DETECTING PASSIVE EAVESDROPPERS IN THE MIMO WIRETAP CHANNEL

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ABSTRACT

The MIMO wiretap channel comprises a passive eavesdropper that attempts to intercept communications between an authorized transmitter-receiver pair, with each node being equipped with multiple antennas. In a dynamic network, it is imperative that the presence of a passive eavesdropper be determined before the transmitter can deploy robust secrecy-encoding schemes as a countermeasure. This is a difficult task in general, since by definition the eavesdropper is passive and never transmits. In this work we adopt a method that allows the legitimate nodes to detect the passive eavesdropper from the local oscillator power that is inadvertently leaked from its RF front end. We examine the performance of non-coherent energy detection as well as optimal coherent detection schemes. We then show how the proposed detectors allow the legitimate nodes to increase the MIMO secrecy rate of the channel.

Index Terms— MIMO wiretap channel, passive eavesdropper, energy detection

1. INTRODUCTION

The broadcast characteristic of the wireless propagation medium makes it difficult to shield transmitted signals from unintended recipients. This is especially true in multiple-input multiple-output (MIMO) systems with multi-antenna nodes, where the increase in communication rate to the legitimate receiver is offset by the enhanced interception capability of eavesdroppers. A three-terminal network consisting of a legitimate transmitter-receiver pair and a passive eavesdropper where each node is equipped with multiple antennas is commonly referred to as the MIMO wiretap or MIMOME channel. The information-theoretic aspects of this scenario led to the development of the notion of secrecy capacity at the physical layer, which quantifies the rate at which a transmitter can reliably send a secret message to the receiver, with the eavesdropper being completely unable to decode it. The secrecy capacity of the multiple-input multiple-output (MIMO) wiretap channel has been studied in [1]-[2], for example.

In the burgeoning literature on the MIMO wiretap channel, a number of transmit precoding techniques have been proposed to improve the channel secrecy rate by exploiting either the instantaneous realizations or statistics of the channel to the eavesdropper [1]-[2]. However, the question of how the legitimate transmitter acquires a passive eavesdropper’s CSI has yet to be answered satisfactorily. More importantly, it is imperative that the presence of a passive eavesdropper be determined before the transmitter can deploy robust secrecy-encoding schemes as a countermeasure. This is a difficult task in a dynamic wireless network, since by definition the eavesdropper is passive and never transmits. Surprisingly, the issue of determining the presence of potential eavesdroppers in the wiretap channel has not been addressed previously to our best knowledge.

In this work we propose a scheme that allows the legitimate nodes to detect the passive eavesdropper from the local oscillator power that is inadvertently leaked from its RF front end. This technique was recently proposed for spectrum sensing in single-antenna cognitive radios (CR) to avoid interfering with primary receivers in [5–7] under AWGN channels. We generalize this technique to MIMO channels in a wiretap scenario, and investigate how the proposed algorithm allows the legitimate nodes to increase the MIMO secrecy rate of the channel. Furthermore, the majority of the existing works on spectrum sensing with multi-antenna CRs model the primary transmitter as a single-antenna terminal, whereas in this work we explicitly consider the detection of a full-rank signal of interest.

2. SYSTEM MODEL

2.1. Network Model

We consider a multi-user network with an \( N_a \)-antenna transmitter (Alice), an \( N_b \)-antenna receiver (Bob), and an unauthorized eavesdropper (Eve) with \( N_e \) antennas. When Alice is transmitting to Bob and Eve is listening in the vicinity, the received signals at Bob and Eve are given by

\[
y_b = \sqrt{d_{ab}} \mathbf{H}_{ba} \mathbf{x} + \mathbf{n}_b, \tag{1}
\]

\[
y_e = \sqrt{d_{ae}} \mathbf{H}_{ea} \mathbf{x} + \mathbf{n}_e, \tag{2}
\]

