A COOPERATIVE APPROACH FOR AMPLIFY-AND-FORWARD DIFFERENTIAL TRANSMITTED REFERENCE IR-UWB RELAY SYSTEMS

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ABSTRACT

This paper proposes a novel cooperative approach for two-hop amplify-and-forward (A&F) relaying that exploits both the signal forwarded by the relay and the one directly transmitted by the source in impulse-radio ultra-wideband (IR-UWB) systems. Specifically, we focus on a non-coherent setup employing a double-differential encoding scheme at the source node and a single differential demodulation at the relay and destination. The log-likelihood ratio based decision rule is derived at the destination node. Numerical simulations show that the proposed system outperforms both the direct transmission with single differential encoding and the non-cooperative multi-hop approach in different scenarios.

Index Terms— Impulse-radio (IR), ultra-wideband (UWB) communications, differential transmitted reference (DTR), amplify and forward (A&F) relaying, log-likelihood ratio test

1. INTRODUCTION

Ultra-wideband impulse radio (UWB-IR) is a promising wireless technology characterized by the transmission of ultra-short pulses that fill up a bandwidth larger than 500 MHz or a fractional bandwidth greater than 20% [1], [2]. The Federal Communications Commission (FCC) regulated power levels are extremely low (below $-41.3$ dBm), which allows UWB technology to share the huge 3.6 – 10.1 GHz band with other existing services.

Because of the very low power involved, it is essential to capture at the receiver as much signal energy as possible. Coherent Rake combining shows the best error performance, but requires accurate channel knowledge, precise timing synchronization, and a large number of fingers [3]. On the other hand, in non-coherent detection schemes the energy of the multipath components is captured by means of autocorrelation and integrate-and-dump (A&D) operations, thus resulting in suboptimal simpler receiver architectures [4].

Hocott and Tomlinson [5] have proposed a UWB transmitted reference (TR) system, in which a reference waveform is sent before each data-modulated pulse for the purpose of determining the current multipath channel response. If the coherence time of the channel is larger than two frame durations, the reference pulse can be employed as a noisy template at the receiver. A differential transmitted reference (DTR) receiver encodes the information sequence with differential modulation, thus being more energy efficient and providing a higher data rate [6].

In order to extend the coverage, amplify-and-forward (A&F) and decode-and-forward (DF) relaying techniques, which were originally proposed for narrowband communications [7], have been also applied to the context of IR-UWB DTR receivers [8], [9]. A critical issue in A&F relaying lies on the length of the overall channel impulse response (CIR) that increases with the number of hops. Compared with the direct transmission, significantly larger guard intervals must be chosen, in order to eliminate intersymbol interference (ISI). A possible solution to the problem consists in performing a multiple-differential encoding at the source node combined with a correlation operation with a noisy template at each intermediate relay. As a result, a significantly better bit error probability (BER) performance is obtained with respect to the direct transmission without extending guard intervals [10]. However, such a non-cooperative system shows its limits when source-relay, or relay-destination links are degraded.

The goal of the present paper is to propose a cooperative approach that exploits the received signals coming from both the relay and the source, thus outperforming the direct transmission and the non-cooperative scheme regardless of the quality of the links. This result is achieved by weighting the decision variables according to the signal to noise ratios (SNRs) of the respective paths, and therefore requires the knowledge of the channel gains at the destination node.

The remainder of the paper is organized as follows. In Section 2, we briefly describe the system model. In Section 3, we derive the log-likelihood ratio based decision rule and an approximated expression for the effective SNR at the destination node. Such a function is maximized using numerical techniques in order to find the optimal power allocation strategy. The performance of the proposed system is compared to the non-cooperative approach and to the basic DTR scheme with single differential encoding in Section 4. Finally we provide some conclusions in Section 5.

2. SYSTEM MODEL

In this paper we are considering a single-user two-hop scenario set up by three devices, i.e. the source $S$ transmitting the original signal, the A&F relay $R$ which constitutes an intermediate forwarding step, and the destination $D$, that is the final node decoding the information bits. We assume that the source does not know the location of the destination and that the relay is always available to cooperate. The communication is time-slotted, in the sense that source and relay transmit alternatively in disjoint time intervals, in order to avoid
interference.

