JOINT TRANSCEIVER DESIGN FOR MISO SWIPT INTERFERENCE CHANNEL

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ABSTRACT
This paper considers a MISO interference channel with simultaneous wireless information and power transfer. We aim to jointly optimizing transmit beamformers and receive power splitting factors to minimize the total transmission power subject to both the signal-to-interference-plus-noise ratio constraints and energy harvesting constraints. We propose relaxation solution to the power minimization problem and provide an easily-checkable sufficient condition to confirm when the relaxation solution is optimum. Moreover, we propose a simple suboptimal solution to the power minimization problem when the sufficient optimality condition does not hold. Simulation results indicate that the proposed solution outperforms the existing suboptimal solution and reaches optimality.

Index Terms— Interference channel, simultaneous wireless information and power transfer, power minimization, power splitting, semidefinite relaxation

1. INTRODUCTION

Recently, simultaneous wireless information and power transfer (SWIPT) has attracted a great deal of attentions [1–10]. By SWIPT, receivers can not only receive wireless information but also wireless energy. It brings new challenges on transceiver design for wireless communication system. Initial research works in the field of SWIPT focused on point-to-point single-antenna SWIPT systems [1–3], while multi-antenna SWIPT systems have been investigated in [4–9]. This paper studies multi-antenna SWIPT interference channel.

[4] considered a MIMO broadcast channel which consists of a transmitter and two separated or co-located receivers: one is information receiver and the other is energy receiver. For such a simple broadcast system, the authors of [4] characterized the rate-energy region and investigated the optimal transmission schemes. By considering bounded channel estimation errors, [5] studied quality of service (QoS)-constrained robust beamforming for a two-user MISO

SWIPT system with separated information/energy receivers. [6] extended [4, 5] and investigated the optimal information/energy beamforming strategy to achieve the maximum harvested energy for multi-user MISO SWIPT system with separated information/energy receivers. [7] considered an orthogonal space-time block codes (OSTBC)-based two-hop multi-antenna amplify-and-forward (AF) relay system with a multi-antenna energy harvesting receiver, and designed optimal source/relay precoder to achieve different tradeoffs between the information rate and power transfer.

A MISO SWIPT interference channel (IFC) with power-splitting-based receivers [4] was considered in [8]. The authors of [8] have tried joint design of transmit beamformers and receive power splitting factors to minimize the transmission power under both the SINR constraints and the energy harvesting constraints. Due to the difficulty in the non-convex power minimization problem, [8] proposed suboptimal beamforming and power splitting methods by considering special zero-forcing and maximum ratio transmission schemes. In [9], the power minimization problem for multi-user MISO SWIPT downlink system was investigated and a globally optimum solution was proposed by using the celebrated semidefinite relaxation (SDR) technique [12]. Although the SDR technique can be straightforwardly extended to the power minimization problem of the MISO SWIPT IFC, unlike the multi-user MISO downlink case, it is difficult (maybe impossible) to prove that the SDR solution must be of rank one for the MISO IFC case.

In this paper, we attempt to solve the power minimization problem of the MISO SWIPT IFC by relaxation techniques. On one hand, we apply the well-known semidefinite relaxation to the power minimization problem to obtain a convex problem. On the other hand, we do a further relaxation to the SDR problem to guarantee a rank one solution. We provide an easily checkable sufficient condition under which the relaxation solution is optimum to the power minimization problem. Moreover, we propose a simple suboptimal solution when the relaxation solution is not optimum to the power minimization problem.

Notations: \( \mathbb{C}^{m \times n} \) denotes the space of \( m \times n \) complex matrices. For a square matrix \( A \), \( \text{Tr}(A) \), \( \text{Rank}(A) \), \( A^H \) denote its trace, rank, conjugate transpose respectively, while \( A \succeq 0 \) (or \( A \succeq 0 \)) means that \( A \) is a positive semidefinite (or

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positive definite) matrix. \( I \) denotes identity matrix whose dimension will be clear from the context. The distribution of a circularly symmetric complex Gaussian (CSCG) random vector variable with mean \( \mu \) and covariance matrix \( C \) is denoted by \( \mathcal{CN}(\mu, C) \), and ‘~’ stands for ‘distributed as’.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a multi-user MISO interference channel where \( K \) pairs of transmitters and receivers operate over a same frequency band. Differently from the conventional IFC, we here consider receivers capable of extracting both information and energy simultaneously from the received signal using a power splitting (PS) scheme [10]. Specifically, as shown in Fig. 1, receiver \( k \) (i.e., \( R_k \)) splits the received signal into two parts. A fraction, \( \rho_k \), of the received signal is used for further signal processing and symbol detection, and the remaining is driven to the energy harvesting (EH) circuit for conversion to DC voltage and energy storage.