where \( \mathbf{x} \in \mathbb{C}^{N_a \times 1} \) is the confidential information signal, \( \mathbf{H}_{ba} \in \mathbb{C}^{N_b \times N_a} \), \( \mathbf{H}_{ea} \in \mathbb{C}^{N_e \times N_a} \) are the deterministic complex MIMO channels from Alice, the distances from Alice to Bob and Eve are \( d_{ab} \) and \( d_{ae} \), respectively, and \( \alpha \) is the path-loss exponent. The i.i.d. additive complex Gaussian noise vectors are distributed as \( \mathbf{n}_b \sim \mathcal{CN}(\mathbf{0}, \mathbf{Z}_b) \), \( \mathbf{n}_e \sim \mathcal{CN}(\mathbf{0}, \mathbf{Z}_e) \). An average power constraint is imposed on Alice’s transmit covariance matrix \( \mathbf{Q} = E \{ \mathbf{x} \mathbf{x}^H \} \) in the form of \( \text{Tr}(\mathbf{Q}) \leq P_a \). Both Alice and Bob are assumed to always have perfect knowledge of the main channel \( \mathbf{H}_{ba} \) irrespective of the potential presence of Eve, since it is used for data communication.

If the input signal \( \mathbf{x} \) is drawn from a Gaussian distribution, the instantaneous MIMO secrecy rate [2] for fixed channels when Eve is present is given by

\[
R_{s,i} = \log_2 \left| \mathbf{I} + \mathbf{H}_{ba} \mathbf{Q} \mathbf{H}_{ba}^H \mathbf{Z}_b^{-1} \right| - \log_2 \left| \mathbf{I} + \mathbf{H}_{ea} \mathbf{Q} \mathbf{H}_{ea}^H \mathbf{Z}_e^{-1} \right|. \tag{3}
\]
The fundamental procedure of detecting the passive node Eve is as follows. We assume all three nodes possess either heterodyne or direct-conversion transceivers. A general impairment in such receivers is that a small portion of the local oscillator (LO) signal back-propagates to the antenna ports and leaks out, even when in passive reception mode [8]. While the LO leakage signal power is on the order of -50 to -90 dBm from a single antenna port, the LO leakage signal is boosted when multiple RF chains are present as in the MIMO wiretap setting, and is consequently easier to detect.

Therefore, we assume that Alice periodically ceases data transmission in order to allow both herself and Bob to independently sense the radio environment. Since the sensing algorithm and process is assumed to be identical at both Alice and Bob, to avoid repetition we focus on the detection process at Bob in the sequel. The binary hypothesis test at Bob during these ‘silent’ periods is

\[ H_0 : \quad y_b (t) = \sqrt{d_{ab}^{\alpha}} H_{ba} w_i (t) + n_b (t) \]

\[ H_1 : \quad y_b (t) = \sqrt{d_{be}^{\alpha}} H_{be} s_i (t) + \sqrt{d_{ab}^{\alpha}} H_{ba} w_i (t) + n_b (t) \]

where \( H_{ba} \in \mathbb{C}^{N_b \times N_a} \) is the complex MIMO leakage channel from Eve to Bob who are separated by distance \( d_{ba} \). The aggregate LO leakage signal from Eve is \( s_i (t) = [ s_1 (t) \ldots s_{N_b} (t) ]^T \).

We model the LO leakage signal from Eve’s \( i^{th} \) antenna port as an unmodulated frequency tone [7]:

\[ s_i (t) = A_i \cos (\omega t + \theta_i) \],

(4)

where \( A_i \) is the amplitude, \( \omega \) is the LO frequency, and \( \theta_i \) is an arbitrary phase. Similarly, the LO leakage signal from Alice is \( w_i (t) = [ w_1 (t) \ldots w_{N_a} (t) ]^T \), where

\[ w_i (t) = B_i \cos (\tilde{\omega} t + \xi_i) \],

(5)

where \( B_i \) is the amplitude, \( \tilde{\omega} \) is her LO frequency, and \( \xi_i \) is an arbitrary phase.

2.2. Secrecy Rate Performance

We consider the following signal transmission model. The overall data transmission period is split into blocks of \( T \) channel uses each. At the beginning of each block, Alice and Bob independently sense the radio environment for the presence of Eve. If the consensus is that Eve is absent, then for the remaining \( T - 1 \) channel uses in that block Alice designs her input covariance \( Q \) to maximize the conventional MIMO secrecy rate to Bob via waterfilling. If Eve is determined to be present, Alice acquires the statistics of her channel \( H_{ca} \) and optimizes \( Q \) by splitting her transmit resources between data and an artificial jamming signal such that the expected value of the MIMO secrecy rate for that block is maximized [1]. The block duration \( T \) is assumed to be long enough in order to invoke information-theoretic random coding arguments.