The transmitted signal is modulated in amplitude (Pulse Amplitude Modulation (PAM)) and the symbols \( a_1[k] \) and \( a_2[k] \) obtained after two steps of differential encoding from the information symbols \( \{s_0[k]\}_{k=-\infty}^{+\infty} \), identically distributed (i.i.d.) and equiprobable in \( \{-1,+1\} \),

\[
a_1[k] = a_1[k-1]a_0[k] \quad \text{and} \quad a_2[k] = a_2[k-1]a_1[k],
\]

where \( a_0[0] = 0 \) and \( a_2[1] \) as initial values.

In order to increase the symbol energy at the receiver, we use \( N_f > 1 \) frames to convey a single information symbol at the cost of a reduced throughput. In formulas,

\[
s_s(t) = \sqrt{\sigma_s} E_g \sum_{k=0}^{+\infty} a_2[k]w_{\text{tx}}(t-jT_f-kT_s),
\]

where \( T_f \) and \( T_s = N_f T_f \) are respectively the frame and the symbol durations, \( w_{\text{tx}}(t) \) is the normalized (\( j_{-\infty}^{+\infty} w_{\text{tx}}^2(t)dt = 1 \)) transmitted pulse, \( E_g \) is the pulse energy, and \( \alpha_s \) denotes the power allocation coefficient of the source node.

For all the links considered, we adopt the IEEE 802.15.3a channel model [11] and the CIR can be written as,

\[
h_{l}(t) = \sum_{i=1}^{N_l} \alpha_{l,i} \delta(t-\tau_{l,i}),
\]

in which \( l \in \{sr, sd, rd\} \) denotes respectively the S-R, S-D, or R-D link, \( N_l \) is the number of multipaths, \( \tau_{l,i} \) is the delay, and \( \alpha_{l,i} \) is the normalized (\( \sum_{i=1}^{N_l} \alpha_{l,i}^2 = 1 \)) amplitude of the i-th ray. The channel gain is affected by log-normal fading and path loss and can be modeled as (cf. [12])

\[
G_l(d) = -10p \cdot \log_{10}(d) + \theta,
\]

where \( G_l(d) \) is the channel gain in dB, \( d \) is the link length, \( p \) is the path loss exponent, and \( \theta \) is the log-normal fading term, namely a Gaussian variable with zero mean and variance \( \sigma_{\text{fading}}^2 \). The shadowing terms associated to different paths are supposed to be uncorrelated.

At the receiver front-end, a bandpass filter \( h_{\text{bp}}(t) \) of bandwidth \( W \) eliminates out-of-band noise. Thus, the received signal can be expressed by

\[
r_{l}(t) = \sqrt{G_l h_{l}(t) * h_{\text{bp}}(t) * s_s(t) + n_l(t)}
\]

\[
= \sqrt{G_l \alpha_s E_g} \sum_{k=0}^{+\infty} a_2[k]w_{\text{tx}}(t-jT_f-kT_s) + n_l(t),
\]

where \( l \in \{sr, sd\} \), \( G_l \) is the channel gain associated to the path of CIR \( h_{l}(t) \), \( n_l(t) \) denotes filtered additive white Gaussian noise (AWGN) with zero mean and zero spectral power density (PSD) \( N_0/2 \), and \( w_{\text{tx}}(t) = w_{\text{tx}}(t) * h_{l}(t) * h_{\text{bp}}(t) \) stands for the received channel template. After filtering, a single differential demodulation is performed by means of a correlation and integration of duration \( T_f \),

\[
\tilde{a}_{1,se}[k] = \sum_{j=0}^{N_f-1} \int_{jT_f+J_f+T_f} r_{l}(t) \delta(t-T_s)dt.
\]

At the relay \( l = sr \), the soft estimates \( \tilde{a}_{1,se}[k] \) are then remodeled and transmitted as

\[
s_r(t) = \sqrt{\sigma_x} E_g \sum_{k=0}^{+\infty} \tilde{a}_{1,se}[k]w_{\text{tx}}(t-jT_f-kT_s),
\]

where \( \gamma_x \) is the power allocation coefficient associated to the relay.

As a result, in two different time slots the destination node receives the signals \( r_{sd}(t) \) (Eq. (5) with \( l = sd \)) from the source and \( r_{rd}(t) \) from the relay as

\[
r_{rd}(t) = \sqrt{G_{rd} + h_{sd}} \gamma_x n_{rd}(t) + n_{rd}(t)
\]

\[
= \sqrt{G_{rd} \alpha_r E_g} \sum_{k=0}^{+\infty} \tilde{a}_{1,se}[k]w_{tx,rd}(t-jT_f-kT_s) + n_{rd}(t),
\]

in which \( w_{tx,rd}(t) = w_{tx}(t) * h_{rd}(t) * h_{bp}(t) \) and \( n_{rd}(t) \) is filtered AWGN with zero mean and PSD \( N_0/2 \).

