MODEL-BASED CEPSTRAL ANALYSIS FOR ULTRASONIC NON-DESTRUCTIVE EVALUATION OF COMPOSITES

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ABSTRACT

The use of model-based cepstral features has been shown as an effective characterization of damaged materials tested with ultrasonic non-destructive evaluation (NDE) techniques. In this work, we focus our study on carbon-fiber reinforced polymer plates and show that the use of signal models with physical meaning can provide a cepstral representation with a high discriminative power. First, we introduce a complete digital signal model based on a physical analysis of wave propagation inside the plate. The resulting model has several drawbacks: a high number of parameters to estimate and the difficulty of expressing it as a classical rational transfer function, which does not allow a model parameter estimation through classical least-squares signal modeling techniques. In order to overcome these problems, we propose two simplifications of the physical model also based on a mechanical analysis of the system. We carry out a set of damage recognition experiments showing that cepstra extracted from these models are more discriminative than other previously used methods such as the LPC cepstrum (all-pole model) or a simple FFT cepstrum.

Index Terms— Signal modeling, feature extraction, non-destructive evaluation, ultrasonics, composites

1. INTRODUCTION

Carbon-fiber reinforced polymers (CFRP) are high performance advanced materials with a growing applicability due to their extreme strength-to-weight and rigidity-to-weight efficiency ratios. However, impact-type damages are hardly visible and can induce severe degradation of the material mechanical properties, while remaining invisible from the surface [1]. Thus, non-destructive evaluation (NDE) techniques are required to guarantee their reliability [2].

An important issue in these NDE systems is the analysis of the acquired signals, which carries out the extraction of relevant information from the tested specimen. Pagodinas [3] reviewed carefully ultrasonic signal processing methods for the detection of defects in composites, such as time-delay estimation methods, deconvolution techniques or the application of wavelet transforms. First works focused on filtering techniques for noise reduction [4, 5], and deconvolution with spectral extrapolation [6] or cepstrum [7]. In particular, the cepstrum has been previously used for ultrasonic signal characterization due to its deconvolutional properties [8]. Other related works tackle dimensionality reduction by means of linear discriminant analysis (LDA), or principal components analysis (PCA) [9, 10]. However, most of the aforementioned studies deal with the simple detection of damages. Thus, the final step of the system is limited to a binary classification between damaged/undamaged states. This requires a huge amount of experimental data and an expensive training process, without providing any quantification of the damage level and location.

This paper presents several digital signal models \(H(z)\) to characterize the specimen being tested. The main innovation consists in including information about flaws in the model parameters. In our previous work [11] we have already shown that all-pole models are suitable for this task. In particular, we showed that the cepstrum extracted from these models provides a suitable representation to discriminate different damage levels. In the present work, we will propose new models which can provide even better discrimination. This analysis leads us to two approaches with physical basis. An important point of these proposals is to provide models with a small number of parameters, whose goal is twofold: simple models are required for fast and practical NDE systems and, also, it helps to obtain smoothed spectra, which can increase the NDE accuracy for damage parameters reconstruction [11]. Also, we will impose that the proposed models must be written down as a classical rational (zero-pole) transfer function, so that a classical signal modeling method can be applied to estimate the model parameters.

The rest of the paper is organized as follows: Section 2 exposes the experimental setup and the proposed CFRP modeling. Section 3 outlines the main aspects of the developed model approximations. Section 4 presents relevant results that validate the proposed models, while section 5 discusses the feasibility of this modeling, concluding with ongoing work issues.

2. EXPERIMENTAL SETUP AND CFRP MODELING

2.1. Experimental setup

The specimen tested is a CFRP symmetric plate that consists of four layers. The damages were generated by applying different free-fall impact energies. The specimen was excited by low-frequency ultrasonic burst sine-waves at a central frequency of 5 MHz, a duration of one cycle (0.2 μs), and an amplitude that amounts to 5 Vpp. The response signal was measured at a point without damage for calibration, and the measurement procedure was repeated forty times on each undamaged and damaged locations, to generate a relevant data set. Each measurement corresponds to the average of 300 signal captures, providing an effective reduction of the noise of the detected response signals, increasing the SNR around 25 dB. The measurements have been discretized at a sample frequency \(f_s\).
of 100 MHz/12 bits, and the corresponding number of samples per capture is 1500 (15 μs). However, the signals have been decimated to \( f_s = 20 \) MHz (300 samples), in order to reduce part of the noise and focus on the frequency band of interest. Figure 1 illustrates the experimental setup used to register the ultrasonic signals [11].

