ESTIMATING AN OPTIMAL SETPOINT TO LESSEN ERRORS IN FILLING WEIGHING SYSTEM BASED ON KALMAN FILTERING

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ABSTRACT

A weighing system in which a sensor is not mounted to a discharger especially in vertical filling gives rise to an excess of weight added to the given target of weight. In addition, the excess is not constant on account of some factors, such as vibration of the machine, flow of the substance, and cycle time of the system. These factors cause the surplus to oscillate. To overcome this problem, Kalman filtering is performed to predict the optimal setpoint to meet the defined target. To illustrate the performance of the proposed technique, the resulting outcome is compared with that of using the conventional statistical method. The results have shown that the proposed approach has significantly increased the speed and lowered the error. It is pointed out that the proposed algorithm may be preferable to the traditional statistical technique due to its effectiveness and its simple implementation.

Index Terms—Estimation, excess of weight, filling weighing system, Kalman filter, prediction, setpoint

1. INTRODUCTION

A filling weighing machine (shown in Fig. 1) is a measuring system used to weigh a substance during filling/packaging process to a desirable quantity. A discharger releases the substance into a container until the sensor detects that the targeted weight has been filled. Generally, such system generates a delay in sensing data because the sensor does not sense the data at the same time as the substance is released. During weighing process, a setpoint of weight cannot equal a desired target of weight because at the time that the sensed weight equals the defined target of weight, there is an excess of weight which is not sensed left in the free fall state from the discharger. To visualize this problem, the mentioned issue is graphically shown in Fig. 2(a).

In Fig. 2(b), the setpoint is compensated by the surplus of weight in order to ensure the actual weight satisfy the defined target of weight. However, the compensation is not that simple since the surplus is not constant (see Fig. 2(a)) due to some interfering factors. These factors are produced by vibration of the machine, fluctuating flow of the substance and cycle time of weight reading of the system. To see why this is problematic, assume the machine in Fig. 1 operates 5 weighing cycles per minute where an averaged 6-grain error is in each cycle and the machine is operated 20 hours a day, 25 days a month. Approximately, the weighing error of one weighing machine is summed up to 2.16 tons of excessive substance a year.

Recently, several works have been presented to seek the appropriate setpoint. In [1], a set of excess of weight is collected within a period of time in order to calculate its average for compensating the setpoint. To use this scheme, some memories are needed to collect data and the recompensation of the setpoint is delayed due to the computation of average excess weight. Besides, the compensation may not be able to adapt to any changes in the surplus. In [2,3], the techniques are concentrated on improving a proportional-integral-derivative controller so that the setpoint and the desired target can be set identically. Since the substance is gradually and progressively filled, the excess weight is fairly low and can be neglected. Although these schemes provide good results, they are time-consuming and require large load processor. Later, the method based on fuzzy logic is proposed in [4] to avoid a mathematical model. Consequently, many rules must be created to cover many situations that the excess weight may occur.

In this work, a new approach which is simpler than the schemes in [1-4] is proposed. Not only the time consumption of the proposed technique is less, but also its memory usage is more efficient. In addition, a new technique for predicting the optimal setpoint is presented to improve the efficiency of the machine in Fig. 1. The proposed technique is compared with the compensation
The organization of this paper is as follows. The compensation design of the proposed algorithm is described in Section 2. The simulation and experimental results of forecasting the optimal setpoint are furnished in Section 3. Section 4 is devoted for the conclusion.

2. COMPENSATION DESIGN

In this section, the proposed technique is explained in details. The block diagram of our compensation design is depicted in Fig. 3.

2.1. Proposed technique

Since Kalman introduced his famous algorithm, Kalman filtering has been ubiquitously utilized in many applications [5]. In the model of Kalman filter, it is assumed that the state of a system at time \( k \) is inherited from the previous state at time \( k-1 \) as expressed in the following equation,

\[
X_k = AX_{k-1} + BU_k + W_k
\]  

(1)

where \( X_k \) is a linear summation of its prior state value plus a control signal and a process noise. The variable \( U_k \) is the control signal to the system, and \( W_k \) is the process noise referred to the vibration of the machine, fluctuating flow of the substance, and cycle time of weight reading of the system. \( A \) is the state transition weight and \( B \) is the control input weight.

