SPECTRUM SHARING BASED ON TWO-PATH SUCCESSIVE RELAYING

Chao Zhai, Wei Zhang
School of Electrical Engineering & Telecom
University of New South Wales
Sydney, Australia

P. C. Ching
Department of Electronic Engineering
Chinese University of Hong Kong
Shatin, Hong Kong SAR of China

ABSTRACT

A spectrum sharing scheme is proposed for overlaid wireless networks based on the two-path successive relaying technique. In this scheme, two secondary transmitters are used to alternately transmit secondary information to their respective receivers whilst relaying the primary signal at the same time. By using superposition coding at the secondary transmitters and successive decoding at the receivers, a diversity gain of 2 is obtained for the primary system without losing the bandwidth efficiency. Outage probabilities of the primary system and secondary system are analyzed. The optimal power allocation factor of the secondary system is determined to minimize the outage probability of the secondary system while maintaining the performance of the primary system. Numerical results show that the proposed scheme can indeed enhance the outage performance of the primary and secondary systems simultaneously.

Index Terms— Cognitive radio, Relay network, Spectrum sharing.

1. INTRODUCTION

Cooperative techniques [1] are widely used in cognitive radio networks for spectrum sensing and sharing [2]. Secondary system can either implicitly access primary channel when it is idle, or explicitly help the data transmission of the primary system in exchange for the sharing of spectrum. In [3], a cooperative spectrum sharing scheme was proposed that requires three phases for the secondary system to relay the primary information and transmit its own information. Two-phase cooperative spectrum sharing schemes were presented in [4] [5], where in the first phase the primary signal is broadcasted and in the second phase the transmission of secondary signal and the relaying of primary signal is simultaneously performed. As multiple phases are required to transmit the same primary signal, spectrum efficiency of the primary system is relatively low.

To address the problem of low spectrum efficiency in relay cooperation, the two-path successive relaying scheme was proposed in [6] [7], which can achieve both cooperative diversity and high bandwidth efficiency. In this scheme, the two relays alternately help forward the source signal to the destination while the source continues its signal transmission without the need of waiting or retransmission. It was recently shown that full diversity and full rate can be achieved with the cooperation of the two relays via space-time coding techniques [8] [9]. Moreover, the achievable rate of the two path successive relaying scheme was investigated in [10], and the diversity-multiplexing tradeoff with amplify-and-forward and decode-and-forward fashion were analyzed in [11] and [12], respectively.

In this paper, to facilitate spectrum sharing of overlaid wireless networks, a two-path successive cognitive relaying scheme is developed for cognitive radio network. While two cognitive transmitters serve as relays to help primary system in a successive fashion, they also send their own information to the intended receivers, respectively. It is achieved by employing superposition coding at secondary transmitters and successive decoding at the receivers. On one hand, the secondary signal transmission introduces interference to the primary system. On the other hand, the secondary users relay the primary signals to achieve a diversity gain of 2. By determining an optimal power allocation between relaying primary signal and transmitting secondary signal, the interference imposed on the primary system can be greatly suppressed by taking advantage of cooperation. Numerical results show that the outage performance of both primary and secondary systems are improved.

The rest of this paper is organized as follows. In Section 2, system model is introduced and a spectrum sharing based on two-path successive relaying scheme is proposed. Section 3 analyzes the outage performance of the scheme for secondary and primary systems. Numerical results are given in Section 4. Section 5 concludes this paper.