### 2.2. Mechanical modeling

To characterize the system, it is necessary to understand how the emitted wave propagates inside the specimen. Two main effects must be taken into account, i.e. the impedance ratios and the attenuation. The first concept is introduced by dealing with the general behavior of two arbitrary successive layers, labeled as \( d_i \) and \( d_{i+1} \), as showed in Figure 2.

When an incident wave \( I_x \) propagates normally through an interface \( J_i \), it generates a transmitted component \( (T_x) \) that propagates into layer \( d_{i+1} \) and a reflected one \( (R_x) \) that propagates into layer \( d_i \). The resulting transmission and reflection coefficients are obtained by applying the energy conservation principle [12],

\[
T_x = \frac{2Z_i}{Z_i + Z_{i+1}}, \quad R_x = \frac{Z_{i+1} - Z_i}{Z_i + Z_{i+1}}
\]

It is noteworthy that the components \( T_x \) and \( R_x \) reach further interfaces and split again into reflection and transmission coefficients, leading to a complex propagation pattern. \( Z \) denotes the impedance of the layers being in contact. The latter is defined as,

\[
Z \ = \ \sqrt{\frac{\rho E(1-\nu)}{(1+\nu)(1-2\nu)}}
\]

where \( E, \nu, \rho \) are the mechanical properties associated to each layer, for instance the Young modulus, the Poisson ratio and the density, respectively. We deal with the second concept by considering a wave that propagates in the same homogeneous layer \( d_i \) from position \( J_{i-1} \) to position \( J_i \). Therefore, the wave amplitude decay is usually defined by the common formula,

\[
\left(\frac{A_i}{A_{i-1}}\right) = e^{-\alpha_i m_i}
\]

where \( A_i \) and \( A_{i-1} \) are the wave amplitudes at the respective positions \( J_i \) and \( J_{i-1} \). \( \alpha \) and \( m \) denote the loss factor and the thickness of the considered layer, respectively.

Both above-mentioned mechanical concepts can be interpreted in terms of signal modeling as gains and transfer functions. Figure 3 depicts the signal model corresponding to the considered specimen. For sake of simplicity, the transducer responses has been left out from the system. The values \( G_{IN} \) and \( G_{OUT} \) refer to the gains of the input and output responses, respectively, and correspond directly to the attenuation due to the transducers. The transfer function \( H_n(z) \) is the response that models the layer \( n \) for both the forward- and backward-propagation paths, assuming that the material behavior is linear. \( G_{Tij} \) and \( G_{Rij} (i,j = 1, ..., 4) \) are the transmission and reflection coefficients due to the interfaces between successive layers, respectively.

### 3. PROPOSED MODELS

The model proposed in the previous section is quite complex and may require a huge amount of parameters. More importantly, it can not be straightforwardly expressed by a rational transfer function, so that classical least-square signal modeling methods are not easy to apply. In this section we propose two simplified models inspired in the mechanical behavior of the plate, which achieve the required model simplicity.

#### 3.1. First approach

The first simplification consists in eliminating the reflection coefficients due to the layers interface. Since the mechanical properties of the layers in the undamaged state are quite close, the resulting transmission coefficients are much larger than the reflection ones \( (T_x \gg R_x) \), as summarized in Table 1.