The idea is to perform the operations of bandpass filtering and demodulation (correlation and integration) on both the received signals and then to combine in an appropriate way the resulting random variables \( \tilde{a}_{1,se}[k] \) (Eq. (6) with \( l = sd \)) and \( \tilde{a}_{0,se}[k] \).

\[
\tilde{a}_{0,se}[k] = \sum_{j=0}^{N_f-1} \int_{jT_f+T_f} r_{rd}(t) \delta(t-T_s)dt.
\]

In the following, we study the decision strategy at the destination.

### 3. Decision Strategy

#### 3.1. Study of the Log-Likelihood Ratio

In order to make the decision on the information bit \( a_0[k] \), the existing non-cooperative method [10] takes into account only the S-R-D path, namely the decision variable \( \tilde{a}_{0,se}[k] \). Our proposal is to exploit the information coming from both the S-R-D and S-D paths, considering the three decision variables \( \tilde{a}_{1,se}[k] \), \( \tilde{a}_{1,se}[k+1] \), and \( \tilde{a}_{0,se}[k] \), which can be well approximated as Gaussian distributed and independent [4], [6]. In formulas,

\[
\begin{align*}
\tilde{a}_{1,se}[k+1] &= \tilde{a}_{sd}[k+1] + z_{sd}[k+1] \\
\tilde{a}_{1,se}[k] &= \tilde{a}_{sd}[k] + z_{sd}[k] \\
\tilde{a}_{0,se}[k] &= \tilde{a}_{rd}[k] + z_{rd}[k],
\end{align*}
\]

in which \( z_{sd}[k] \) and \( z_{rd}[k] \) are Gaussian variables with zero mean and variances \( \sigma_{z_{sd}}^2 \) and \( \sigma_{z_{rd}}^2 \) respectively. If \( E_{s} = N_f E_{r} \) is the total transmitted energy associated to one information symbol shared by source and relay, \( \delta = W N_f T_f N_0^2/2 \) a second noise term, and \( \gamma_{l} = G_{l} \bullet T_{T_{l}} w_{\text{tx}}^2(t)dt \) the percentage of power captured after the bandpass filter, it is possible to show that [10]

\[
\begin{align*}
\beta_{l} &= E_{l} \gamma_{s} \gamma_{s}, & l \in \{sd, sr\} \\
\beta_{rd} &= E_{r} \gamma_{rd} \gamma_{rd} \\
\beta_{sd} &= \beta_{rd} \gamma_{rd} \\
\sigma_{z_{sd}}^2 &= \beta_{sd} \gamma_{s} \gamma_{r} + \beta_{sd} \gamma_{rd} \gamma_{rd} + \gamma_{rd} \gamma_{rd} \gamma_{rd} + \delta \\
\gamma_{s} &= \gamma_{s} + \alpha_{s} (\beta_{sd} \gamma_{sd} + \gamma_{rd} \gamma_{rd} \gamma_{rd} = 1)
\end{align*}
\]

Notice that \( \delta \) cannot be neglected, as \( W N_f T_s \gg 1 \) and that the last equation represents the total transmitted power constraint.

The optimal decision rule on \( a_0[k] \) is based on the log-likelihood ratio, that is

\[
\Lambda(\tilde{a}[k]) = \Lambda(\tilde{a}_{1,se}[k+1], \tilde{a}_{1,se}[k], \tilde{a}_{0,se}[k]) = \begin{cases} 
\frac{f_{\Lambda}(\tilde{a}_{1,se}[k+1], \tilde{a}_{1,se}[k], \tilde{a}_{0,se}[k]| a_0[k] = 1)}{f_{\Lambda}(\tilde{a}_{1,se}[k+1], \tilde{a}_{1,se}[k], \tilde{a}_{0,se}[k]| a_0[k] = -1)}
\end{cases}
\]