2. SYSTEM MODEL

We consider an overlaid wireless network as shown in Fig. 1, where a pair of primary transmitter (PT) and receiver (PR) coexist with two pairs of secondary transmitters (ST1, i = 0, 1) and receivers (SR1, i = 0, 1). The positions of all nodes are fixed. The small-scale distance between any two secondary users is assumed to be much shorter than the large-scale distance between any primary user and any secondary user. The channel between any transmitter and receiver is assumed to undergo independent Rayleigh block fading, which remains unchanged in one data frame but changes independently from one frame to another. The Rayleigh fading channel from transmitter u to receiver v is denoted by $h_{u,v}$. The channel power gain is denoted by $G_{u,v} = |h_{u,v}|^2$, which is exponentially distributed with mean $G_{u,v} = d_{u,v}^{-\alpha}$, where $d_{u,v}$ is the distance between u and v and $\alpha$ is the path loss exponent. The channel between u and v is symmetric, i.e., $h_{u,v} = h_{v,u}$. For each link, the channel is perfectly known at the receiver only.

2.1. Cognitive Relay for Spectrum Sharing

PT sends its signal continuously to PR, while ST0 and ST1 alternately send their own signals to their intended receivers, respectively. The transmission power of the primary and secondary system is de-

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noted by $P_p$ and $P_s$, respectively. Due to the concurrent transmission of secondary signal, the primary user is interfered. In order to remedy the impairment of the interference imposed on the primary user, $ST_0$ and $ST_1$ also help relay the primary signals by employing a two-path successive relay technique. When the primary signal is correctly received by both $ST_0$ and $ST_1$, the secondary transmitters will use superposition coding to encode their own signals along with the primary signals and then alternate them forward them to $PR$ and the secondary receivers. If at least one ST fails to receive the primary signal, both $ST_0$ and $ST_1$ will keep silent. From the perspective of information theory, the signal is said to be correctly received if the channel capacity is larger than the transmission rate. Throughout the paper, we fix the transmission rate of the primary system and channel capacity is larger than the transmission rate. Throughout information theory, the signal is said to be correctly received if the channel capacity is larger than the transmission rate.

To expound the proposed spectrum sharing scheme with two-path successive relay, we consider the signal transmission during $L$ time slots as below.

- **Time slot 1**: $PT$ transmits $x_p[1]$; $ST_0$, $SR_0$, $SR_1$ receive and detect $x_p[1]$; $ST_1$ keeps silent; $PR$ receives $x_p[1]$.


- The progress of **Time slot 2 and 3** repeats till **Time slot $L$**.

- **Time slot $L+1$**: $ST_1$ transmits the composite signal $x_c[L+1] = f(x_p[L], x_1[L+1])$; $SR_1$ cancels $x_p[L]$, then detects $x_1[L+1]$; $ST_0$ and $SR_0$ are silent; $PR$ receives $x_c[L+1]$ and then jointly decodes the primary signals $x_p[1], x_p[2], \cdots, x_p[L]$ once by regarding secondary signals as interference.

It can be seen that totally $L + 1$ time slots are used to transmit $L$ primary symbols. The spectral efficiency of the primary system is $L/(L+1)$, which approaches 1 for large $L$.

### 2.2. Signal Model

We consider time slot $k (k = 2, \ldots, L)$, when $ST_i$ ($i = \text{mod}(k,2)$) and $PT$ simultaneously do the transmissions. For $ST_i$, the transmitted composite signal is generated by linearly combining the previous primary signal and its current secondary signal, e.g.,

$$x_c[k] = f(x_p[k-1], x_i[k]) = \sqrt{\beta} x_p[k-1] + \sqrt{1-\beta} x_i[k],$$  

(1)

where $\beta \in [0, 1]$ is the power allocation factor, which is the same for both $ST_0$ and $ST_1$. If $\beta = 0$, $ST_i$ becomes selfish and only transmits its own signal. If $\beta = 1$, $ST_i$ is selfless and only relays the primary signal. We set $\beta \geq 0.5$ because more power is allocated to relay the primary signal and less interference is introduced.