![Fig. 1. A power splitting-based MISO SWIPT IFC model.](image)

We assume that each transmitter of multiple antennas transmits signals to its intended receiver through beamforming. Let \( N_k \geq 1 \) be the number of antennas at transmitter \( k \), \( \mathbf{v}_k \in \mathbb{C}^{N_k \times 1} \) be the transmit beamforming vector employed by transmitter \( k \), and \( s_k \sim \mathcal{CN}(0,1) \) be the transmitted symbol intended for receiver \( k \). Then the received signal at receiver \( k \) before power splitting can be mathematically expressed as

\[
y_k = \sum_{j=1}^{K} \mathbf{h}_{kj}^H \mathbf{v}_j s_j + n_k, \quad \forall k, \tag{1}
\]

where \( n_k \sim \mathcal{CN}(0, \sigma_n^2) \) is an additional white Gaussian noise (AWGN) introduced by the receiving antenna, \( \mathbf{h}_{kj}^H \) denotes the channel between transmitter \( j \) and receiver \( k \).

After power splitting, the signal for information decoding at receiver \( k \) is modeled as

\[
y_k^{ID} = \sqrt{\rho_k} \left( \sum_{j=1}^{K} \mathbf{h}_{kj}^H \mathbf{v}_j s_j + n_k \right) + z_k, \quad \forall k, \tag{2}
\]

where \( z_k \sim \mathcal{CN}(0, \sigma_z^2) \) is an AWGN due to RF to baseband signal conversion [10], and the signal for energy harvesting is

\[
y_k^{EH} = \sqrt{1 - \rho_k} \left( \sum_{j=1}^{K} \mathbf{h}_{kj}^H \mathbf{v}_j s_j + n_k \right), \quad \forall k. \tag{3}
\]

We assume all the noises \( n_k \)'s, \( z_k \)'s and symbols \( s_k \)'s are independent from each other. Then the SINR metric for symbol detection at user \( k \) is given by:

\[
\text{SINR}_k = \frac{\rho_k | \mathbf{h}_{kk}^H \mathbf{v}_k |^2}{\sum_{j \neq k} \rho_k | \mathbf{h}_{kj}^H \mathbf{v}_j |^2 + \rho_k \sigma_k^2 + \delta_k^2}, \quad \forall k, \tag{4}
\]

and the harvested power at user \( k \) is expressed as

\[
E_k = \zeta_k (1 - \rho_k) \left( \sum_{j=1}^{K} | \mathbf{h}_{kj}^H \mathbf{v}_j |^2 + \sigma_k^2 \right), \quad \forall k, \tag{5}
\]

where \( \zeta_k \in (0,1) \) denotes the conversion efficiency of the energy harvesting unit.

In this paper, we jointly design transmit beamformers and PS factors to minimize the total transmission power under both the SINR constraints and EH constraints. Mathematically, it can be expressed as

\[
\begin{align*}
\min_{\{\mathbf{v}_k, \rho_k\}} & \quad \sum_{k=1}^{K} ||\mathbf{v}_k||^2 \\
\text{s.t.} & \quad \frac{\rho_k | \mathbf{h}_{kk}^H \mathbf{v}_k |^2}{\sum_{j \neq k} \rho_k | \mathbf{h}_{kj}^H \mathbf{v}_j |^2 + \rho_k \sigma_k^2 + \delta_k^2} \geq \gamma_k, \quad \forall k, \\
& \quad \zeta_k (1 - \rho_k) \left( \sum_{j=1}^{K} | \mathbf{h}_{kj}^H \mathbf{v}_j |^2 + \sigma_k^2 \right) \geq e_k, \quad \forall k, \\
& \quad 0 \leq \rho_k \leq 1, \quad \forall k
\end{align*} \tag{6}
\]

where \( \gamma_k \)'s and \( e_k \)'s are the SINR and EH targets respectively.