Define \( P_{dc} \) and \( P_{lc} \) as the overall consensus detection and false alarm probabilities derived via an arbitrary fusion rule from the local decisions at Alice and Bob. If Eve is modeled as being present in a particular transmission block with probability \( \beta \), the expected value of the MIMO secrecy rate for an arbitrary block is written as

\[ \bar{R}_e = R_b P_{dc} (1 - \beta) + R_b P_{lc} \beta + (R_b - R_e) (1 - P_{dc}) \beta + \bar{R}_e P_{lc} (1 - \beta) \],

(6)

where \( R_b \) is the ergodic MIMO secrecy rate, \( R_e \) is the information rate leaked to Eve upon missed detection, and \( \bar{R}_e \) is the sub-optimal rate to Bob when some resources are mistakenly allocated for secrecy encoding by Alice.

3. EAVESDROPPER DETECTION

The authors in [5] mainly focus on the use of a coherent matched filter detector [9] for determining the presence of the primary receiver. However, the matched filter approach requires phase synchronization at Bob as well as estimation of \( H_{ba} \), which is exceedingly difficult given the very low LO leakage power. Park et al. propose noncoherent envelope detection in the frequency domain by applying a discrete Fourier transform (DFT) to the down-converted and sampled received signal [6,7]. In this work we focus on detection in the time domain, and assume that Eve’s LO frequency \( \omega \) (or a good estimate of it) is known a priori to both legitimate terminals to enable downconversion to baseband. If Eve employs a direct-conversion receiver then evidently her LO frequency is known exactly. The same is true if all terminals have an identical heterodyne architecture.

3.1. Preliminaries

After downconverting and sampling, the hypothesis test at Bob based on \( M \) discrete-time vector observations (with a slight abuse of notation) is

\[ H_0 : \quad y_b [n] = m_A [n] + n_b [n] \], \quad n = 0, \ldots, M - 1

\[ H_1 : \quad y_b [n] = m_E [n] + m_A [n] + n_b [n] \], \quad n = 0, \ldots, M - 1

where

\[ m_A [n] = \sqrt{d_{ab}^{\alpha}} H_{ba} w_d [n] \]

\[ m_E [n] = \sqrt{d_{be}^{\alpha}} H_{be} s_d [n] \]

\[ w_d [n] = [ B_1 e^{j \omega D n + \xi_1} \ldots B_{N_b} e^{j \omega D n + \xi_{N_b}} ]^T \]

\[ s_d [n] = [ A_1 e^{j \omega D n + \theta_1} \ldots A_{N_b} e^{j \omega D n + \theta_{N_b}} ]^T \].

The deterministic MIMO channels \( H_{ba} \) and \( H_{be} \) are assumed to be constant over the detection process. In the sequel, we assume the absence of external interference at the receiver(s), such that the background noise is spatially uncorrelated: \( n_b [n] \sim \mathcal{CN} (0, \sigma_n^2 I) \). It is assumed that Bob’s own leakage signal is removed via filtering and does not contaminate the detection process [7]. The received signal has the following multivariate normal distribution:

\[ y_b [n] \sim \mathcal{CN} (m_A [n], \sigma_A^2 I) \quad \text{under } H_0 \]

\[ y_b [n] \sim \mathcal{CN} (m_E [n] + m_A [n], \sigma_A^2 I) \quad \text{under } H_1 \]  

(7)

For convenience we aggregate the samples into a \((N_b \times M)\) observation matrix

\[ Y_b = [ y_b [0] \ldots y_b [M - 1] ] \]

(8)

which follows a matrix-variate normal distribution [10] under both hypotheses:

\[ Y_b \sim \mathcal{CN} (M_A, \sigma_A^2 I) \quad \text{under } H_0 \]

\[ Y_b \sim \mathcal{CN} (M_E + M_A, \sigma_A^2 I) \quad \text{under } H_1 \]  

(9)

where we define

\[ M_A = [ m_A [0] \ldots m_A [M - 1] ] \]

\[ M_E = [ m_E [0] \ldots m_E [M - 1] ] \].