The signal received by $SR_i$ and $SR_j$ in time slot $k$ is written as $y_{SR_i}[k] (i = 0 \text{ or } j)$,

$$y_{SR_i}[k] = \sqrt{P_{PT,SR_i}} x_p[k] + \sqrt{P_{ST_i,SR_i}} x_i[k] + n_{SR_i}[k].$$  

(2)

The desired signal for $SR_i$ is $x_i[k]$. However, both $SR_i$ and $SR_j$ need to have $x_p[k]$ for the interference cancelation in the next time slot. Hence, the successive decoding is performed as follows. First, the previous primary signal $x_p[k-1]$ is canceled. Then, the secondary signal $x_i[k]$ is detected and canceled. Finally, the current primary signal $x_p[k]$ is detected.

The signal received by another secondary transmitter $ST_j$ ($j = 1 - i$) is given by

$$y_{ST_j}[k] = \sqrt{P_{PT,ST_j}} x_p[k] + \sqrt{P_{ST_i,ST_j}} x_i[k] + n_{ST_j}[k].$$  

(3)

The secondary transmitter $ST_j$ needs to know the current primary signal $x_p[k]$ for the relaying in the next time slot. As the secondary users are close to each other, the channel between $ST_0$ and $ST_1$ is stronger than the channel between $PT$ and $ST_i$. Hence, the composite signal $x_c[k]$ can be firstly detected and canceled. Then, the current primary signal is detected. In the detection of $x_c[k]$, as $\beta > 0.5$ the previous primary signal $x_p[k-1]$ is firstly detected and canceled, and then the secondary signal $x_i[k]$ is extracted.

For the primary receiver $PR$, all the received signals in $L + 1$ time slots are written as a vector $y$ as follows.

$$y = H x_p + w.$$  

(4)

where $x_p = [x_p[1], x_p[2], \ldots, x_p[L]]^T$ is the primary signal, and $H$ is the equivalent MIMO channel with size $(L + 1) \times L$, given by

$$H = \begin{bmatrix}
0 & \cdots & 0 & 0 \\
0 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & 0 \\
0 & \cdots & 0 & 0 \\
\end{bmatrix}$$

$$c_{PT,PR} = \begin{bmatrix}
0 \\
c_{ST_0,PR} \\
c_{ST_1,PR} \\
\vdots \\
c_{ST_0,PR} \\
c_{ST_1,PR} \\
\end{bmatrix}$$

$$c_{ST_0,PR} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
0 \\
\end{bmatrix}$$

$$c_{ST_1,PR} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
0 \\
\end{bmatrix}$$

$$y = \begin{bmatrix}
t_{PT,PR} x_0[2] \\
t_{ST_0,PR} x_0[3] \\
t_{ST_1,PR} x_0[4] \\
\vdots \\
t_{PT,PR} x_0[L] \\
t_{ST_0,PR} x_0[L+1] \\
\end{bmatrix} + \begin{bmatrix}
n_{PT,PR}[1] \\
n_{PT,PR}[2] \\
n_{PT,PR}[3] \\
\vdots \\
n_{PT,PR}[L] \\
n_{PT,PR}[L+1] \\
\end{bmatrix}$$

where $c_{PT,PR} = \sqrt{P_{PT,PR}}$, $c_{ST_0,PR} = \sqrt{P_{ST_0,PR}}$, $c_{ST_1,PR} = \sqrt{P_{ST_1,PR}}$. The vector $w$ denotes the interference plus the noise, given by

$$w = \sqrt{(1-\beta)} P_s.$$
Obviously, a diversity gain of 2 is achieved for the primary system because each primary signal passes through two independent fading channels.

3. PERFORMANCE ANALYSIS

Let the event of both secondary transmitters correctly detecting the primary signal be $E_c$. In such case, the two-path successive relaying will be properly activated. Otherwise, the secondary transmitters will keep silent. The outage probabilities of the primary and secondary system are denoted by $P^P_{out}$ and $P^S_{out}$, respectively. Then,

$$
P^P_{out} = \Pr(E_c)P^P_{out,c} + [1 - \Pr(E_c)]P^P_{out,d},
$$

$$
P^S_{out} = \Pr(E_c)P^S_{out,c} + [1 - \Pr(E_c)]P^S_{out,d},
$$

where $P^P_{out,c}$ and $P^S_{out,c}$ represent the outage probabilities of the primary system and secondary system using two-path successive relay, respectively. $P^P_{out,d}$ and $P^S_{out,d}$ denote the outage probabilities of the primary system and secondary system with secondary transmitters being silent. As the secondary system keeps silent in this case, it has $P^S_{out,d} = 1$.