Clearly, problem (6) is nonconvex and difficult to solve even when \( \rho_k \)'s are fixed. [8] has considered problem (6). But the authors of [8] proposed only suboptimal solutions with fixed beamformers. In this paper, we attempt to solve problem (6) by using relaxation techniques. Before we proceed to present our relaxation solution, we first study the feasibility of problem (6). The following lemma shows that the feasibility of problem (6) is independent of PS factors and EH constraints.

**Lemma 2.1.** [9, Lemma 3.1 & 3.2] Problem (6) is feasible if and only if problem (7) is feasible.

\[
\text{find } \{\mathbf{v}_k\} \\
\text{s.t.} \quad \frac{| \mathbf{h}_{kk}^H \mathbf{v}_k |^2}{\sum_{j \neq k} | \mathbf{h}_{kj}^H \mathbf{v}_j |^2 + \sigma_k^2 + \delta_k^2} \geq \gamma_k, \quad \forall k. \tag{7}
\]
It was shown in [11] that the feasibility of problem (7) can be easily tested through a simple iterative algorithm based on uplink-downlink duality theory. Hence, the feasibility of problem (6) can be easily checked. Without loss of generality, in the rest of this paper, we assume that problem (6) is feasible.

3. THE PROPOSED SOLUTION TO PROBLEM (6)

In this section, we first derive a relaxation of problem (6) by doing twice relaxations. Then we show an easily-checkable sufficient condition under which the relaxation solution is optimal to problem (6). Finally, we summarize the proposed algorithm at the end of this section.

3.1. A Relaxation of Problem (6)

Define $X_k = v_k v_k^H$. Similar to [9], by using SDR, problem (6) can be relaxed as the following convex problem

$$\min \left\{ \sum_{k=1}^K \text{Tr}(X_k) \right\}$$

s.t. $\frac{1}{\gamma_k} h_k^H X_k h_k - \sum_{j \neq k} h_j^H X_j h_j \geq \sigma_k^2 + \frac{\delta_k^2}{\rho_k}, \ \forall k,\ (8)$

$$\sum_{j=1}^K h_j^H X_j h_j \geq \frac{e_k}{\zeta_k(1-\rho_k)} - \sigma_k^2, \ \forall k,$$

$$X_k \succeq 0, \ \forall k,$$

$$0 \leq \rho_k \leq 1, \ \forall k.$$ 

Problem (8) can be solved by some off-the-shelf convex optimization tools (e.g., CVX [13]). However, due to the different channel structure from that of the multiuser MISO SWIPT downlink case [9], it is very difficult (maybe impossible) to prove that the solution to problem (8) must be of rank one. Furthermore, if the solution to problem (8), $\{X_k\}$, is not of rank one, we need to recover a feasible solution from $\{X_k\}$ for problem (6), which is obviously not easy. Hence, we will do a further relaxation to problem (8) in order to get an easy feasible solution recovery method.

By replacing the second constraint with the sum of the first and second constraints in problem (8), we obtain the following convex problem

$$\min \left\{ \sum_{k=1}^K \text{Tr}(X_k) \right\}$$

s.t. $\frac{1}{\gamma_k} h_k^H X_k h_k - \sum_{j \neq k} h_j^H X_j h_j \geq \sigma_k^2 + \frac{\delta_k^2}{\rho_k}, \ \forall k,\ (9)$

$$\left( 1 + \frac{1}{\gamma_k} \right) h_k^H X_k h_k \geq \frac{e_k}{\zeta_k(1-\rho_k)} + \frac{\delta_k^2}{\rho_k}, \ \forall k,$$

$$X_k \succeq 0, \ \forall k,$$

$$0 \leq \rho_k \leq 1, \ \forall k,$$

which is clearly a further relaxation of problem (8). Unlike problem (8), we can prove that problem (9) must have rank one solution.

3.2. A Sufficient Optimality Condition

The following proposition shows that the solution to problem (9) must be of rank one. Moreover, a sufficient optimality condition is provided to confirm when the solution to the relaxation problem (9) is optimal to problem (6).

**Proposition 3.1.** Let $\{\tilde{\rho}_k, \tilde{X}_k\}$ denote a solution to problem (9). We have

1) $\text{Rank}(\tilde{X}_k) = 1, \ \forall k.$ Moreover, $\{\tilde{X}_k\}$ can be expressed as $\tilde{X}_k = \tilde{v}_k \tilde{v}_k^H, \ \forall k.$

2) If $\Delta \triangleq \min_k \left( \tilde{\rho}_k - \frac{\delta_k}{\sqrt{\zeta_k + \delta_k}} \right) > 0, \ \forall k,$ then $\{\tilde{v}_k\}$ is an optimal solution to problem (6).

