OPTIMAL TRANSMIT POWER ALLOCATION IN WIRELESS SENSOR NETWORKS
PERFORMING FIELD RECONSTRUCTION

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ABSTRACT

In our previous work, we developed efficient field reconstruction methods in wireless sensor networks. In this paper, we use an amplify-and-forward to transmit the sensor measurements to the fusion center and we derive the mean square error (MSE) of the reconstructed field as a function of the measurement and receive SNR and of the sensor positions. We propose to allocate the sensor node transmit powers such that the sum power is minimized subject to an MSE target and we phrase this approach as a convex optimization problem that can be numerically solved in an efficient manner. For the case of critical sampling we derive a closed-form expression for the optimal power allocation. We illustrate the power savings achieved with the proposed power allocation schemes both for Gaussian and Rayleigh fading channels.

Index Terms— Wireless sensor network, field reconstruction, power allocation

1. INTRODUCTION

Wireless sensor networks (WSN) have recently attracted considerable attention for diverse monitoring applications [1]. With WSN, sensor nodes are deployed in the region to be monitored and communicate wirelessly in order to collect and process information about the phenomenon of interest. In this paper, we consider the problem of power allocation in the context of the system architecture introduced in [2] for distributed sampling and reconstruction of a two-dimensional (2-D) physical fields.

Power scheduling for decentralized estimation in sensor networks based on uncoded quadrature amplitude modulated (QAM) and on analog transmission was studied respectively in [3] and [4]. The optimal power allocation in those papers is similar to water-filling, i.e., sensor nodes with poor channel gains or noisy observations remain inactive to save power. These results were extended to the case of distributed estimation of a random field in [5]. A suboptimal power allocation scheme for the estimation of a random parameter in the presence of noisy links was proposed in [6].

In this paper, we investigate the problem of power allocation in a WSN using amplify-and-forward transmissions to the fusion center (FC) as in [4]. However, contrary to the scalar model in [4], we consider a matrix-vector model that we used for field reconstruction based on shift-invariant spaces in our previous work [7]. Different from [4], the optimal power allocation for our model depends on the sensor node positions. In order to maximize network lifetime, our aim is to minimize the transmit sum power subject to a prescribed estimation accuracy. We formulate this objective as a convex optimization problem that can be solved numerically using standard techniques. For the special case of critical sampling, we derive a closed-form solution for this problem.

Our paper is organized as follows. Section 2 reviews our system model for field reconstruction in WSN. In Section 3, we study the optimal power allocation problem. Section 4 shows numerical results illustrating the performance of our power allocation scheme, and Section 5 provides concluding remarks.

2. SYSTEM MODEL

2.1. Measurement and Transmission Model

We consider a WSN consisting of I sensor nodes deployed over a given region $\mathcal{A}$ to monitor a 2-D physical field $h(x, y)$. Here, $x$ and $y$ are the spatial coordinates. The position of node $i$ is denoted by $(x_i, y_i)$ and its measurement is given by $h_i + v_i$ where $h_i = h(x_i, y_i)$. Here, $v_i$ denotes spatially white measurement noise with (node-dependent) variance $\sigma^2_{v_i}$. We assume that the physical field belongs to a shift-invariant space $V(g)$ (see [2,8] for details), i.e.,

$$h(x, y) = \sum_{(k,l) \in \mathcal{K}} c_{k,l} g(x - k, y - l).$$  \hfill (1)

Here, $g(x,y)$ is a generator function with compact support $\mathcal{S}$ (e.g., a B-spline function) and $\mathcal{K} = \mathbb{Z}^2 \cap (\mathcal{A} + \mathcal{S})$. We further assume that the field has mean power $\sigma^2_f$.