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where \( \tilde{a}[k] = (\tilde{a}_{1,sd}[k+1], \tilde{a}_{1,sd}[k], \tilde{a}_{0,srd}[k]) \) and \( f_\mathbf{X}(\cdot) \) denotes the joint probability density function (PDF) of the random vector \( \mathbf{X} \). We decide for \( a_0[k] = 1 \) if \( \Lambda(\tilde{a}[k]) > 0 \) and for \( a_0[k] = -1 \) otherwise. Using formula (10), the independent Gaussian assumption and the fact that for \( i \in [-1, 1] \)

\[
p(a_1[k + 1] = -1, a_1[k] = -i | a_0[k] = i) = p(a_1[k + 1] = 1, a_1[k] = i | a_0[k] = i) = 1/2,
\]

one finds that

\[
\Lambda(\tilde{a}[k]) = \frac{2\beta_{srd} - a_{0,srd}[k] + \beta_{sd}}{2\beta_{srd} - a_{0,srd}[k]} + \ln \left( \frac{\beta_{sd}(\tilde{a}_{1,sd}[k + 1] + \tilde{a}_{1,sd}[k])}{\sigma_{sd}^2} \right).
\]

To make formula (14) more tractable, we consider the following Jacobi approximation which is common in digital signal processing,

\[
\ln(e^{a_1} + e^{a_2}) = \max(a_1, a_2) + \ln(1 + e^{-|a_1-a_2|}) \approx \max(a_1, a_2).
\]

As a result, we obtain the approximated decision variable

\[
\Lambda(\tilde{a}[k]) \approx X[k] = \frac{2\beta_{srd} - a_{0,srd}[k] + \beta_{sd}}{2\beta_{srd} - a_{0,srd}[k]} \times (|\tilde{a}_{1,sd}[k + 1] + \tilde{a}_{1,sd}[k]| - |\tilde{a}_{1,sd}[k + 1] - \tilde{a}_{1,sd}[k]|).
\]

### 3.2. Optimal Power Allocation Coefficients

The study of an optimal power allocation strategy is far more challenging in the cooperative case than in the non-cooperative scenario simply because there are two different paths and three decision variables to be taken into account. The proposed solution consists of the numerical maximization of a closed-form analytical expression for the effective SNR at the destination node.

The non-cooperative case considers only the S-R-D path and \( \text{SNR}_{nc} = \beta_{srd}^2/\sigma_{sd}^2 \). Instead, in the cooperative scenario we expect \( \text{SNR} \) to depend on the received SNRs of both the possible paths, i.e. \( \beta_{srd}^2/\sigma_{sd}^2 := \gamma_{sd} \).

According to formula (16), \( X[k] \) is the sum of a Gaussian variable, and absolute values of the sum and difference of other two Gaussian variables. If \( \gamma_{sd} \) is high, i.e. the S-D path is reliable, then the noise terms \( z_{sd}[k+1] \) and \( z_{sd}[k] \) are significantly smaller than the corresponding signal terms and \( |\tilde{a}_{1,sd}[k + 1] \pm \tilde{a}_{1,sd}[k]| \) are approximately Gaussian distributed. If instead \( \gamma_{sd} \) is small, this last approximation is not more accurate, but \( \|\tilde{a}_{1,sd}[k + 1] \pm \tilde{a}_{1,sd}[k]\| \) is weighted by \( \gamma_{sd} \) and the Gaussian variable \( \sigma_{sd}^{-2}a_{0,srd}[k] \) dominates. Since this heuristic argument is also supported by numerical simulations, it is reasonable to take

\[
\text{SNR}_c = \frac{E^2(X[k])}{\text{Var}(X[k])},
\]

where the closed-forms for the mean and variance of \( X[k] \) are given by

\[
E^2(X[k]) = (\gamma_{srd} + \gamma_{sd} - \frac{\sqrt{\gamma_{sd}}}{\pi}(1 - e^{-\gamma_{sd}}) - 2\gamma_{sd}Q(\sqrt{2\gamma_{sd}}))^2
\]

\[
\text{Var}(X[k]) = \gamma_{srd} + \gamma_{sd} \left( \frac{1}{\pi} + \gamma_{sd} \right) - \left( \frac{1}{\pi}e^{-\gamma_{sd}} - 2\sqrt{\gamma_{sd}}Q(\sqrt{2\gamma_{sd}} + \sqrt{\gamma_{sd}}) \right)^2,
\]

with \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2}du \). Notice that for \( \gamma_{srd} \gg \gamma_{sd} \), \( \text{SNR}_c \approx \text{SNR}_{nc} \) as in this case the direct path is not reliable and our decision is mainly based on the S-R-D path.

Now, we have to find

\[
\alpha_{s, opt} = \arg \max \text{SNR}_c, \quad \text{s.t.} \quad \alpha_s + \alpha_c(\beta_{srd}^2 + \sigma_{sd}^2) = 1. \tag{19}
\]

Due to the complexity of equations (11) linking the optimal power allocation coefficients to the parameters of formula (17), the maximization of \( \text{SNR}_c \) is performed numerically by means of an exhaustive search.