The measurement \( Z_k \) of the system can be modeled as

\[
Z_k = HX_k + V_k
\]  

(2)

where \( Z_k \) is the measurement, \( H \) is the transformation weight and \( V_k \) is the measurement noise. The noises \( W_k \) and \( V_k \) are assumed to be zero-mean Gaussian with variances \( Q \) and \( R \), respectively.

In designing Kalman filter, a mathematical model is required to estimate the incoming value based on its current value. The state transition weight \( A \) and the control input weight \( B \) should be modeled into the mathematical model as well. However, no essential data is provided by the machine to do so. Hence, the state transition weight \( A \) is set to one, \( A=I \), owing to the system of the machine does not change from step to step. The control input weight \( B \) is also assigned to one, \( B=I \). In this system, the control signal \( U_k \) is not needed, so it is set to zero, \( U_k = 0 \). The only factor that can be mathematically modeled is the transformation weight \( H \).

In this work, the target weight values are chosen at 3005 grams, 3305 grams, 3505 grams, 4005 grams and 5005 grams. The total of 200 samples (40 samples for each target weight) are employed to determine the relationship between the value of excess weight and the target weight, as depicted in Fig. 4. According to the information given in Fig. 4, each selected weight value, the new setpoint is achieved by subtracting the average surplus from the selected weight value. The ratio of the selected target and the new setpoint is given in Table 1. In addition, noise variance and standard deviation of the average surplus are calculated. The relationship between the selected target-to-the new setpoint ratio and the new setpoint is plotted in Fig. 5. From Fig. 5, an exponential model is fitted to the data to extract the transformation weight \( H \) as expressed by

\[
H = 0.1855e^{0.0001(T_e)} + 1.000e^{-2.8548e^{-0.0007(T_e)}}
\]  

(3)

where \( T_e \) is a weight target. Substituting the variable \( T_e \) with 3005g, 3305g, 3505g, 4005g and 5005g, respectively, into (3), resulting in the transformation weights \( H \) given in Table 2.

2.2. Kalman filtering procedure

To achieve convergence, the Kalman filter estimates the process using feedback control where the feedback is obtained in a form of measurement. Accordingly, the Kalman filter involves two steps, namely the prediction update and correction update. The prediction update and correction update can be described by the following mathematical equations.
According to the assigned conditions, the obtained optimal setpoint varies between upper boundary and lower boundary so that the resulted setpoint is feasible. 

In the next section, the proposed technique is verified using MATLAB simulation and machine implementation.

### 3. Simulation and Experiment

#### 3.1. MATLAB simulation

In the MATLAB simulation, the desired target, \( T_p \), is set to 3005g. According to Table 1 and 2, \( R \) is 19.5537, \( H \) is 1.0130, and \( Q \) is 1. Since the proposed algorithm estimates the optimal setpoint from the previous setpoint, \( x_{00} \) is assigned to 2966g (from Table 1) and \( P_{00} \) is initially defined as 1. Additionally, upper boundary and lower boundary are set to 2985g and 2945g, respectively, using soft thresholding process.

To demonstrate the performance of finding the setpoint, a set of predefined 200 sensed weight values is arbitrarily generated and utilized in the proposed scheme and the technique in [1]. The comparison of simulation results of both techniques is depicted in Fig. 6 and the statistical data of the simulated results are listed in Table 3. As can be seen in the simulation result shown in Fig. 6, the proposed technique provides less error in excess weight than the result of the technique in [1]. In addition, although the values of the predefined sensed weight are oscillated (either higher or lower than the desired target), the setpoint of the proposed technique does not fluctuate as much as the result of the technique in [1].

#### 3.2. Experiment using the machine

In experimenting using the machine depicted in Fig.1, two targets, namely 3005g and 5005g, are experimented. At each desired target, the efficiency of the proposed technique and the technique of [1] in experiment is compared. It is also noted that 200 samples are collected in each method. Fig. 7 and 8 are the experimental results for the desired targets of 3005g and 5005g, respectively. The parameters employed in the proposed technique are listed in Table 4.

