is not flat fading, we design a joint and iterative procedure to calculate the precoding matrix and the decoding matrix for each subcarrier. Simulation results show that the proposed algorithm outperforms the existing transmission strategies for MIMO FBMC/OQAM systems. It is also observed that by employing the proposed coordinated beamforming scheme, the MIMO FBMC/OQAM system achieves a similar bit error rate (BER) performance as its orthogonal frequency division multiplexing with the cyclic prefix insertion (CP-OFDM) based counterpart while exhibiting superiority in terms of a higher spectral efficiency, a greater robustness against synchronization errors, and a lower out-of-band radiation.

Index Terms — MIMO FBMC/OQAM, coordinated beamforming, frequency selective channel

1. INTRODUCTION

As a promising alternative to orthogonal frequency division multiplexing with the cyclic prefix insertion (CP-OFDM), filter bank based multi-carrier modulation (FB-MC) has received great research attention in recent years. By using spectrally well-contained synthesis and analysis filter banks at the transmitter and at the receiver [1], [2], FB-MC has an agile spectrum. Thereby, the out-of-band radiation is lower compared to CP-OFDM, and it is consequently beneficial to choose FB-MC over CP-OFDM for asynchronous scenarios [3], [4], or to achieve an effective utilization of spectrum holes [5], [6]. Moreover, in systems where filter bank based multi-carrier with offset quadrature amplitude modulation (FBMC/OQAM) is employed, the fact that the insertion of the CP is not required as in CP-OFDM based systems leads to a higher spectral efficiency.

In FBMC/OQAM systems, the real and imaginary parts of each complex-valued data symbol are staggered by half of the symbol period [1], [7] such that the desired signal and the intrinsic interference are separated in the real domain and in the pure imaginary domain, respectively. In [8] and [9] where receive processing techniques have been developed for multiple-input-multiple-output (MIMO) FBMC/OQAM systems, it is assumed that the channel frequency responses of adjacent subcarriers do not vary. Consequently, the intrinsic interference is canceled by taking the real part of the resulting signal after the equalization. To alleviate the constraint on the frequency selectivity of the channel, a zero forcing (ZF) based approach has been proposed in [10] for multi-stream transmissions in a MIMO FBMC/OQAM system where the channel is not restricted to flat fading. More details of the performance analysis of this algorithm have been presented in [11]. However, the work in [10] and [11] is limited to the case where the number of receive antennas does not exceed the number of transmit antennas. In addition, the authors have shown numerically and have also pointed out that their proposed approach only provides a satisfactory performance in an asymmetric configuration, i.e., when the number of transmit antennas is larger than the number of receive antennas.

In the context of the multi-user MIMO downlink with space division multiple access (SDMA), coordinated beamforming techniques [12], [13] have been proposed to cope with the dimension constraint imposed on block diagonalization based precoding algorithms [14]. Inspired by these works, we design a coordinated beamforming algorithm for point-to-point MIMO FBMC/OQAM systems without restricting the number of transmit antennas and the number of receive antennas. Assuming perfect channel state information at the transmitter, the precoding matrix and the decoding matrix are calculated jointly in an iterative procedure for each subcarrier. Different choices of the decoding matrix in the initialization step are recommended for different scenarios. Moreover, the application of the proposed algorithm in multi-user MIMO downlink settings is also addressed. Via numerical results, the performance superiority of the proposed approach over the state-of-the-art is demonstrated.

The remainder of the paper is organized as follows: Section 2 introduces the data model of a point-to-point MIMO FBMC/OQAM system and reviews two state-of-the-art transmission strategies for such a system. The proposed coordinated beamforming algorithm is described in detail in Section 3. Numerical results are presented in Section 4, before conclusions are drawn in Section 5.

