DISTRIBUTED COOPERATIVE SPECTRUM SENSING WITH ADAPTIVE COMBINING

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ABSTRACT

This paper proposes a novel two-step distributed detection scheme for cooperative spectrum-sensing networks. In the first step, individual contributions from nodes within a neighborhood are fused through an adaptive combiner, which updates the weights and makes local decisions iteratively. In the second step, local decisions are shared within a neighborhood to yield a consensus decision. Results are presented in terms of complementary receiver operating characteristic curves and show the good behavior of the proposed scheme when compared to the optimal linear fusion rule, even if correlated node contributions are considered.

Index Terms— Cognitive radio, spectrum sensing, distributed detection, adaptive signal processing, least mean square algorithms

1. INTRODUCTION

Cognitive radio (CR) technology arises as a promising alternative for the spectrum allocation problem, since it allows a secondary user (SU) to share and opportunistically use the same band assigned to a primary user (PU) [1]. The efficiency of this dynamic spectrum management depends on how reliably the SU identifies the presence (or absence) of legacy users. Methods for spectrum sensing may involve energy detection, matched filter detection or feature detection. Matched filters are the best option if the SU knows the PU’s signal shape a priori, but energy detection offers easy implementation and requires less a priori information [2].

Detection performance can be improved by spatial cooperation among SUs, for a combination of their contributions can produce a more reliable decision. This scheme will be referred to herein as cooperative spectrum sensing. Some strategies consider the use of a fusion center, whose task is to collect all the individual sensing information, fuse them and make the decision. Several optimal and suboptimal strategies for centralized data fusion can be found in the literature. In [3] the authors suggest an algorithm for maximizing the probability of detection for linear fusion based on semidefinite programming. An online adaptive linear fusion based on orthogonal projections onto convex sets (POCS) is proposed in [4].

The centralized scheme may not be a good solution if the system is composed of a large number of nodes. Information centered at only one point would require the fusion center to be able to process a very large amount of data, in addition to being more sensitive to link failures. Furthermore, increasing distance between nodes require the radios to use more power and consequently increases network energy consumption [5]. Therefore, a distributed approach arises as a good alternative for the final decision made by each of several small neighborhoods [5]. Data is shared only among nodes within a neighborhood.

Although a rich literature can be found on the distributed spectrum sensing, few works relate to adaptive node cooperation strategies, which can offer better results under dynamic propagation environments [4]. In their recent research, Cattivelli and Sayed [6] proposed a distributed spectrum sensing technique with adaptive cooperation among nodes by reformulating the detection problem as a parameter estimation problem. However, their work considers that information about the user’s signal is available at every node, which may not be a valid assumption in some cases (e.g., several signal models sharing the same spectrum).

In this paper, we propose a novel cooperative network featuring distributed two-step combining with online adaptive cooperation. Each node employs a simple and conventional energy detector. The main goal of the suggested two-step combining is to distribute the decision tasks taking advantage of spatial diversity information from different neighborhoods. In its turn, the proposed online adaptive linear combining produces decisions after each update instant thus avoiding the need to wait until a large amount of data is processed.

The performance of the proposed network is investigated with simulations with uncorrelated and correlated node contributions. The results are presented by means of complementary receiver operating characteristic (C-ROC) curves and compared to the optimal linear fusion rule described in [3].

2. TWO-STEP DISTRIBUTED NETWORK

We consider a network of $M$ spatially distributed secondary users (nodes) that sense the environment under two hypotheses: $\mathcal{H}_0$ (absence of primary signal) or $\mathcal{H}_1$ (presence of primary signal). During a detection interval, each node, say node $k$, first produces a local energy estimate $y_k$, which is shared with its own neighbors defined by index set $\mathcal{N}_k$. The neighborhood $\mathcal{N}_k$ of node $k$ is here taken as the set of nodes, including itself, linked to it, say within a transmission radius [6]. Upon receiving the local energy estimates from the nodes in the neighborhood, a local (binary) decision $u_k$ is made based on a weighted combination of local energy estimates, referred to as the soft combining step. Finally, the local binary decisions are shared among the nodes within the neighborhood and are combined within each node to yield a local consensus decision, referred to as hard combining step. The following sections describe the two above mentioned steps, i.e., the soft and hard combining steps.

2.1. First Step: Soft Combining

Let us consider the neighborhood $\mathcal{N}_k$ of node $k$ with node degree $M_k = |\mathcal{N}_k|$. The main idea of this first step is to employ soft com-
bining with a linear fusion rule. That is, the local test statistic of node \( k \) is formed as

\[
T(y_k) = \sum_{i \in N_k} w_i y_i = w_k^T y_k = w_k^T \begin{bmatrix} u_k \end{bmatrix} \geq \gamma_k
\]

(1)

where \( w_k = [w_1, w_2, \ldots, w_{M_k}]^T \) and \( y_k = [y_1, y_2, \ldots, y_{M_k}]^T \) contain the respective weights and energy estimates in \( N_k \). The resulting test statistic \( T(y_k) \) is then compared to a local threshold \( \gamma_k \) to yield a local binary decision, \( u_k \).