3.1. The Occurrence Probability $Pr(E_c)$

For $ST_i$, before detecting the current primary signal, the previous primary signal forwarded from $ST_j$ ($j = i - 1$) should first be located and canceled. To correctly detect the previous primary signal, it requires the transmission rate $r_0$ be no larger than the achievable rate $R_{ST_i}^3$, which is given by

$$
R_{ST_i}^3 = \log_2 \left(1 + \frac{\beta \rho \eta \tilde{G}_{ST_i,ST_j}}{\eta G_{PT,ST_i} + (1 - \beta) \rho \eta \tilde{G}_{ST_i,ST_j} + 1}\right),
$$

where the numerator is the power of the previous primary signal, and the denominator represents the power of interference plus noise. In particular, $\eta = \frac{P_t}{N_0}$ and $\rho = \frac{P_s}{P_t}$.

After the successful detection and cancelation of the previous primary signal, the current secondary signal is detected and canceled. To guarantee the correct detection of the current secondary signal, it would require $r_1$ be no larger than the achievable rate $R_{ST_i}^2$, which is given by

$$
R_{ST_i}^2 = \log_2 \left(1 + \frac{\beta \rho \eta \tilde{G}_{ST_i,ST_j}}{\eta G_{PT,ST_i} + 1}\right).
$$

After the successful detection and cancelation of the secondary signal, the achievable rate of the current primary signal becomes

$$
R_{ST_i}^1 = \log_2 (1 + \eta \tilde{G}_{PT,ST_i}).
$$

Based on the above analysis, the occurrence probability of the two-path successive relaying process being activated is

$$
\Pr(E_c) = \Pr \left\{ R_{ST_i}^1 \geq r_0, R_{ST_i}^2 \geq r_1, R_{ST_i}^3 \geq r_0, R_{ST_i}^1 \geq r_0, R_{ST_i}^2 \geq r_1, R_{ST_i}^3 \geq r_0 \right\}.
$$

3.2. Outage Probability of the Primary System

The rate for direct transmission between $PT$ and $PR$ without relay is $R_{pd} = \log_2 \left(1 + \eta G_{PT,PR} \right)$. Then, the outage probability is given by

$$
P^P_{out,d} = \Pr \left\{ R_{pd} < r_0 \right\} = 1 - \exp \left[-T/\eta\right],
$$

where $T = 2^{r_0} - 1$. Distance between any two nodes is normalized with respect to the distance between $PT$ and $PR$, so $d_{PT,PR} = 1$.

For two-path successive relaying, according to the equivalent MIMO signal model (4), the rate of the primary system is [11]

$$
R_{pc} = \frac{1}{L + 1} \log \left| \det(I + \tilde{H}H^H) \right|,
$$

where $\tilde{H} = C^{-1/2}H$ is the normalized channel matrix, with $C = E[ww^H] = \text{diag}\{N_0, N_0, \ldots, N_0\}$, where $\lambda_0 = (1 - \beta)P_t \tilde{G}_{ST_i,PR} + N_0$ and $\lambda_1 = (1 - \beta)P_t \tilde{G}_{ST_i,PR} + N_0$. The matrix in (12) is a trigonal matrix, whose determinant is not easy to obtain. But the approximate rate $R_{pc}$ can be obtained by dividing $\tilde{H}$ into interference-free blocks with size $3 \times 2$, as follows.