2. DATA MODEL AND STATE-OF-THE-ART

We consider a point-to-point MIMO FBMC/OQAM system where the channel on each subcarrier can be treated as flat fading [10], [11]. The number of transmit antennas and the number of receive antennas are denoted by $M_T$ and $M_R$, respectively. The received signal on the $k$th subcarrier and at the $n$th time instant is written as follows

$$y_k[n] = H_k[n]F_k[n]d_k[n] + \sum_{i=n-3}^{n+3} \sum_{\ell=k-1}^{k+1} H_{\ell}[i]F_{\ell}[i]c_{\ell}d_{\ell}[i] + n_k[n], \quad (\ell, i) \neq (k, n),$$

(1)
where \( \mathbf{d}_k[n] \in \mathbb{R}^d \) is the desired signal on the \( k \)th subcarrier and at the \( n \)th time instant when \( (k + n) \) is even\(^1\), and \( d \) denotes the number of spatial streams. The terms \( c_{\ell,i} \mathbf{d}_k[n] \) contribute to the intrinsic interference and are pure imaginary, where \( \ell = k - 1, k + 1, i = n - 3, \ldots, n + 3 \), and \( (\ell, i) \neq (k, n) \). The coefficients \( c_{\ell,i} \) represent the system impulse response determined by the synthesis and analysis filters. The PHYDYAS prototype filter [15] is used, and the overlapping factor is chosen to be \( K = 4 \). For more details about FBMC/OQAM systems, the reader is referred to [7]. Here \( \mathbf{H}_k \in \mathbb{C}^{M_T \times M_T} \) contains the frequency responses of the channels between each transmit antenna and each receive antenna, and \( \mathbf{n}_k[n] \) denotes the additive white Gaussian noise vector with variance \( \sigma_n^2 \). In addition, \( \mathbf{F}_k[n] \in \mathbb{C}^{M_T \times d} \) represents the precoding matrix that maps the spatial streams to the transmit antennas.

### 2.1. Straightforward extension of the transmission strategy as in case of CP-OFDM

In several publications on MIMO FBMC/OQAM systems, such as [8] and [9], it is assumed that the channels on adjacent subcarriers are almost the same. The received signal on the \( k \)th subcarrier and at the \( n \)th time instant can be accordingly written as

\[
y_k[n] = \mathbf{H}_k[n] \mathbf{F}_k[n] \mathbf{d}_k[n] + \mathbf{n}_k[n],
\]

where \( \mathbf{d}_k[n] \) contains the real-valued desired signal and the pure imaginary interference

\[
\mathbf{d}_k[n] = \mathbf{d}_k[n] + \sum_{n+3}^{n+1} \sum_{\ell=k-1}^{K+1} c_{\ell,i} \mathbf{d}_k[i], \quad (\ell, i) \neq (k, n).
\]

Considering \( \mathbf{d}_k[n] \) as an equivalent transmitted signal, (2) resembles the data model of a MIMO CP-OFDM system. Consequently, transmission strategies that have been developed for MIMO CP-OFDM systems can be straightforwardly extended to MIMO FBMC/OQAM systems where only one additional step is required, i.e., taking the real part of the resulting signal after the multiplication by the decoding matrix

\[
\tilde{\mathbf{d}}_k[n] = \text{Re} \left\{ \mathbf{D}^{\text{eq}}[n] y_k[n] \right\},
\]

where \( \mathbf{D}_k[n] \in \mathbb{C}^{M_T \times d} \) is the decoding matrix on the \( k \)th subcarrier and at the \( n \)th time instant. Here \( \text{Re} \{ \cdot \} \) symbolizes the real part of the input argument, while \( \text{Im} \{ \cdot \} \) is used in the following text to represent the imaginary part.

### 2.2. Zero forcing based approach

In [10] and [11], the precoding matrix is designed such that the intrinsic interference (corresponding to the second term on the right hand side of (1)) can be canceled by taking the real part of the received signal. Let us expand the real part of the received signal on the \( k \)th subcarrier and at the \( n \)th time instant

\[
\text{Re} \{ y_k[n] \} = \text{Re} \{ \mathbf{H}_k[n] \mathbf{F}_k[n] \mathbf{d}_k[n] \} + \text{Re} \{ \mathbf{n}_k[n] \} + (-1)^n \prod_{i=3}^{n+3} \text{Im} \{ \mathbf{H}_k[i] \mathbf{F}_k[i] \} \text{Re} \{ c_{\ell,i} \mathbf{d}_k[i] \}
\]

\[
+ \text{Re} \{ \mathbf{n}_k[n] \}, \quad (\ell, i) \neq (k, n).
\]

\(^1\)For the case where \( (k + n) \) is odd, the desired signal on the \( k \)th subcarrier and at the \( n \)th time instant is pure imaginary, while intrinsic interference is real. As the two cases are essentially equivalent to each other, we only take the case where \( (k + n) \) is even to describe the proposed algorithm in this paper.