The reconstruction of $h(x, y)$ from the noisy samples $h_i + v_i$ thus amounts to estimating the coefficients $c_{k,l}$. To this end, we augment the least-squares approach from [2] with an amplify-and-forward (AF) transmission protocol in which sensor node $i$ transmits the scaled measurement $s_i = \sqrt{\gamma_i}(h_i + v_i)$ to the FC, which requires an average transmit power of

$$P_i \triangleq E[s_i^2] = p_i(\sigma^2_h + \sigma^2_{v_i}).$$  \hfill (2)

The transmissions of the individual nodes are over orthogonal channels and the signals received by the FC are given by

$$r_i = \sqrt{\gamma_i} s_i + w_i = \sqrt{\gamma_i} p_i h_i + z_i.$$  \hfill (3)

Here, $\gamma_i$ is the channel gain, $w_i$ is white receiver noise, with variance $\sigma^2_{w_i}$ and $z_i = \sqrt{\gamma_i} p_i v_i + w_i$. Below we will consider Gaussian channels (modeled via $\gamma_i = 1$) and flat Rayleigh fading channels (modeled by exponentially distributed $\gamma_i$). We next formulate the system model using matrices and vectors. To this end, let $(k_0, l_0)$ and $(k_1, l_1)$ denote the smallest and largest...
indices in \( A \), respectively, such that \( J = KL \) with \( K \triangleq k_1 - k_0 + 1 \) and \( L \triangleq (l_1 - l_0 + 1) \). We define the block-banded \( I \times J \) matrix \( G \) with elements
\[
[G]_{i,p} = g(x_i - k_p, y_i - l_p)
\]
and the vectors \( r, v \) (length \( I \)), and \( c \) (length \( J \)) with elements
\[
[r]_i = r_i, \quad [v]_i = v_i, \quad \text{and} \quad [c]_p = c_{k_p, l_p},
\]
where \( k_p = k_0 + ((p-1) \mod K) \), and \( l_p = l_0 + \left\lfloor \frac{p - 1}{K} \right\rfloor \). The received signals can be rewritten in matrix-vector form as
\[
r = AGc + z = \tilde{G}c + z,
\]
where \( A \triangleq \text{diag} \{\sqrt{\gamma_i}\} \), \( \tilde{G} \triangleq AG \), and the aggregate noise vector \( z \) has covariance matrix \( C_z \triangleq \text{diag} \{\sigma_i^2, \sigma_w^2\} \).

2.2. Field Reconstruction and Reconstruction Performance

We determine the field coefficients \( \hat{c} \) in the linear system model (4) using the best linear unbiased estimator (BLUE) [9] with the noise covariance matrix \( C_z \) as weight, i.e.,
\[
\hat{c} \triangleq \arg \min_{c} \|Gc - r\|^2_{C_z^{-1}} = (G^T C_z^{-1} G)^{-1} G^T C_z^{-1} r.
\]
Note that the computation of the coefficient estimates \( \hat{c} \) requires that the noise statistics \( C_z \) and the matrix \( G \) (i.e., the sensor node positions and channel gains) be known at the FC. In order for the estimator (5) to exist, the matrix \( G \) must have full rank, which in turn requires \( I \geq J \), i.e., that there are at least as many sensors as unknown coefficients (in addition, the sensor nodes need to be sufficiently closely spaced). Technically, the sensor node positions \( (x_i, y_i) \), have to form a so-called stable sampling set [8]. The case \( I = J \) will be referred to henceforth as critical sampling.