The information of the results given in Fig. 7 and 8 is summarized in Table 5. In addition, the results obtained from the proposed algorithm and the technique in [1] are compared in terms of the accuracy of weight and the speed of operation (per minute), as reported in Table 6. The accuracy of weight and the speed of operation are computed using the following equations,

\[
\%\text{error} = \frac{|W_{act} - W_{pt}|}{W_{pt}} \times 100
\]

\[
\%\text{SpdPrf} = \frac{S_{pt} - S_{111}}{S_{111}} \times 100
\]

where

- \( W_{act} \) is the actual weight,
- \( W_{pt} \) is the target of weight,
- \( SpdPrf \) is the speed performance,
- \( S_{pt} \) is the speed obtained by the proposed technique, and
- \( S_{111} \) is the speed acquired by the method of [1].

### Table 1. The relation of targets and setpoints

<table>
<thead>
<tr>
<th>Weight Targets</th>
<th>Selected weight targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>3005g</td>
<td>3305g</td>
</tr>
<tr>
<td>3305g</td>
<td>3505g</td>
</tr>
<tr>
<td>3505g</td>
<td>4005g</td>
</tr>
<tr>
<td>4005g</td>
<td>5005g</td>
</tr>
<tr>
<td>Average Surplus</td>
<td>39g</td>
</tr>
<tr>
<td>New Setpoint</td>
<td>2966g</td>
</tr>
<tr>
<td>Ratio</td>
<td>1.0131</td>
</tr>
<tr>
<td>Measurement Noise Variance</td>
<td>19.5537g^2</td>
</tr>
<tr>
<td>Standard Deviation of the Average Surplus</td>
<td>±4.4220g</td>
</tr>
</tbody>
</table>

### Table 2. Transformation weight at each selected target weight

<table>
<thead>
<tr>
<th>Transformation Weight</th>
<th>Weight Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0130</td>
<td>1.0108</td>
</tr>
<tr>
<td>1.0097</td>
<td>1.0076</td>
</tr>
<tr>
<td>1.0056</td>
<td></td>
</tr>
</tbody>
</table>

2.2.1. Prediction update

\[ \hat{x}_{k|k-1} = \hat{x}_{k|k-1} + w_k \]

\[ P_{k|k-1} = AP_{k|k-1}A^T + Q \]

where \( \hat{x}_{k|k-1} \) is the predicted state value at time \( k \), given corrected state at time \( k-1 \), \( \hat{x}_{k|k-1} \). \( P_{k|k-1} \) is the predicted error variance at time \( k \), given corrected error variance at time \( k-1 \), \( P_{k|k-1} \).

2.2.2. Correction update

\[ K_k = P_{k|k-1}H^T\left(HP_{k|k-1}H^T + R\right)^{-1} \]

\[ \hat{x}_k = \hat{x}_{k|k-1} + K_k( z_k - H\hat{x}_{k|k-1} ) \]

\[ P_k = (I - K_kH)P_{k|k-1} \]

where \( K_k \) is Kalman gain, \( \hat{x}_k \) is the corrected state at time \( k \), \( P_k \) is the corrected estimate of error variance at time \( k \).

2.3. Soft thresholding

For circumventing either too high or too low value of the surplus which can cause distortion of the setpoint value, soft thresholding is introduced to produce a set of conditions. These conditions are based on the measurement noise variance provided in Table 1 and are expressed as follows.

---

%error = \frac{|W_{act} - W_{pt}|}{W_{pt}} \times 100

%SpdPrf = \frac{S_{pt} - S_{111}}{S_{111}} \times 100

---

\[ W_{act} \] is the actual weight,
\[ W_{pt} \] is the target of weight,
\[ SpdPrf \] is the speed performance,
\[ S_{pt} \] is the speed obtained by the proposed technique, and
\[ S_{111} \] is the speed acquired by the method of [1].
Clearly, the experimental results indicate that the proposed algorithm is superior to the technique in [1] for both targets, 3005g and 5005g, in terms of accuracy and speed. This is because the proposed technique adjusts its setpoints to the optimal setpoints whereas [1] gradually adjusts it towards a constant setpoint. However, such constant setpoint may not be an optimal value. Therefore, the surplus occurred using the technique in [1] is still more than that using the proposed method.

4. CONCLUSION

In this paper, the new scheme for obtaining the optimal setpoint in filling weighing system is proposed. This technique provides a solution to predict an optimal setpoint based on Kalman filtering and soft thresholding. The proposed scheme is verified not only by simulation but also by testing with an actual machine. It is observed that the new algorithm significantly enhances the accuracy and speed when compared with the conventional statistical method. The obtained simulation and experimental results suggest that this new technique may be an alternative solution for a real application.