The precoding matrix \( \mathbf{F} \) for each subcarrier and each time instant is calculated such that

\[
\text{Im} \{ \mathbf{H} \mathbf{F} \} = 0,
\]

where \( \mathbf{H} \) represents the channel matrix on the same subcarrier and at the same time instant, and the time as well as the subcarrier indices are ignored, as the precoding concept is on a per-subcarrier basis. Define a matrix \( \bar{\mathbf{H}} \) as

\[
\bar{\mathbf{H}} = \left[ \begin{array}{c} \text{Im} \{ \mathbf{H} \} \\ \text{Re} \{ \mathbf{H} \} \end{array} \right] \in \mathbb{R}^{M_T \times 2M_T}.
\]

The stacked version of the real part and the imaginary part of the precoding matrix \( \mathbf{F} \), i.e., \( \left[ \text{Re} \{ \mathbf{F} \} \quad \text{Im} \{ \mathbf{F} \} \right]^T \), should lie in the null space of \( \bar{\mathbf{H}} \). However, this approach is designed only for scenarios where \( M_T \geq M_R \). In addition, it is observed in the numerical results presented in [11] and is also pointed out in [10] that in a symmetric case where \( M_T = M_R \), this scheme does not lead to a good performance.

### 3. COORDINATED BEAMFORMING

Therefore, we propose to jointly and iteratively update the precoding matrix and the decoding matrix to alleviate the dimension constraint on the precoding algorithm in [10], i.e., scenarios where \( M_T \leq M_R \) will be dealt with. First, an equivalent channel matrix \( \mathbf{H}_{eq} \) is defined as

\[
\mathbf{H}_{eq} = \mathbf{D}^T \mathbf{H} \in \mathbb{C}^{d \times M_T},
\]

where \( \mathbf{D} \in \mathbb{R}^{M_T \times d} \) is the real-valued decoding matrix. In addition, we decouple the precoding matrix into two parts, i.e.,

\[
\mathbf{F} = \mathbf{F}_1 \mathbf{F}_2 \in \mathbb{C}^{M_T \times d},
\]

where \( \mathbf{F}_1 \in \mathbb{C}^{M_T \times M_T} \) and \( \mathbf{F}_2 \in \mathbb{R}^{M_T \times d} \).

The proposed coordinated beamforming algorithm is summarized as follows:

- **Step 1:** Initialize the decoding matrix \( \mathbf{D}^{(0)} \in \mathbb{R}^{M_T \times d} \), set the iteration index \( p \) to zero, and set a threshold \( \epsilon \) for the stopping criterion. The decoding matrix is generated randomly if the current subcarrier is the first one; otherwise set the decoding matrix as the one calculated for the previous subcarrier [12].

- **Step 2:** Set \( p = p + 1 \) and calculate the equivalent channel matrix \( \mathbf{H}_{eq}^{(p)} \) in the \( p \)th iteration as

\[
\mathbf{H}_{eq}^{(p)} = \mathbf{D}^{(p-1)^T} \mathbf{H} \in \mathbb{C}^{d \times M_T}.
\]

Define a matrix \( \tilde{\mathbf{H}}_{eq}^{(p)} \)

\[
\tilde{\mathbf{H}}_{eq}^{(p)} = \left[ \begin{array}{c} \text{Im} \{ \mathbf{H}_{eq}^{(p)} \} \\ \text{Re} \{ \mathbf{H}_{eq}^{(p)} \} \end{array} \right] \in \mathbb{R}^{d \times 2M_T}.
\]

