ITERATIVE INTERFERENCE ALIGNMENT FOR CELLULAR SYSTEMS WITH USER SELECTION

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ABSTRACT

An iterative interference alignment algorithm for cellular systems with multiple signaling dimensions is introduced. The main invention is that we combine iterative interference alignment with user selection, which naturally increases the system sum rate by exploiting the multiuser diversity. Extensive simulations show significant gains over single cell processing and joint/coherent transmit schemes, also with partial channel state information.

1. INTRODUCTION

One of the key challenges in the design of future cellular systems is to overcome the interference barrier. Intra cell interference and out of cell interference substantially limit the reusability of spectral resources. In [1] coherent transmission from multiple base stations to multiple users was considered as a promising technique to achieve enormous gains in spectral efficiency. Under practical considerations one major drawback of coherent transmission is that the base stations must share data. Another possibility to overcome the interference barrier is coordinated scheduling. Coordinated scheduling requires only the exchange of scheduling information and no data sharing. In this work, we consider coordinated scheduling and exploit the possibilities of interference alignment in cellular systems.

In the landmark paper [2] interference alignment was used to show that the capacity of the interference channel with time varying channels is much higher than previously believed. The interference alignment scheme in [2] uses long symbol extensions to approach arbitrarily close to the optimal degrees of freedom. Interference alignment over a limited number of dimensions, we proposed algorithms that, under certain conditions, achieve the optimal degrees of freedom. Independently, In [6] similar algorithms are proposed and, for the case of a single data stream per user, necessary conditions for zero interference are found. Recently, in [7] we characterized the degrees of freedom of symmetric cellular systems with constant channels and introduced a user selection scheme that is based on this new insights.

In this paper we propose an interference management algorithm that selects a subset of users for each transmission and optimizes the transmit precoders and receive filters for the active users. We demonstrate the superior performance of our algorithm by comparing it with other popular transmit schemes for cellular systems. Finally, we show that the algorithm is robust even with channel state information (CSI) uncertainties at the transmitters.

2. PROBLEM SETUP

We consider a cellular system with B base stations and K users. The base stations are equipped with N antennas and the users are equipped with M antennas. The users are uniformly distributed over the network area and each base station provides wireless service to a subset of users \( B_b \subseteq \{1, 2, \ldots, K\} \) with \( B_b \cap B_l = \emptyset \) for all \( b \neq l \). All base stations are connected via a backhaul network which enables them to share scheduling information and channel state information (CSI).

We assume a block fading channel model where the channel remains constant for a certain time and then jumps to a new independent channel realization. Transmission is performed over \( \mu \) independent channel realizations, such that, the channel from base station \( l \) to user \( k \) in cell \( b \) can be given by a block diagonal matrix \( H_{b,k}^{[l]} \in \mathbb{C}^{\mu M \times \mu N} \). In the down-
link the $\mu M$ dimensional signal vector received by user $k$ in cell $b$ in one transmission interval (time index dropped) is

$$y_{b,k} = H_{b,k}x^{[b]} + \sum_{l \neq b} H_{b,k}^l x^{[l]} + n_k,$$

where $n_k \sim N_c(0, I_{\mu M})$ is zero mean unit variance circular symmetric additive white Gaussian noise. In each transmit interval each base station transmits to a subset of users $S_b \subseteq B$, hence, the transmitted signal vector is

$$x^{[b]} = V^{[b]}s^{[b]} = \sum_{k \in S_b} v_{b,k}^{[b]}s_k^{[b]}.$$

Here we assume a single data stream per user, such that, the number of streams transmitted by base station $b$ is $S^{[b]} = |S_b|$. Further, we defined $V^{[b]} \in \mathbb{C}^{\mu N \times S^{[b]}}$ as the precoder that maps the data symbols $s^{[b]} \in \mathbb{C}^{S^{[b]} \times 1}$ intended for the users in cell $b$ to $\mu N$ transmit dimensions. The transmit signal is power constrained such that $|x^{[b]}|^2 \leq \mu P$ and power allocation between different users is uniform. Receiver $k \in S_b$ obtains an estimate $\hat{s}_k^{[b]}$ of the transmitted data symbol $s_k^{[b]}$ by multiplying the received signal vector with the receive filter $u_{b,k} \in \mathbb{C}^{M \times 1}$, i.e., $\hat{s}_k^{[b]} = (u_{b,k})^H y_{b,k}$. In the following we consider the problem of maximizing the system sum rate

$$R_S(S) = \frac{1}{\mu} \sum_{b=1}^B \sum_{k \in S_b} \log (1 + \gamma_{b,k}). \quad (1)$$