TABLE 3. STATISTICAL DATA OF THE SETPOINT (SIMULATION)

<table>
<thead>
<tr>
<th>Compensation Scheme</th>
<th>Av.M</th>
<th>Avg.SP</th>
<th>Ex</th>
<th>Var</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Technique</td>
<td>3005.5g</td>
<td>2967g</td>
<td>38g</td>
<td>7.790</td>
<td>2.7891</td>
</tr>
<tr>
<td>Method of [1]</td>
<td>2955g</td>
<td>51g</td>
<td>55.2552</td>
<td>7.4334</td>
<td></td>
</tr>
</tbody>
</table>

Av.M-Average Measured Weight, Avg.SP-Average Setpoint, Ex-Average Excess, Var-Variance of Setpoint, STD-Standard Deviation of Setpoint

TABLE 4. SETTING PARAMETERS (EXPERIMENT)

<table>
<thead>
<tr>
<th>Desired Target</th>
<th>H</th>
<th>x_40</th>
<th>P_0</th>
<th>Up.Bou</th>
<th>Lo.Bou</th>
</tr>
</thead>
<tbody>
<tr>
<td>3005g</td>
<td>1.0130</td>
<td>2966g</td>
<td>1</td>
<td>2985g</td>
<td>2945g</td>
</tr>
<tr>
<td>5005g</td>
<td>1.0056</td>
<td>4978g</td>
<td>1</td>
<td>4985g</td>
<td>4945g</td>
</tr>
</tbody>
</table>

Up.Bou-Upper Boundary, Lo.Bou-Lower Boundary,

**The remaining parameters are set similarly to the MATLAB simulation.**

TABLE 5. STATISTICAL DATA OF THE SETPOINT (EXPERIMENT)

<table>
<thead>
<tr>
<th>Compensation Scheme</th>
<th>Av.M</th>
<th>Avg.SP</th>
<th>Ex</th>
<th>Var</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Technique</td>
<td>3005.5g</td>
<td>2967g</td>
<td>39g</td>
<td>2.8606</td>
<td>1.6913</td>
</tr>
<tr>
<td>Method of [1]</td>
<td>3009.8g</td>
<td>2946g</td>
<td>64g</td>
<td>26.0502</td>
<td>5.1039</td>
</tr>
<tr>
<td>Proposed Technique</td>
<td>5009.3g</td>
<td>4982g</td>
<td>27g</td>
<td>5.7485</td>
<td>2.3976</td>
</tr>
<tr>
<td>Method of [1]</td>
<td>5011.1g</td>
<td>4946g</td>
<td>65g</td>
<td>28.1665</td>
<td>5.3072</td>
</tr>
</tbody>
</table>

TABLE 6. ACCURACY AND SPEED COMPARISON (EXPERIMENT)

<table>
<thead>
<tr>
<th>Compensation Scheme</th>
<th>Tg</th>
<th>Av.M</th>
<th>Var_w</th>
<th>STD_w</th>
<th>Error%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Technique</td>
<td>3005g</td>
<td>3005.5g</td>
<td>6.2814</td>
<td>2.5063</td>
<td>0.0166</td>
</tr>
<tr>
<td>Method of [1]</td>
<td>3009.8g</td>
<td>17.0629</td>
<td>4.1307</td>
<td>0.1604</td>
<td></td>
</tr>
<tr>
<td>Proposed Technique</td>
<td>5005g</td>
<td>5009.3g</td>
<td>12.7562</td>
<td>3.5716</td>
<td>0.0851</td>
</tr>
<tr>
<td>Method of [1]</td>
<td>5011.1g</td>
<td>14.4884</td>
<td>3.8064</td>
<td>0.1218</td>
<td></td>
</tr>
</tbody>
</table>

Tg-Target, Var_w-Variance of Measured Weight, STD_w-Standard Deviation of Measured Weight, SpdPrf-Speed Performance

Fig. 6. MATLAB simulation results.

Fig. 7. Optimal setpoints predicted by the proposed algorithm (left) vs setpoints obtained by the method of [1] (right) at 3005g.

Fig. 8. Optimal setpoints predicted by the proposed algorithm (left) vs setpoints obtained by the method of [1] (right) at 5005g.
5. REFERENCES