- **Step 3:** Calculate the precoding matrix \( \mathbf{F}^{(p)} = \mathbf{F}_1^{(p)} \mathbf{F}_2^{(p)} \) for the \( p \)th iteration. First, we perform the singular value decomposition (SVD) of \( \tilde{\mathbf{H}}_{eq}^{(p)} \) as

\[
\tilde{\mathbf{H}}_{eq}^{(p)} = \mathbf{U}_1^{(p)} \Sigma_1^{(p)} \mathbf{V}_1^{(p)^T}.
\]

Denoting the rank of \( \tilde{\mathbf{H}}_{eq}^{(p)} \) as \( r^{(p)} \), we define \( \mathbf{V}_1^{(p)} \in \mathbb{R}^{2M_T \times M_T} \) as containing the last \( M_T = 2M_T - r^{(p)} \) right singular vectors that form an orthonormal basis for the null
space of $\hat{H}^{(p)}_{eq,1x}$. Hence, the first part of the precoding matrix for the $l$th iteration $F_{l}^{(p)}$ can be obtained via

$$V_{1,0}^{(p)} = \begin{bmatrix} \text{Re}\{F_{1}^{(p)}\} \\ \text{Im}\{F_{1}^{(p)}\} \end{bmatrix} \in \mathbb{R}^{2M_T \times M_d}. \quad (13)$$

To further calculate the second part of the precoding matrix in the $l$th iteration $F_{l}^{(p)}$, the following equivalent channel matrix after the cancellation of the intrinsic interference for the $l$th iteration is defined as

$$\hat{H}^{(p)}_{eq,1x} = \text{Re}\left\{H^{(p)}_{eq,1x}F_{1}^{(p)}\right\} \in \mathbb{R}^{d \times M_d}. \quad (14)$$

Further calculate the SVD of $\hat{H}^{(p)}_{eq,1x}$ and define $V_{2,1}^{(p)} \in \mathbb{R}^{M_d \times d}$ as containing the first $d$ right singular vectors. Thereby, $F_{l}^{(p)}$ is obtained as $F_{l}^{(p)} = V_{2,1}^{(p)}.$

**Step 4:** Update the decoding matrix based on the equivalent channel matrix after the cancellation of the intrinsic interference where only the processing at the transmitter is considered

$$\tilde{H}^{(p)}_{eq,1x} = \text{Re}\left\{H^{(p)}F_{l}^{(p)}\right\} \in \mathbb{R}^{d \times M_d}. \quad (15)$$

When the MMSE receiver\(^2\) is used, the decoding matrix has the following form

$$D^{(p)} = \tilde{H}^{(p)}_{eq,1x} \left(\tilde{H}^{(p)\dagger}_{eq,1x} \tilde{H}^{(p)}_{eq,1x} + \sigma_n^2 I_d\right)^{-1} \quad (16)$$

**Step 5:** Calculate the term $\Delta(F)$ defined as

$$\Delta(F) = \left\| F^{(p+1)} - F^{(p)} \right\|^2_F, \quad (17)$$

which measures the change of the precoding matrix $F$. If $\Delta(F) < \epsilon$, the convergence is achieved, and the iterative procedure terminates. Otherwise go back to **Step 2**.

It is important to note that in the special case where $M_T = M_R = d + 1$, we propose to compute the decoding matrix in the initialization step as follows. Then it is observed that the coordinated beamforming technique only needs two iterations to converge. First, calculate the SVD of $\tilde{H} \in \mathbb{R}^{M_R \times 2M_T}$ as defined in (7), and let $V_{1,0}^{(0)}$ contain the last $(2M_T - M_R)$ right singular vectors. Defining $F_{1}^{(0)}$ via

$$V_{1,0}^{(0)} = \begin{bmatrix} \text{Re}\{F_{1}^{(0)}\} \\ \text{Im}\{F_{1}^{(0)}\} \end{bmatrix}, \quad (18)$$

we compute $U_{2,1}^{(0)} \in \mathbb{R}^{M_R \times d}$ from the SVD of $\text{Re}\left\{H^{(0)}F_{1}^{(0)}\right\}$ such that it contains the first $d$ left singular vectors. Then the decoding matrix for the initialization step is chosen as

$$D^{(0)} = U_{2,1}^{(0)}. \quad (19)$$

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**Application in multi-user MIMO downlink systems**