Here, $\gamma_{b,k}$ is the signal-to-interference-plus-noise ratio (SINR) of user $k$ in cell $b$ and $S = \{S_1, \ldots, S_B\}$ is a list of all scheduled users. The SINR is a function of the channels, the precoders, the receive filters and the scheduled users. The algorithm presented in the next section aims at maximizing the system sum rate $R_S(S)$ by scheduling certain users and optimizing the transmit precoders and receive filters.

### 3. Algorithm and Receive Filter Design

The algorithm presented in this section is based on the idea of alternating the optimization of receive filters and transmit precoders and includes a greedy user selection. Different design criteria can be used for the receive filters and transmit precoders. Below we introduce two different design criteria, the first aims on minimizing the intra cell and out of cell interference and the second aims on maximizing the SINR of each user. In the sequel we assume that a central scheduling unit performs the user selection, optimizes the transmit precoders and receive filters and distributes the scheduling information among all base stations. Alternatively, the algorithm can be modified such that processing can be performed by the base stations in a distributed fashion.

The transmit protocol for the proposed algorithm can be summarized as follows. First, each base station transmits orthogonal common pilots which are used by all users $k$ to measure the channels $H_{b,k}^{[l]}$ for all base stations $l$. The channels are quantized and fed back, such that, base station $b$ has knowledge of the channels $\tilde{H}_{b,k}^{[l]} = Q_b(H_{b,k}^{[l]})$ for all $k \in S_b$ and all $l = 1, \ldots, B$, where $Q_b : \mathbb{C}^{\mu M \times M} \rightarrow \mathbb{C}^{\mu M \times M}$ is the quantizer. Next, all base stations forward this CSI to the central scheduling unit, which uses our algorithm to select users for transmission and optimize the transmit precoders and receive filters. Finally, dedicated (i.e. precoded) pilots are transmitted by all base stations, such that the selected users can compute the optimal receive filter.

**Iterative Algorithm:** For the ease of presentation we split the algorithm in an outer loop and an inner loop. For fixed receive filters the inner loop performs the user selection and the optimization of transmit precoders. The outer loop optimizes the receive filters for a fixed set of users and fixed precoders. The outer loop is summarized in Algorithm 1. It is performed until some termination condition is satisfied, e.g. a maximum number of iterations is reached or the maximum residual interference falls below a certain threshold. In the outer loop all base stations are processed in an arbitrary ordering, the effective channels $\tilde{H}_{b,k}^{[l]} v_k^{[l]}$ are determined and the optimal receive filters $u_{b,k}$ are computed, as described below. If no precoders are specified (i.e. in the first iteration) isotropic interference from the base stations is assumed.

The inner loop is summarized in Algorithm 2. It comprises of the user selection and the computation of transmit precoders. In every iteration step the inner loop starts with no selected users. All base stations are processed in an arbitrary ordering until no new users are added to the system. For each base station $b$ a new user is added if this increases the system sum rate $R_S$. Each time a user is added to the system all precoders are updated such that the new interference situation is considered. The precoders $v_k^{[b]}$ are obtained by computing the

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**Algorithm 1 – Outer loop: Iteration and receive filter opt.**

1. Set $v_k^{[b]} = 1/\sqrt{N}[1, 1, \ldots, 1]^T$ for all $b$, $k$ and $S_b = \emptyset$ for all $b$.
2. **repeat**
   3. **for** $b = 1$ to $B$ and all $k \in B_b$ **do**
   4. Determine effective channels $\tilde{H}_{b,k}^{[l]} v_k^{[l]}$ for all $b$, $k$.
   5. **if** $k \in S_b$ **then**
   6. Compute receive filter $u_{b,k}$.
   7. **else**
   8. Receive filter is $u_{b,k} = \tilde{H}_{b,k}^{[l]} v_k^{[l]}$.
   9. **end if**
   10. **end for**
11. **Inner loop:** User selection and precoder optimization
   (Input: $u_{b,k}$ for all $b$, $k$. Output: $S_b$ and $V^{[b]}$ for all $b$).
12. **until** some termination condition is satisfied.
Algorithm 2 – Inner loop: User selection and precoder opt.