**Proof.** Let $\tilde{\lambda}_k$ and $\tilde{\mu}_k$ be the optimal dual variables respectively associated with the first and second constraint of problem (9). Let $\bar{Y}_k$ be the optimal dual variable associated with the constraint $\tilde{X}_k \succeq 0.$ The optimal primal variable $\{\tilde{\rho}_k, \tilde{X}_k\}$ and the optimal dual variable $\{\tilde{\lambda}_k, \tilde{\mu}_k, \bar{Y}_k\}$ must satisfy the KKT condition\(^1\) of problem (9), i.e.,

$$\bar{Y}_k = \left( 1 + \sum_{j \neq k} \tilde{\lambda}_j h_j^H h_k^H - \beta_k h_k h_k^H \right) \geq 0, \ \ \ (10a)$$

$$\left( \tilde{\lambda}_k + \tilde{\mu}_k \right) e_k = \frac{\tilde{\mu}_k e_k}{\zeta_k(1-\rho_k)^2}, \ \ (10b)$$

$$\bar{Y}_k \tilde{X}_k = 0, \ \ (10c)$$

$$\tilde{\lambda}_k \left( \frac{1}{\gamma_k} h_k^H \tilde{X}_k h_k - \sum_{j \neq k} h_j^H \tilde{X}_j h_j - \sigma_k^2 - \frac{\delta_k^2}{\rho_k} \right) = 0, \ \ (10d)$$

$$\tilde{\mu}_k \left( 1 + \frac{1}{\gamma_k} \right) h_k^H \tilde{X}_k h_k - \frac{e_k}{\zeta_k(1-\rho_k)} - \frac{\delta_k^2}{\rho_k} = 0, \ \ (10e)$$

$$\tilde{\lambda}_k, \tilde{\mu}_k \geq 0, \ \forall k, \ \ (10f)$$

where $\beta_k \triangleq \left( \frac{\tilde{\lambda}_k}{\tilde{\mu}_k} + \frac{1}{\gamma_k} \right) \tilde{\mu}_k.$

First, (10b) and (10f) imply that $\tilde{\mu}_k > 0$ and thus $\beta_k > 0, \ \forall k,$ otherwise we have $\bar{Y}_k > 0$ and thus $\tilde{X}_k = 0$ (i.e., a contradiction) according to (10a) and (10c). It follows that $\bar{Y}_k$ is rank $N_k - 1$ and thus $\text{Rank}(\tilde{X}_k) = 1, \ \forall k.$ This proves part 1.

Next, we prove part 2). From (10b), we have

$$\tilde{\lambda}_k = \tilde{\mu}_k \left( \frac{e_k \tilde{\mu}_k^2}{\zeta_k(1-\rho_k)^2} - \delta_k^2 \right), \ \forall k. \ \ (11)$$

By assumption that $\tilde{\rho}_k > \frac{\delta_k}{\sqrt{\zeta_k + \delta_k}}$ for all $k$, we have

$$\frac{e_k \tilde{\rho}_k^2}{\zeta_k(1-\rho_k)^2} - \delta_k^2 > 0, \ \forall k,$$

\(^1\)The detailed derivation of KKT condition is omitted due to page limitation.
which, together with (11) and the fact that \( \bar{\mu}_k > 0 \) for all \( k \), implies \( \lambda_k > 0, \forall k \). It follows that

\[
\frac{1}{\gamma_k} h_k^H X_k h_k - \sum_{j \neq k} h_j^H \bar{X}_j h_k - \sigma_k^2 \frac{e_k}{\zeta_k(1 - \bar{\rho}_k)} - \frac{\delta_k^2}{\bar{\rho}_k} = 0, \quad \forall k, \quad (12a)
\]

\[
(1 + \frac{1}{\gamma_k}) h_k^H \bar{X}_k h_k - \frac{e_k}{\zeta_k(1 - \bar{\rho}_k)} = 0, \quad \forall k \quad (12b)
\]

by noting (10d), (10e) and \( \bar{\lambda}_k, \bar{\mu}_k > 0, \forall k \). By subtracting (12a) from (12b), we have

\[
\sum_{j=1}^{K} h_j^H \bar{X}_j h_k + \sigma_k^2 - \frac{e_k}{\zeta_k(1 - \bar{\rho}_k)} = 0, \quad \forall k. \quad (13)
\]