In case of a multi-user MIMO downlink system where SDMA is employed, one base station equipped with $M_{R}^{(BS)}$ transmit antennas transmit to $Q$ users at the same time and on the same frequency. The number of receive antennas of the $q$th user is denoted by $M_{t,q}$, and the total number of receive antennas of all users severally simultaneously is then $M_{R}^{(tot)} = \sum_{q=1}^{Q}M_{t,q}$. The joint receive vector on the $k$th subcarrier and at the $n$th time instant is denoted by $y_{k,n} = \left[y_{k,1}^{(q)}[n] \quad y_{k,2}^{(q)}[n] \quad \ldots \quad y_{k,Q}^{(q)}[n]\right]^T \in \mathbb{C}^{M_{R}^{(tot)}}$, where the received signals of all $Q$ users are stacked and can also be represented by $\mathbf{y}$. Unlike the single-user case, $\mathbf{H}_{k}^{(q)} \in \mathbb{C}^{M_{R}^{(tot)} \times M_{t,q}^{(BS)}}$ denotes the joint channel matrix of all $Q$ users\(^3\) and is written as

$$\mathbf{H}_{k}^{(q)} = \left[ \mathbf{H}_{k,1}^{(q)}[n] \quad \mathbf{H}_{k,2}^{(q)}[n] \quad \ldots \quad \mathbf{H}_{k,Q}^{(q)}[n]\right]^T, \quad (20)$$

where $\mathbf{H}_{k,q}^{(q)} \in \mathbb{C}^{M_{R}^{(tot)} \times M_{t,q}^{(BS)}}$ represents the channel frequency response between the base station and the $q$th user, $q = 1, 2, \ldots, Q$. The data vector $d_{k}^{(q)} \in \mathbb{R}^d$ with $d = \sum_{q=1}^{Q}d_{q}$ is expressed as

$$d_{k}^{(q)} = \left[ d_{k,1}^{(q)} \quad d_{k,2}^{(q)} \quad \ldots \quad d_{k,Q}^{(q)}\right]^T, \quad (21)$$

where $d_{k,q} \in \mathbb{R}^{d_{q}}$ denotes the signal for the $q$th user on the $k$th subcarrier and at the $n$th time instant. Moreover, $F_{k}^{(q)} \in \mathbb{C}^{M_{R}^{(tot)} \times d_{q}}$ contains the precoding matrices for all users

$$F_{k}^{(q)}[n] = \left[ F_{k,1}^{(SU)}[n] \quad F_{k,2}^{(SU)}[n] \quad \ldots \quad F_{k,Q}^{(SU)}[n]\right], \quad (22)$$

where $F_{k,q}^{(SU)}[n] \in \mathbb{C}^{M_{R}^{(tot)} \times d_{q}}$, $q = 1, 2, \ldots, Q$, are calculated to mitigate the multi-user interference by employing, e.g., block diagonalization (BD) [14], and a multi-user MIMO system is decoupled into parallel equivalent single-user transmissions. Then $F_{k,q}^{(SU)}[n] \in \mathbb{C}^{M_{R}^{(tot)} \times d_{q}}$ is the transmit beamforming matrix for the resulting equivalent single-user system. When the equivalent number of transmit antennas $M_{T}^{(eq)} = M_{R}^{(BS)} - \sum_{q=1}^{Q}d_{q}$ (when BD is used) is equal to $M_{t,q}$, the proposed coordinated beamforming technique can be applied to compute $F_{k,q}^{(SU)}[n]$ for the $q$th user. Note that this is also a scenario where the BD based approach [16] designed for the FBMC/OQAM based multi-user MIMO downlink fails to provide a satisfactory performance.