### 4. NUMERICAL RESULTS

In the following, we present simulation results in order to show the validity of Eq. (17) and analyze the performance of the proposed cooperative approach.

The pulse \( w_{ts}(t) \) is a normalized version of the widely employed Scholtz’s monocycle [13], i.e.

\[
w_{ts}(t) = \left( 1 - 4\pi \frac{t - v_p}{v_m} \right)^2 e^{-2\pi(t-v_p)/v_m^2},
\]

with \( v_p = 0.35 \) ns and \( v_m = 0.2877 \) ns. The pulse and frame durations are respectively \( T_p = 0.7 \) ns and \( T_f = 70 \) ns, in order to eliminate ISI effects. We focus on the channel model CM1, considering a path loss exponent \( p = 3 \) and a standard deviation of the log-normal fading term \( \sigma_{fad} = 2.5 \). The number of frames used to transmit each symbol is \( N_f = 2 \), the bandwidth of bandpass filter is \( W = 5 \) GHz, and the integration time is \( T_i = 5.25 \) ns [10].

Let \( d_{rd}, d_{sr} \) and \( d_{dr} \) be the source-destination, source-relay, and relay-destination distances. We take into account two possible dispositions of these three nodes favoring the S-R-D and S-D path respectively:

1. \( d_{rd} = 1 \) m, \( d_{sr} = 0.4 \) m, and \( d_{dr} = 0.6 \) m, i.e. relay close to the middle point of the segment joining source and destination;
2. \( d_{rd} = 1 \) m, \( d_{sr} = 0.8 \) m, and \( d_{dr} = 0.8 \) m, in which the three nodes form an isosceles triangle.

In Figure 1, we consider the transmission of \( 10^6 \) symbols with fixed CIRs and channel gains, and plot the BER as \( \alpha_s \) varies in [0, 1] with a step size equal to 0.01. Two values of the ratio \( E_b/N_0 \) are chosen for each scenario, in order to obtain a minimum BER close to \( 10^{-2} \) and \( 10^{-4} \). In all the cases the value of \( \alpha_s \) that maximizes

![Fig. 1. Bit error probability as a function of the power allocation coefficient \( \alpha_s \). The optimal value of \( \alpha_s \) given by Eq. (19) yields a BER close to the absolute minimum in all the cases considered.](image-url)
SNRc yields a BER close to the absolute minimum. In addition, even if SNRc depends on $E_o/N_0$, we notice that the power allocation strategy is slightly affected by changes in this ratio.

Figures 2 and 3 present a performance comparison between the direct transmission with single differential encoding, A&F non-cooperative relaying with double differential encoding and the power allocation strategy described in [10], and the proposed cooperative approach with $\alpha_s = 0.5$ (equal power allocation) and $\alpha_r$, given by Eq. (19). We perform $10^4$ different Monte Carlo trials of the transmission of $10^3$ symbols, selecting randomly three channels from a set of 100 sample CIRs and generating each time independent channel gains. First of all, we have to remark that the Jacobi approximation of Eq. (16) yields basically almost the same results of the optimal decision rule given by Eq. (14). The first scenario with the relay aligned with source and destination favors the non-cooperative scheme. Nevertheless, the exploitation of the direct S-D link allows the cooperative approach to obtain a gain of about 1 dB (2 dB) when BER = $10^{-2}$ (BER = $10^{-3}$) with respect to non-cooperative relaying [10]. Notice that, when no channel knowledge is available at the source node, which means that we have to set $\alpha_s = 0.5$, the cooperative approach still outperforms the non-cooperative scheme. On the contrary, the second scenario is unfavorable for the S-R-D path and, if $E_o/N_0$ is low, the direct transmission behaves best, since a single differential encoding is employed. However, as $E_o/N_0$ increases, the proposed scheme yields the best error performance, achieving a gain of about 1 dB (4 dB) for BER = $10^{-2}$ (BER = $10^{-3}$).

5. CONCLUSIONS

In this paper, we present a new decision strategy that exploits both the signal forwarded by the relay and the one directly transmitted by the source in a two-hop amplify-and-forward IR-UWB system. To achieve this aim, we discuss an easy to implement, reliable approximation of the log-likelihood ratio and propose a semi-analytical power allocation strategy. Numerical results prove the effectiveness of the presented scheme in different scenarios, showing gains of at least 2-4 dB with respect to both the direct transmission with single differential encoding and the non-cooperative A&F relaying.

6. REFERENCES