Energy detectors [8] implemented at each node of the neighborhood collect information from the environment during the same sensing interval. Assuming that they use a sufficiently large number of samples for computing \( y_i, i \in N_k \), such estimates may be considered Gaussian variables under each hypothesis [8], then the test statistics \( T(y_k) \) is also Gaussian [3]:

\[
T(y_k) \sim \begin{cases} 
\mathcal{N}(w_k^T \mu_{k,0}, w_k^T \Sigma_{k,0} w_k), & \text{for } H_0 \\
\mathcal{N}(w_k^T \mu_{k,1}, w_k^T \Sigma_{k,1} w_k), & \text{for } H_1
\end{cases}
\]

(2)

where \( \mu_{k,0} \) and \( \Sigma_{k,0} \) are the mean vector and covariance matrix of \( y_k \) under hypothesis \( H_0(H_1) \), as defined in [9].

The detection performance of node \( k \) after employing soft combination can thus be evaluated with help of the following expressions for probabilities of false alarm (\( P_{f,k,1} \)) and detection (\( P_{d,k,1} \)):

\[
P_{f,k,1} = P(T(y_k) \geq \gamma_k | H_0) = Q\left( \frac{\gamma_k - w_k^T \mu_{k,0}}{\sqrt{w_k^T \Sigma_{k,0} w_k}} \right)
\]

(3a)

\[
P_{d,k,1} = P(T(y_k) \geq \gamma_k | H_1) = Q\left( \frac{\gamma_k - w_k^T \mu_{k,1}}{\sqrt{w_k^T \Sigma_{k,1} w_k}} \right)
\]

(3b)

where \( Q(\cdot) \) is the complementary cumulative distribution function.

2.2. Second Step: Hard Combining

In the second step, each node shares its local binary decision (obtained in the first step) with its neighbors. Each node then combines the received decisions with its local decision using conventional hard combining (OR-fusion rule) to render a local consensus decision [10]. The OR-fusion rule implies that node \( k \) decides \( H_1 \) if at least one of the \( M_k \) nodes has suggested \( H_1 \).

To evaluate the detection performance at node \( k \) after this second step, the Bahadur-Lazarsfeld expansion [11] is used to calculate the probabilities of false alarm and detection considering correlation among neighbors’ decisions \( u_i, i \in N_k \), made in the first step. Let \( u_k \in \{0, 1\}^{|N_k|} \) be the local decisions of the neighborhood \( N_k \).

The expressions of \( P_{f,k,2} \) and \( P_{d,k,2} \) for the OR-fusion rule are then given by

\[
P_{f,k,2} = 1 - P(u_k = [0, 0, \ldots, 0] | H_0) \quad (4a)
\]

\[
P_{d,k,2} = 1 - P(u_k = [0, 0, \ldots, 0] | H_1) \quad (4b)
\]

In order to write \( P(u_k | H_h) \) in a convenient form, we need first to normalize the binary random variables \( u_i, i \in N_k \), conditioned on hypothesis \( H_h, h \in \{0, 1\} \), as follows [11]:

\[
z_h^k = \frac{u_k - P(u_k = 1 | H_h)}{\sqrt{P(u_k = 1 | H_h)[1 - P(u_k = 1 | H_h)]}}
\]

(5)

Thus, a general expression for \( P(u_k | H_h) \), according to Bahadur-Lazarsfeld, is [11]

\[
P(u_k | H_h) = \prod_{i,j,l,\ldots} P(u_i | H_h) \left[ 1 + \sum_{i<j} \rho_{ij} z_h^i z_h^j + \sum_{i<j<l} \rho_{ijk} z_h^i z_h^j z_h^l + \ldots \right]
\]

(6)

where \( \rho^h \) are the correlations of the normalized random variables \( z_h^i \), computed as \( \rho_{ij}^h = E[z_h^i z_h^j] \) \( i,j,\ldots \in N_k [11] \). We note, with help of (5), that \( \rho^h \) are equivalent to the correlation coefficients of the neighbors’ decisions \( u_i \) conditioned on hypothesis \( H_h \). Also note in (6) that for the hypothesis \( H_0 \) the term \( P(u_i = 0 | H_0) \) corresponds to \( (1 - P_{f,i,1}) \), whereas for \( H_1 \) the term \( P(u_i = 0 | H_1) \) is equal to \( (1 - P_{d,i,1}) \).