**4. SIMULATION RESULTS**

In this section, we evaluate the bit error rate (BER) performance of the proposed coordinated beamforming technique. For all examples, the number of subcarriers is 1024, and the total bandwidth is 10 MHz. In case of CP-OFDM, the length of the CP is set to 1/4 of the symbol period. The ITU Veh-A channel [17] is adopted. Moreover, the PHYDYAS prototype filter [15] with the overlapping factor $K = 4$ is employed. The data symbols are drawn from a 16 QAM constellation. First, we consider a point-to-point system where $M_{T} = M_{R} = 6$, and $d = 5$. The BER performances of three schemes for MIMO FBMC/OQAM systems are presented and also compared to that of a MIMO CP-OFDM system in Fig. 1.

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\(^2\)Other receivers, such as zero forcing or maximum ratio combining, can also be employed in this coordinated beamforming algorithm.

\(^3\)Here we only provide the formulas of the channel matrices, precoding matrices, and data vectors on the $k$th subcarrier and at the $n$th time instant explicitly due to limited space. In case of the $i$th subcarrier and the $l$th time instant, the corresponding expressions can be obtained by replacing $k$ and $n$ with $i$ and $l$, respectively.
can be found that the proposed coordinated beamforming technique yields the best performance. Note that the decoding matrix is initialized as (19). In this case, we observe that it takes only two iterations to converge. On the other hand, the transmission scheme that is a straightforward extension of the CP-OFDM case as described in Section 2.1 relies on the assumption that the channel frequency responses remain the same across adjacent subcarriers. As the ITU Veh-A channel exhibits frequency selectivity and such an assumption is therefore violated, the performance of this scheme degrades especially in the high signal-to-noise ratio (SNR) regime. It can also be observed that in this symmetric scenario, the ZF based precoding cannot be employed in such a scenario. In case of the proposed coordinated beamforming scheme, the receiver is informed of the decoding matrix. As an example, the performance degradation due to the frequency selectivity of the channel as the SNR increases.

In the last experiment, a multi-user MIMO downlink setting is considered, where the base station is equipped with $M_{BS} = 8$ transmit antennas, each of the two users has 4 receive antennas, and the number of spatial streams transmitted to each user is 3. The multi-user interference is first mitigated by using the BD algorithm [14], and the four schemes considered in the first example are employed in the resulting $4 \times 4$ equivalent point-to-point transmission.

In the implementation of this algorithm, only the ZF based step that ensures the cancellation of the intrinsic interference is considered. The remaining part of the transmit processing (spatial mapping) and the receive processing (MMSE receiver) are chosen to be the same as the other schemes for the purpose of a fair comparison. Note that the precoding algorithm proposed in [10] is dominated by the ZF based step.

**Fig. 1.** Comparison of the BER performances of different schemes in a system where $M_T = M_R = 6$, and the ITU Veh-A channel is considered; CBF - coordinated beamforming.

**Fig. 2.** Comparison of the BER performances of different schemes in a system where $M_T = 0$, $M_R = 7$, $d = 5$, and the ITU Veh-A channel is considered; CBF - coordinated beamforming.

**Fig. 3.** Comparison of the BER performances of different schemes in a multi-user MIMO system where $Q = 2$, $M_T^{(BS)} = M_R^{(tot)} = 8$, $d = 6$, the ITU Veh-A channel is considered, and BD [14] is employed as the multi-user predcing algorithm to mitigate multi-user interference; CBF - coordinated beamforming.

5. CONCLUSIONS

In this work, we have developed a coordinated transmit beamforming algorithm for MIMO FBMC/OQAM systems to alleviate the dimension constraint or the strict requirement for flat fading channels that limit the state-of-the-art schemes. In the proposed approach, the calculation of the precoding matrix and the decoding matrix is carried out jointly in an iterative manner. Nevertheless, it is not required that the receiver is informed of the decoding matrix. As an example, the MMSE receiver of the resulting equivalent channel can be used. Numerical results show that the proposed precoding technique outperforms the state-of-the-art algorithms with only a moderate additional complexity in single-user MIMO as well as multi-user MIMO downlink systems. For a special scenario where $M_T = M_R = d+1$, it is observed that the coordinated beamforming algorithm requires only two iterations to converge.
6. REFERENCES