(12a), (13) and \( \bar{X}_k = \bar{\nu}_k \bar{v}_k^H, \forall k \), implies that \( \{ \bar{\nu}_k \} \) satisfies the SINR and EH constraints of problem (6) with equality. Moreover, it is known that problem (9) is a relaxation of problem (6). Hence, we conclude that \( \{ \bar{\nu}_k, \bar{\rho}_k \} \) is an optimal solution to problem (6). This completes the proof. \( \square \)

Remark 3.1. The optimal value of problem (8) is upper bounded by the optimal value of problem (6) and lower bounded by the optimal value of problem (9). Hence, if \( \{ \bar{\nu}_k \} \) (obtained by solving problem (9) and eigen-value decomposition) is optimal to problem (6), the optimal value of problem (8) is the same as the optimal values of problem (6) and (9).

3.3. The Proposed Algorithm

It is seen from the proof of Proposition 3.1 that, the optimal solution to problem (9) must satisfy the second constraint of problem (9) with equality but may not for the first constraint. However, solving problem (9) can provide at least a feasible solution, i.e., \( \{ \bar{\nu}_k, \bar{\rho}_k \} \), such that the SINR constraints of problem (6). This helps us to develop a very simple suboptimal solution to problem (6) based on \( \{ \bar{\nu}_k, \bar{\rho}_k \} \) when \( \Delta = 0 \). The key idea of the suboptimal solution is to scale up \( \{ \bar{\nu}_k \} \) by a common factor \( \sqrt{\alpha} \) to satisfy all the constraints of problem (6) while the transmission power \( \sum_{k=1}^{K} \alpha ||\bar{\nu}_k||^2 \) is kept as small as possible. The optimal \( \alpha \) can be simply obtained in a closed-form

\[
\hat{\alpha} = \max_k \frac{e_k}{\zeta_k(1 - \bar{\rho}_k)} - \frac{\sigma_k^2}{\sum_{j=1}^{K} ||h_k^H \bar{v}_j||^2}
\]

To summarize, the proposed algorithm consists of the following two steps.

1) solve problem (9) and obtain \( \{ \bar{\nu}_k, \bar{\rho}_k \} \).

2) quit if \( \Delta > 0 \); otherwise, calculate \( \hat{\alpha} \) and obtain \( \{ \sqrt{\hat{\alpha}} \bar{v}_k, \bar{\rho}_k \} \).

Step 1 provides an optimal solution to problem (6) if \( \Delta > 0 \), while Step 2 provides a suboptimal solution to problem (6) in general. It is worth mentioning that, \( \{ \sqrt{\hat{\alpha}} \bar{v}_k, \bar{\rho}_k \} \) is an optimal solution to problem (6) if \( \hat{\alpha} = 1 \) in the second step.

4. NUMERICAL EXAMPLES

In this section, we numerically evaluate the performance of the proposed algorithm by comparing with the existing zero-forcing (ZF) beamforming based suboptimal algorithm [8] and the lower bound of problem (6) given by the optimal value of problem (8). The same parameters \( \delta^2, \sigma^2, e, \zeta, \gamma \) are set for different users. Moreover, we set \( \sigma^2 = \delta^2 = -10 \) dBm and \( \zeta = 1 \) in all our simulations. All channel coefficients are randomly and independently generated according to an identical CSCG distribution with zero mean and unit variance. Each result in our plots is averaged over 100 channel realizations.

In our simulations, we find that \( \Delta \) is almost always positive\(^2\). Figure 2 shows that the performance of the proposed solution coincides with the lower bound. This implies that the relaxation solution is optimal to problem (6). Moreover, the proposed solution outperforms the ZF solution and their performance gap decreases with the SINR target \( \gamma \). This is because that when \( \gamma \) is extremely large, the beamforming scheme must satisfy zero-forcing condition to make the SINR constraints of problem (6) feasible.

![Figure 2: Transmission power increases with \( e \) and \( \gamma \).](image-url)

5. CONCLUSION

This paper considers the QoS-constrained power minimization problem for MISO SWIPT IFC. We have proposed relaxation techniques to tackle the problem and give a sufficient optimality condition on the relaxation solution. The results of this paper is useful for practical design of transceiver in MISO SWIPT IFC.

\(^2\)This part of simulation results are omitted due to page limitation.
6. REFERENCES