Finally, with \( P(u_k | H_h) \) calculated in (6), the performance of the final two-step network can be evaluated in (4). Although only local decisions of neighboring nodes are combined, each node \( i \) has previously made its local decision \( u_i \) after taking into account contributions (the vector of test statistics \( y_i \)) within its own neighborhood \( N_i \) (see Section 2.1). Therefore this second step is, in the final analysis, a fusion of information among neighborhoods.

3. ADAPTIVE WEIGHT UPDATE ALGORITHM

The linear combination in (1) must be done in such a way that it guarantees desired probabilities of false alarm and detection in (3). Therefore detector performance at first step depends on both the set of weights and the threshold chosen. Quan et al. proposed in [3] that obtaining these optimal parameters can be treated as an unconstrained optimization problem of (3b) for a given desired probability of false alarm in (3a). This technique, referred to herein as optimal linear fusion, offers performance comparable to the optimal likelihood-ratio test (LRT) rule, but its solution is not trivial [3].

An alternative (suboptimal) approach is based on the observation that the sum of energy estimates obtained at node \( k \) in each instant \( n \) varies around the sum of their means under both hypotheses. This can be viewed as an optimization problem for minimizing

\[
E[e_k^2] = E[(r_k - w_k^T y_k)^2] \quad (7)
\]

the mean squared error (MSE) between the local test statistic \( T(y_k) \) and a reference signal, defined as

\[
r_k = \begin{cases} 
1^T \mu_{k,0} & \text{for } H_0 \\
1^T \mu_{k,1} & \text{for } H_1
\end{cases}
\]

(8)

where \( 1 \) is the \( M_k \times 1 \) vector with all elements equal to 1.
In order to minimize iteratively the MSE function given in (7), the least-mean squares (LMS) algorithm is used [12]. For the proposed adaptive combiner, the LMS algorithm can be summarized as

\[ T(y_k[n]) = w_k^T[n]y_k[n] \] (9)

\[ e_k[n] = r_k[n] - T(y_k[n]) \] (10)

\[ w_k[n+1] = w_k[n] + 2\mu e_k[n]y_k[n] \] (11)

Moreover, the feasibility on achieving statistical information (\(\mu\) and \(\Sigma\)) about neighbors’ estimates [9] allows the node \(k\) to find the optimum instantaneous threshold \(\gamma_k[n]\) with the actual weight vector \(w_k[n]\),

\[ \gamma_k[n] = w_k^T[n]\mu_{k,0} + Q^{-1}(\epsilon)\sqrt{w_k^T[n]\Sigma_{k,0}w_k[n]} \] (12)

and thus perform an online hypothesis testing with \(T(y_k[n])\) to reach \(u_k[n]\), according to (1). Note that the algorithm is capable of make decisions at each iteration step without the need to wait until convergence has taken place. Furthermore, the threshold \(\gamma_k[n]\) is estimated such that it guarantees a fixed predefined probability of false alarm, \(P_f,k,1 = \epsilon\), at each iteration [4].

We would like to emphasize that the algorithm minimizes an objective function different of that for optimal linear fusion in [3]. This proposed structure is simple, a good feature for a scenario in which every node (not only a fusion center) performs soft combination. Moreover, it provides online decisions and iterative weight adaptation under dynamic channel environments [4].

### 3.1. Generating an Estimate of Reference \(r_k\)

The exact reference signal \(r_k[n]\), as modeled in (8), is not available since achieving the precise information about \(H_0\) or \(H_1\) at each instant is the main purpose of spectrum sensing systems. To obtain a sufficiently accurate estimate \(\hat{r}_k[n]\), we use the structure depicted in Fig. 2. According to this structure, during the first step, each node not only processes an energy estimate, \(y_i\), but also makes a binary decision, \(d_i\), employing a local hypothesis testing with its individual test statistic, as described in [13]. Both data are then shared among neighbors.

For the neighborhood \(N_k\), the node \(k\) collects the vector of estimates \(y_k\), which is fed into an adaptive filter employing the LMS algorithm and the vector of individual decisions \(d_k = [d_1, d_2, \ldots, d_k]^T\) used to estimate the reference \(r_k\) through an OR-logic fusion rule (see Fig. 2). If the output of the OR-logic is 0 (\(H_0\)), then \(\hat{r}_k = 1^T\mu_{k,0}\); otherwise, if the output is 1 (\(H_1\)), \(\hat{r}_k = 1^T\mu_{k,1}\). The drawback of this approach is that the individual thresholds for obtaining the binary decisions \(d_i\) of each node should be carefully chosen to guarantee a good \(\hat{r}_k\) for the LMS to converge. This is a difficult task in cases of estimates \(y_i\) with low signal-to-noise (SNR) ratio.