1: Input: $\mathbf{u}_{b,k}$ for all $b, k$
2: Set: $S_b = \emptyset$ for all $b$
3: repeat
   4: for $b = 1$ to $B$
      5: for $k \in B_b \setminus S_b$
      6: $T_b = \{k\} \cup S_b$
      7: Compute precoder for potential new user $\mathbf{v}_k^{[b]} = \mathbf{u}_{b,k}$ for all $k \in T_b$.
      8: Update precoder of all previously selected users $\mathbf{v}_l^{[l]} = \mathbf{u}_{l,m}$ for all $l \neq b$ and all $m \in S_l$.
      9: Compute system sum rate $R_k = R_{31}(\ldots \cup S_{b-1} \cup T_b \cup S_{b+1} \cup \ldots)$. 
    10: end for
    11: Add one new user if sum rate increases.
    12: end for
13: until No new users added at any base station $b$.

Interference Minimization: To minimize (or even cancel) all interference we proceed as follows. At the receiver side we consider only the out of cell interference and at the transmitter we consider the intra cell and the out of cell interference. Intuitively, this implies that at each receiver the intra cell interference must align with the out of cell interference.

The receive filter optimization works as follows. Assume that all transmit precoders $V^{[b]}$ for all base stations $b$ are fixed. The receive filter that minimizes the out of cell interference seen by user $k \in B_b$ is given by $u_{b,k} = \nu_{\min}(O_{b,k})$, where $\nu_{\min}(X)$ is defined as the eigenvector corresponding to the smallest magnitude eigenvalues of the matrix $X$ and the out of cell interference covariance matrix is defined as

$$O_{b,k} = \sum_{l=1}^{B} H^{[l]}_{b,k} V^{[l]} (H^{[l]}_{b,k} V^{[l]})^H.$$ 

The transmit precoder optimization is performed in the reciprocal network. In the reciprocal network the channel from user $k$ in cell $b$ to base station $l$ is given by $H^{[l]}_{b,k} = (H^{[l]}_{b,k})^H$ and the transmit precoder used by user $k \in S_k$ is $\mathbf{v}_{b,k} = \mathbf{u}_{b,k}$. In the reciprocal network the receive subspace with minimum out of cell interference is $\tilde{\mathbf{U}}^{[b]} = \nu_{\min}(\tilde{O}_b)$, where $\nu_{\min}(S^{[b]})$ are the eigenvectors corresponding to the $S^{[b]}$ smallest magnitude eigenvalues and the out of cell interference covariance matrix in the reciprocal network is defined as

$$\tilde{O}_b = \sum_{l \neq b} \sum_{k \in S_l} H^{[l]}_{l,k} \mathbf{v}_{l,k} (H^{[l]}_{l,k} \mathbf{v}_{l,k})^H.$$ 

The intra cell interference is canceled by an additional zero-forcing step, i.e. the transmit precoder for user $k$ in cell $b$ is $\mathbf{v}_k^{[b]} = \tilde{\mathbf{U}}^{[b]} \mathbf{w}_k^{[b]}$, where $\mathbf{w}_k^{[b]} \in \text{ker}\left[\tilde{H}_{b,l}^{[b]} \tilde{U}^{[b]}_l\right]_{l \in S_b \setminus \{k\}}$.

**SINR Maximization:** To maximize the SINR of each user we proceed as follows. Similar to the interference minimization approach we consider only out of cell interference at the receivers and intra cell and out of cell interference at the transmitter. Hence, at each receiver the intra cell interference must align with the out of cell interference, but since we focus on maximizing the SINR a certain amount of interference is possible if at the same time the desired signal strength increases.