With the optimal coefficient estimates (5), the field can be reconstructed for any position \((x, y) \in A\) according to (cf. (1))
\[
\hat{h}(x, y) = \sum_{(k,l) \in A} \hat{c}_{k,l} g(x-k, y-l).
\]
To assess accuracy of (6), we next derive the mean-square field reconstruction error within \( A \). To this end, we define the length-\( I \) vector
\[
[g]_p(x, y) = g(x - k_p, y - l_p),
\]
with \( k_p \) and \( l_p \) as in Section 2.1, and the associated Gramian \( G_z = \int_{A} g(x, y) g^T(x, y) \, dx \, dy \). Taking the expectation with respect to the noise, with the sensor positions and channel gains fixed, we obtain
\[
\epsilon \triangleq \mathbb{E} \left\{ \int_{A} \left( \hat{h}(x, y) - h(x, y) \right)^2 \, dx \, dy \right\}
\]
\[
= \int_{A} \mathbb{E} \left\{ g^T(x, y) \hat{c} - \hat{c} c^T \right\} g(x, y) \, dx \, dy
\]
\[
= \int_{A} \text{tr} \left\{ C_{\hat{c}} g(x, y) g^T(x, y) \right\} \, dx \, dy = \text{tr} \left\{ C_{\hat{c}} G_z \right\}.
\]
Here, \( C_{\hat{c}} = \text{cov} \{ \hat{c} \} \triangleq \mathbb{E} \{ (\hat{c} - c)(\hat{c} - c)^T \} \) denotes the covariance matrix of the (unbiased) coefficient estimates \( \hat{c} \). For compactly supported generator functions, the Gramian \( G_z \) can be shown to be a symmetric block-banded Toeplitz matrix. The covariance matrix of the coefficient estimates can be further developed as
\[
C_{\epsilon} = \text{cov} \{ G_z^{-1} \hat{G} \}^{-1} G_z^{-1} z
\]
\[
= (G_z^{-1} \hat{G} \hat{G}^T G_z^{-1})^{-1} = (G_T A^T C_z^{-1} AG)^{-1}
\]
\[
= (G_T DG)^{-1}.
\]
Here, we used the diagonal matrix
\[
D \triangleq A^T C_z^{-1} A = \text{diag} \{d_i\}, \quad \text{with} \quad d_i = \frac{1}{\sigma_i^2 + \frac{1}{\eta_p \epsilon}}.
\]
Inserting (8) and (9) into (7), we finally obtain the MSE as
\[
\epsilon = \text{tr} \left\{ C_{\hat{c}} G_z \right\} = \text{tr} \left\{ (G_T DG)^{-1} G_z \right\}.
\]
This expression captures the dependence of the reconstruction MSE on the channel gains \( \gamma_i \), the sensor node positions \( (x_i, y_i) \), the AF factors \( p_i \), and the noise variances \( \sigma_{\epsilon}^2 \) and \( \sigma_w^2 \).

3. OPTIMAL POWER ALLOCATION

In the context of sensor networks, one of the main concerns is network lifetime, which in turn is directly related to energy efficiency. For that reason, we propose to keep the total transmit power of the sensor nodes as low as possible while satisfying prescribed requirements for the reconstruction quality. Hence, we aim at allocating the power scaling factors \( p_i \) such that the transmit sum power
\[
P \triangleq \sum_{i=1}^J P_i = \sum_{i=1}^J p_i (\sigma_i^2 + \frac{1}{\eta_p \epsilon})
\]
is minimized subject to a given MSE target \( \epsilon_{\text{max}} \). Defining the length-\( J \) vectors \( p = (p_1 \ldots p_J)^T \) and \( q = (q_1 \ldots q_J)^T \) with \( q_i = \sigma_i^2 + \frac{1}{\eta_p \epsilon} \), we have \( P = p^T q \) and the optimal power allocation problem is given by (cf. (10))
\[
\min_{p \in R^J_+} \quad p^T q
\]
subject to \( \text{tr} \left\{ (G_T D(p)G)^{-1} G_z \right\} \leq \epsilon_{\text{max}} \).

Here, we made the dependence of \( D \) on \( p \) explicit by writing \( D(p) \). The optimization problem (11) can be shown to be convex (see [10]). Note that the MSE target \( \epsilon_{\text{max}} \) cannot be chosen arbitrarily small. In fact, even with infinite transmit powers, the MSE is lower bounded by a strictly positive number due to the presence of measurement noise, i.e.,
\[
\epsilon \geq \epsilon_{\text{min}} \triangleq \lim_{p_i \to \infty} \epsilon = \text{tr} \left\{ (G_T \text{diag} \{\sigma_i^{-2}\} G)^{-1} G_z \right\}.
\]
Hence the MSE target has to be chosen larger than \( \epsilon_{\text{min}} \) in order for (11) to have a solution. While the problem in general has no closed-form solution, it can be solved numerically in an efficient manner using standard algorithms [11]. Clearly, the power allocated to node \( i \) depends on the local measurement noise variance \( \sigma_{\epsilon}^2 \), on the channel gain \( \gamma_i \), and (through the matrix \( G \)) on the sensor node positions.
3.1. Critical Sampling