(10)
4. NONCOHERENT DETECTION

Energy detection (ED) is a low-complexity noncoherent technique that obviates the need to estimate the leakage signal parameters and channels, and only requires an accurate estimate of the background noise variance \( \sigma_b^2 \). The ED test statistic is given by:

\[
T_{ED} (Y_b) = \text{Tr} \left( Y_b^H Y_b \right) = \sum_{n=0}^{M-1} ||y_b[n]||^2. \tag{11}
\]

The ED hypothesis test compares the test statistic to a threshold \( \eta \) to determine the presence of Eve:

\[
T_{ED} (Y_b) \begin{cases} \geq \eta & H_1 \text{ (Eve Present)} \\ < \eta & H_0 \text{ (Eve Absent)} \end{cases} \tag{12}
\]

where \( \eta \) is determined by a pre-specified probability of false alarm constraint \( P_{FA} \).

From (11), under both hypotheses \( T_{ED} (Y_b) \) has a noncentral chi-square distribution, since it is the sum of the squares of \( 2MN_b \) real and independent nonzero-mean Gaussian random variables:

\[
H_0 : T_{ED} (Y_b) \sim \frac{2}{\sigma_b^2} \chi^2_{2MN_b} (\lambda_0) \\
H_1 : T_{ED} (Y_b) \sim \frac{2}{\sigma_b^2} \chi^2_{2MN_b} (\lambda_1) \tag{13}
\]

with associated noncentrality parameters:

\[
\lambda_0 = \left( \frac{2}{\sigma_b^2} \right) \text{Tr} \left( \Re \left\{ M_E^H M_A \right\} \right) \\
\lambda_1 = \left( \frac{2}{\sigma_b^2} \right) \text{Tr} \left( \Re \left\{ (M_E + M_A)^T (M_E + M_A) \right\} \right),
\]

respectively. Under the null hypothesis, \( T_{ED} (Y_b) \) has the density function:

\[
f_T (t; H_0) = \frac{e^{-(t/2)\sigma_b^2}}{\sqrt{\lambda_0} \sqrt{\pi} \sigma_b^2} \frac{t^{(MN_b-1)/2}}{\Gamma(MN_b-1)} \tag{17}
\]

and the probability of false alarm is calculated as:

\[
P_{FA} = Q_{MN_b} \left( \sqrt{\lambda_0} \sqrt{\frac{2\eta}{\sigma_b^2}} \right), \tag{14}
\]

where \( Q_k (a, b) \) is the generalized Marcum Q-function [9, 11]. Similarly, the probability of detection is:

\[
P_D = Q_{MN_b} \left( \sqrt{\lambda_1} \sqrt{\frac{2\eta}{\sigma_b^2}} \right). \tag{15}
\]

The value of the threshold \( \eta \) that corresponds to a particular \( P_{FA} \) can be computed numerically, or from the approximate inversion of the Marcum Q-function [11].

\[\text{5. OPTIMAL DETECTOR}\]

In contrast to energy detection, in this section we consider the optimal Neyman-Pearson detector when all parameters of the leakage signals are assumed to be known to Bob. From (7)-(9), the likelihood function under the null hypothesis is:

\[
f (Y_b; H_0) = \prod_{n=0}^{M-1} f (y_b[n]; H_0) = \prod_{n=0}^{M-1} \frac{1}{\sigma_b^2} \exp \left[ -\frac{(y_b[n] - m_A[n])^H (y_b[n] - m_A[n])}{\sigma_b^2} \right]
\]

with the corresponding log-likelihood function:

\[
L_0 (Y_b) = -MN_b \ln (\pi \sigma_b^2) - \frac{1}{\sigma_b^2} \text{Tr} \left\{ (Y_b - M_A)^H (Y_b - M_A) \right\}, \tag{16}
\]