First, consider the receive filter optimization. Assume that all transmit precoders $V^{[b]}$, $b = 1, \ldots, B$ are fixed. The receive filter of user $k \in S_b$ is given by $u_{b,k} = \nu_{\max}(\Theta_{b,k}^{-1} G_{b,k})$, where $\nu_{\max}(X)$ is defined as the eigenvector corresponding to the largest magnitude eigenvalue of the matrix $X$. The gain covariance matrix $G_{b,k}$ and the out of cell interference plus noise covariance matrix $\Theta_{b,k}$ are defined as

$$G_{b,k} = \tilde{H}^{[b]}_{b,k} \nu_{b,k} \left(\tilde{H}^{[b]}_{b,k} \nu_{b,k}\right)^H,$$

$$\Theta_{b,k} = \sum_{l=1}^{B} H^{[l]}_{b,k} V^{[l]} (H^{[l]}_{b,k} V^{[l]})^H + I_{M\mu}.$$ 

The transmit precoder optimization is performed in the reciprocal network, i.e. $\nu_{b,k} = \mathbf{u}_{b,k} = \nu_{\max}(\tilde{G}_{b,k}^{-1} G_{b,k})$. In the uplink the interference plus noise covariance matrix $\tilde{\Theta}_{b,k}$ and the gain covariance matrix $\tilde{G}_{b,k}$ are defined as

$$\tilde{G}_{b,k} = \tilde{H}^{[b]}_{b,k} \mathbf{v}_{b,k} \left(\tilde{H}^{[b]}_{b,k} \mathbf{v}_{b,k}\right)^H,$$

$$\tilde{\Theta}_{b,k} = \sum_{l=1}^{B} H^{[l]}_{b,m} \mathbf{v}_{l,m} (H^{[l]}_{b,m} \mathbf{v}_{l,m})^H + \sum_{m \in S_b \setminus \{k\}} H^{[b]}_{b,m} \mathbf{v}_{b,m} (H^{[b]}_{b,m} \mathbf{v}_{b,m})^H + I_{N\mu}.$$ 

### 4. SIMULATIONS

In this section we present LTE system level simulations. The channels are modeled using the SCME (urban macro scenario) with a velocity of 3 km/h and high angular spread. The physical layer is configured according to the LTE FDD downlink see [8] and references therein. The system setup is as follows: $B = 3$ base stations with $N = 5$ antennas, $K = 15$ users with $M = 5$ antennas, all antenna arrays are correlated (wavelength/2 spacing), system bandwidth 10 MHz at 2.1 GHz carrier frequency, average receive SNR is 25 dB. The performance measure is the system sum rate $R_{31}$ defined in (1).
Our proposed algorithm is compared to a single cell processing scheme and a coherent transmission scheme. The single cell processing scheme is zero-forcing (ZF) beamforming with greedy user selection as described in [9]. The coherent transmission scheme is block diagonalization (BD) with greedy user selection as described in [10]. Block diagonalization requires data sharing between the base stations.

In Figure 1 we assume perfect CSI at the transmitters and compare the network spectral efficiency of our maximum SINR algorithm with ZF and BD. We observe that the proposed algorithm performs close to BD, which is a remarkable result since, in contrast to BD, no data sharing is required. As expected we observe that the network spectral efficiency of our scheme increases with the number of iterations. If we perform only one iteration the complexity of our algorithm is comparable to that of ZF. Another astonishing result is that with only 1 iteration our scheme clearly outperforms ZF.

In Figure 2 we consider partial CSI at the transmitters. We assume each user quantizes the six most significant channel taps which are assumed to lay within the first 100 samples. The real and imaginary parts of each of the six channel taps are quantized using $n_B$ bits and fed back along with the tap positions. This results in a feedback load per feedback message of $R_{FB} = 12NM(\log_2(100) + n_B)$. We observe that for a huge fraction of feedback rates the proposed algorithm performs close to block diagonalization. This is a remarkable result since our requirements on the backhaul network are significantly smaller than with block diagonalization.

5. CONCLUSION

We introduced an algorithm that optimizes the transmit precoders and receive filters in cellular systems. The algorithm includes a greedy user selection, which exploits the multiuser diversity inherent to cellular systems. The algorithm requires the base stations to exchange a small amount of scheduling information but no data sharing is needed. We saw that our algorithm performs very close to other schemes that require data sharing and outperforms schemes without data sharing.

6. REFERENCES