### 4. RESULTS AND DISCUSSION

In this section, we evaluate the detection performance of the proposed structure. We assume that \(H_0\) and \(H_1\) occur with equal probability. Comparisons are made between centralized and distributed schemes under both uncorrelated and correlated node contributions. The 12-node network topology shown in Fig. 1 is used in the simulations, with twelve different neighborhoods. A total of \(10^5\) samples (Gaussian energy estimates) were generated for each node, according to the statistical model for \(y_i\) proposed in [9]. For the uncorrelated case, the covariance matrices \(\Sigma_{k,0}\) and \(\Sigma_{1,1}\) are identity matrices for all \(k\). For the correlated case, it is assumed

\[ E[(y_i - \mu_{i,h})(y_j - \mu_{j,h})] = 0.5 \]

where \(i\) and \(j\) are index of spatially adjacent nodes (not necessarily belonging to the same neighborhood), and \(\mu_{i,h}(\mu_{j,h})\) is the mean of the random variable \(y_i(y_j)\) under hypothesis \(H_k\), \(h \in \{0, 1\}\).

In order to compare both centralized and distributed schemes, each neighborhood in Fig. 1 performs only the first step described in Section 2.1 to simulate the centralized case, whereas the full two-step procedure is considered for simulating the distributed case. In the first step, every neighborhood runs an LMS algorithm with step size \(\mu = 1 \times 10^{-5}\) for the simulation with uncorrelated samples, and \(\mu = 2 \times 10^{-5}\) for the simulation with correlated samples, in order to achieve equal quality of convergence for both cases. The LMS performance is also compared to the optimal linear fusion rule described in [3].

The resulting complementary receiver operating characteristic (C-ROC) curves of two particular nodes (nodes 4 and 9) are presented in the following. The neighborhood \(N_4\) is composed of nodes 4, 5, 6 and 9, with individual SNR ratios equal to -1.94 dB, 0 dB, 1.58 dB and 5.1 dB, respectively (according to the expression for individual SNR in [3]). Fig. 3 shows the detection performance of node 4 after employing single detection, centralized (1-step) and distributed (2-step) cooperation within neighborhood \(N_4\), where energy estimates are uncorrelated throughout the network. We see that both cooperative structures outperform the single detector, and the performance obtained with adaptive combiner is close to that achieved with optimal linear fusion. The proposed two-step distributed detection structure also offers better performance than the centralized scheme.
Fig. 4. C-ROC performance at node 4 employing single detection, centralized (1-step) and distributed (2-step) cooperation within neighborhood \(\mathcal{N}_4\): correlated case.

Fig. 4 plots the C-ROC curves for node 4 for the case of correlated energy estimates. We can see the performance degradation due to the correlation between the adjacent nodes. In particular, the performance of the proposed two-step combiner is highly dependent of the correlation coefficients \(\rho_i\) among the neighbors’ binary decisions (see (6)). If there is a correlation among nodes’ soft contributions \(y_i\), this correlation will also appear among their hard contributions \(u_i\), degrading the two-step system performance.

The neighborhood \(\mathcal{N}_9\) is composed of nodes 3, 4, 7, and 9, and their individual SNR ratios are \(-4.47\) dB, \(-1.94\) dB, 2.92 dB and 5.1 dB, respectively. The detection performance comparisons for the node 9 under uncorrelated and correlated node estimates are presented in Fig. 5 and Fig. 6, respectively. We can verify from both figures the good performance of the adaptive combiner when compared to that of the optimal linear combiner. In this neighborhood, the proposed two-step distributed structure offers a more pronounced performance improvement compared to \(\mathcal{N}_4\). This is because the neighbors in \(\mathcal{N}_9\) are more spatially distributed, i.e., there will be less correlation between neighbors in \(\mathcal{N}_9\) than in \(\mathcal{N}_4\).

5. CONCLUSION

This paper proposed a distributed two-step combining for cooperative spectrum sensing purposes. A good estimate of a reference signal through conventional OR-fusion rule allows the employment of an online adaptive linear fusion, a suboptimal but simple alternative to the optimal linear fusion rule.

Fig. 5. C-ROC performance at node 9 employing single detection, centralized (1-step) and distributed (2-step) cooperation within neighborhood \(\mathcal{N}_9\): uncorrelated case.

Other recent works have proposed algorithms for adaptive cooperation among nodes, but in a centralized scheme. Results showed that the suggested two-step distributed structure can improve the performance of cooperative spectrum sensing networks, especially in neighborhoods with low correlation among nodes’ contributions.

6. REFERENCES