Define \( M_1 \triangleq M_E + M_A \). Under the alternative hypothesis \( H_1 \), a similar analysis yields:

\[
f (Y_b; H_1) = \frac{1}{(\sigma_b^2)^{MN_b}} \times \exp \left[ -\frac{1}{\sigma_b^2} \text{Tr} \left\{ (Y_b - M_1)^H (Y_b - M_1) \right\} \right],
\]

\[
L_1 (Y_b) = -MN_b \ln (\pi \sigma_b^2) - \frac{1}{\sigma_b^2} \text{Tr} \left\{ (Y_b - M_1)^H (Y_b - M_1) \right\}. \tag{17}
\]

The optimal Neyman-Pearson test compares the log-likelihood ratio to a threshold that corresponds to a particular \( P_{FA} \):

\[
T_{op} (Y_b) = \text{Tr} \left\{ \Re \left\{ (M_E^H Y_b) \right\} \right\} \overset{H_1}{\geq} \varepsilon, \tag{19}
\]

where \( \varepsilon = 0.5\sigma_b^2 \varepsilon' + 0.5 \text{Tr} \left\{ M_E^H (M_E + M_A) + M_E^H M_E \right\} \). Therefore, the optimal detection rule is observed to be a replica-correlator or equivalently a matched filter, which is the expected outcome for detecting a known complex deterministic signal in Gaussian noise [9].

Next, we note that the test statistic is distributed as:

\[
H_0 : T_{op} (Y_b) \sim \mathcal{N} \left( \text{Tr} \left\{ (M_E^H M_A) \right\}; \frac{\sigma_b^2}{2} \text{Tr} \left\{ M_E^H M_E \right\} \right) \\
H_1 : T_{op} (Y_b) \sim \mathcal{N} \left( \text{Tr} \left\{ (M_E^H M_1) \right\}; \frac{\sigma_b^2}{2} \text{Tr} \left\{ M_E^H M_E \right\} \right),
\]

from which we can derive the probabilities of detection and false alarm.

A more realistic scenario in practice would be the case where some or all of the eavesdropper’s leakage signal parameters are unknown. This would necessitate the use of a generalized likelihood ratio detector [12] for example, the treatment of which is omitted due to limitations of space.
6. NUMERICAL RESULTS

For several network scenarios we present simulation results obtained by averaging over 1000 i.i.d. Rayleigh channel fading instances. In each instance the eavesdropper is present with probability $\beta = 0.5$, and we set the number of antennas as $N_a = N_b = N_e = 4$ with an Alice-Bob separation of $d_{ab} = 10$ m, and assume $d_{ae} = d_{be}$. The leakage amplitude is set to -50 dBm/antenna with an IF frequency of 200 kHz and unit noise power for all users, and the number of samples is fixed at $M = 10^5$.

[Graph of eavesdropper detection probability versus distance]

In Fig. 1, the detection probability of ED and MF at Bob is compared for a target $P_{fa} = 0.1$ versus the distance to the eavesdropper. While the MF detector expectedly outperforms ED when Eve is proximate, both detectors ultimately become useless as $d_{be}$ grows. Increasing the number of observation samples $M$ as a countermeasure detracts from the time available for data transmission, while increasing $N_a$ or $N_b$ will improve the interception capability of Eve. On the other hand the interception capability of Eve is degraded as $d_{be} = d_{ae}$ grows, which makes the interplay of these tradeoffs worthy of further study.

[Graph of ergodic secrecy rate versus transmit power $P_a$]

Fig. 2 depicts $R_e$ versus Alice’s total power constraint $P_a$ for ED and MF detectors, as well as a non-adaptive scheme which pessimistically assumes that Eve is always present. Eve is located 10 m away from both legitimate terminals. The local eavesdropper detection decisions at Alice and Bob are combined using an OR fusion rule. It is seen that the eavesdropper detection schemes outperform the non-adaptive strategy by reducing the unnecessary allocation of resources for secure transmission when the eavesdropper is absent.

7. CONCLUSIONS

In the MIMO wiretap channel, it is critical that the presence of a passive eavesdropper be determined so as to enable robust secrecy-encoding schemes as a countermeasure. In this work we adopt a method that allows the legitimate nodes to detect the eavesdropper from the local oscillator power that is inadvertently leaked from its RF front end. We analyze the performance of non-coherent energy detection as well as optimal coherent detection. We then show how the proposed detectors allow the legitimate nodes to increase the MIMO secrecy rate of the channel.

8. REFERENCES