$$
R_{pc} = \frac{L}{2L + 1} \log_2 \left\{ 1 + \frac{\eta \rho \eta \tilde{G}_{ST_i,ST_j}}{1 + (1 - \beta) \rho \eta \tilde{G}_{ST_i,ST_j} + 1} + \frac{\eta \rho \eta \tilde{G}_{ST_i,ST_j}}{1 + (1 - \beta) \rho \eta \tilde{G}_{ST_i,ST_j} + 1} \right\},
$$

where $x = G_{PT,PR}, y = G_{ST_i,PR}$, and $z = G_{ST_i,PR}$. The approximate outage probability is computed as

$$
P^P_{out,c} = \Pr \left\{ R_{pc} < r_0 \right\}.
$$

3.3. Outage Probability of the Secondary System

Consider a certain time slot $t$ ($2 \leq t \leq L - 1$), in which $PT$ transmits the current primary signal and $ST_i$ ($i \equiv \text{mod}(t, 2)$) transmits the composite signal simultaneously. After the cancelation of the previous primary signal, $SR_i$ should first detect its desired secondary signal by viewing the current primary signal as noise. To get the correct detection of the secondary signal, it requires $r_1$ be no larger than the achievable rate $R_{SR_i}^1$, which is given by

$$
R_{SR_i}^1 = \log_2 \left(1 + \frac{(1 - \beta) \rho \eta \tilde{G}_{ST_i,SR_i}}{\eta G_{PT,SR_i} + 1}\right).
$$

In time slot $t + 1$, $PT$ and $ST_i$ ($j = i - 1$) will transmit at the same time. After the cancelation of the previous primary signal, the secondary receiver $SR_i$ needs to detect and cancel the secondary signal of $ST_j$ followed by the detection of the current primary signal. To guarantee the correct detection and cancelation of the secondary signal, the transmission rate $r_1$ must be no larger than the achievable rate $R_{SR_i}^2$, which is given by

$$
R_{SR_i}^2 = \log_2 \left(1 + \frac{(1 - \beta) \rho \eta \tilde{G}_{ST_j,SR_i}}{\eta G_{ST_i,SR_i} + 1}\right).
$$

For any time slot, after cancelation of the secondary signal, the primary signal needs to be detected and it will be used for interference cancelation at the next time slot. This requires the transmission rate $r_0$ to be smaller than or equal to the achievable rate $R_{SR_i}^2$, which is given by

$$
R_{SR_i}^2 = \log_2 \left(1 + \eta \tilde{G}_{PT,SR_i}\right).
$$

Therefore, the outage probability of the secondary system ($ST_i, SR_i$) is

$$
P^S_{out,c} = 1 - \Pr \left\{ R_{SR_i}^1 \geq r_1, R_{SR_i}^2 \geq r_1, R_{SR_i}^3 \geq r_0 \right\}.
$$

Finally, we obtain

$$
P^S_{out,c} = \frac{1}{2} \left(P^S_{out,c} + P^S_{out,d}\right).
$$


It can be seen from Fig. 2 that, with the increase of the secondary transmission power, the performance of secondary system becomes better, while the performance of the primary system is greatly improved. The performance becomes better with the decrease of the target rate. With the increase of the large-scale distance, the performance becomes worse, because the average channel quality gets worse. Moreover, numerical results coincide well with the simulation results for the secondary system. It reveals that the proposed spectrum sharing can well improve the performance of both primary and secondary system if an optimal power allocation factor $\beta$ is used. Although the secondary signal transmission will interfere primary system, the secondary system also makes contribution to the primary system by relaying the primary signals. It is a win-win solution to both primary and secondary systems.

5. CONCLUSION

A spectrum sharing scheme based on two-path successive relaying was proposed in this paper. By combining the primary and secondary signals, the two secondary transmitters not only support the secondary signal transmission, but also help relay primary signal transmission via a diversity approach. The outage probabilities of the primary and secondary systems were analyzed and the optimal power allocation factor was determined. Numerical results were shown to validate the fact that the proposed spectrum sharing scheme can enhance the outage performance of both primary and secondary systems.

6. REFERENCES