We next show that for the special case of critical sampling the power allocation problem (11) has a closed-form solution. In this case, there are as many sensor nodes as unknown coefficients, i.e., $I = J$. We furthermore assume that the node positions form a stable sampling set such that $G$ is a square invertible matrix. In this case, (10) can be specialized as

$$\varepsilon = \text{tr} \left\{ (G^T DG)^{-1} Gg \right\} = \text{tr} \left\{ D^{-1} (G^T G)^{-1} Gg \right\}$$

$$= \sum_{i=1}^{I} \frac{1}{d_i} g_i + \sum_{i=1}^{I} \frac{\sigma^2_v g_i}{\gamma_i p_i},$$

where we defined $g_i = \left[(G^T G)^{-1} G_i G^{-1}\right]_i$ and used that $D^{-1} = \text{diag} \{d_i^{-1}\}$ with $d_i^{-1} = \left(\frac{\sigma^2_v + \sigma^2_w}{\gamma_i p_i} \right)$ (cf. (9)). Then the side-constraint in the optimization problem in (11) simplifies to

$$\sum_{i=1}^{I} \frac{\sigma^2_v g_i}{\gamma_i p_i} \leq \varepsilon_{\text{max}} \triangleq \varepsilon_{\text{max}} - \varepsilon_{\text{min}},$$

where $\varepsilon_{\text{min}} = \sum_{i=1}^{I} \sigma^2_v g_i$ and $\varepsilon_{\text{max}} \geq \varepsilon_{\text{min}}$ ensures a nonempty feasible set. The Lagrangian associated to the optimum power allocation problem (11) in this case equals

$$L(p, \lambda) = p^T q + \lambda \left( \sum_{i=1}^{I} \frac{\sigma^2_v g_i}{\gamma_i p_i} - \varepsilon_{\text{max}} \right)$$

$$= \sum_{i=1}^{I} \left(\frac{p_i q_i}{\gamma_i} + \lambda \frac{\sigma^2_v g_i}{\gamma_i p_i} \right) - \lambda \varepsilon_{\text{max}}$$

and the Lagrangian dual function reads

$$g(\lambda) = L(p^*, \lambda) = \inf_{p} L(p, \lambda) = \sum_{i=1}^{I} \left(\frac{p_i q_i}{\gamma_i} + \frac{\sigma^2_v g_i}{\gamma_i p_i} \right) - \lambda \varepsilon_{\text{max}}$$

$$= 2\sigma_w \sqrt{\lambda} \sum_{i=1}^{I} \sqrt{\frac{g_i q_i}{\gamma_i}} - \lambda \varepsilon_{\text{max}},$$

where

$$p_i^* = \arg\inf_{p_i} \left(\frac{p_i q_i}{\gamma_i} + \frac{\sigma^2_v g_i}{\gamma_i p_i} \right) = \frac{\lambda \sigma^2_v g_i}{\gamma_i q_i}.$$

We therefore have the Lagrange dual problem

$$\max \lambda \quad g(\lambda)$$

subject to $\lambda \geq 0$,

whose solution is given by

$$\lambda^* = \left( \frac{2\sigma^2_w}{\varepsilon_{\text{max}}} \right)^{2}.$$
Fig. 1. Performance comparison of different transmit power allocation schemes in terms of transmit sum power versus number of sensor nodes in Gaussian and Rayleigh fading channels: (a) normalized MSE target of $-20$ dB (b) normalized MSE target of $-10$ dB.

Fig. 2. Comparison of different power allocation schemes in terms of transmit sum power versus normalized MSE target for 100 sensor nodes and Gaussian and Rayleigh fading channels.

5. CONCLUSIONS

We considered field reconstruction in WSN based on shift-invariant spaces and an AF protocol for the transmission of sensor node measurements to the fusion center. We derived the MSE achieved by this scheme and developed optimal schemes for allocating transmit power to the sensor nodes in order to minimize the sum power while maintaining a desired MSE performance. For the case of critical sampling, we derived closed-form expressions for the optimal power allocation. Numerical simulations for Gaussian and Rayleigh fading channels showed that the proposed power allocation schemes have the potential for several dB of power savings compared to uniform power allocation.

6. REFERENCES