<table>
<thead>
<tr>
<th>( J_1 )</th>
<th>( J_2 )</th>
<th>( J_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>( R_{12} = 0.0078 )</td>
<td>( R_{23} = 0 )</td>
</tr>
<tr>
<td>( T )</td>
<td>( T_{12} = 0.9922 )</td>
<td>( T_{23} = 1 )</td>
</tr>
</tbody>
</table>

**Table 1. Transmission and reflection coefficients.**

Under this assumption we obtain the model depicted in Figure 4, where each layer is modeled by a transfer function \( H_n(z) \) associated to a delay of \( z^{-m_n} \). Function \( H_{1w}(z) = \prod_n H_n(z) \) represents the one-way propagation path. Assuming that \( H_{1w}(z) \) is all-pole and neglecting a global delay factor \( z^{-m_w} \), the global (closed-loop) transfer function \( H(z) \) can be obtained as,

\[
H(z) = \frac{\sum_{k=0}^{2} b_k z^{-k}}{1 + \sum_{k=1}^{2M+2} a_k z^{-k}}
\]
where $M$ denotes the number of samples corresponding to the specimen width ($M = 12$), $p + s$ determines the number of coefficients used in the denominator, and $q$ corresponds to the number of parameters in the numerator. This transfer function corresponds to a zero-pole model. It is worth pointing out that the coefficients $a_k$ are not consecutive and represent both short-term and long-term correlations with orders $p$ and $s$, respectively.

### 3.2. Second approach

The first approximation has the disadvantage that it does not fit well parts of the signals that correspond to the inner reflections that suffer the wave within the specimen. Obviously, the strong assumption of neglecting this kind of reflections leads to a lack of accuracy.

Thus, a more accurate model requires taking into account these reflections somehow. For this reason, we propose a second approach which tries to account for the aforementioned reflections with a single virtual interface $J_v$. This idea is depicted at figure 5. Under this approach, the model behaves quite similarly as a bi-layered model. In a physically consistent model, wave component $R_{E1}$ should generate both reflected and transmitted components at the virtual interface. However, in our approach, the reflected one is neglected in order to obtain an even simpler model. This approximation is justified since the reflected part of wave component $R_{E1}$ is weak: (i) in the undamaged case, $T_{E1} \gg R_{I1}$, and thus the transmission produced by $R_{E1}$ at the virtual interface $J_v$ is much larger than the reflection. (ii) in a damaged case, $T_{E1} > R_{I1}$, and thus $R_{E1}$ is quite weak, and its corresponding reflection even weaker.

With the introduced approximation, the signal model of our second approach will be that depicted in figure 6, whose transfer function can be expressed as follows,

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 + \sum_{k=1}^{p} a_k z^{-k} + \sum_{k=2m}^{2m+r} a_k z^{-k} + \sum_{k=2M}^{2M+s} a_k z^{-k}}$$

where $m$ and $r$ denote the virtual interface position and the number of coefficients used to model this interface, respectively. This is again a zero-pole model whose denominator includes short-term, medium-term and long-term sums with orders $p$, $r$ and $s$, respectively. The two last sums, namely medium-term and long-term, account for the reflections at the virtual and transducer-specimen interfaces.

### 4. EXPERIMENTAL RESULTS

In order to evaluate the discriminative capability of the proposed models, a damage classification system based on cepstral distances has been developed, based on our previous work [11]. For an optimal use of the available data set, the training/test is performed using the leaving-one-out technique. Therefore, 39 signals are used to train a reference cepstral vector corresponding to a certain damage level, while the remaining signal is used for the test. Rotating the measurements enables us to train the system always with 39 signals, while testing it with $6 \times 40 = 240$ signals. In addition to the classification error ($\text{Err}[%]$), the efficiency of the system is evaluated by defining a weighted error factor. Let the results of the test be a confusion table $R(i,j)$, with $i, j = 1, \ldots, 6$, where $R(i,j)$ represents the number of measurements at damage level $i$ that have been classified as a damage level $j$. The weighted error factor is then defined as,

$$w_{err}[%] = 100 \times \frac{\sum_{i=1}^{6} \sum_{j=1}^{6} R(i,j) \cdot |i-j|}{240} .$$


Thus, when the erroneously recognized class corresponds to a damage close to that of the correct class, the error has less influence on the error rate.

Table 2 shows the results obtained for different cepstrum-based techniques. Each experiment has been conducted twice using rectangular and Hamming windowing of 300 samples in the time-domain. The tested techniques employ the real cepstrum $c(n)$, which is defined by means of the following expression,

$$\log |H(\omega)| = \sum_{n=-\infty}^{\infty} c(n) \cdot e^{j\omega n}$$

where $H(\omega)$ is the obtained spectrum estimate. In fact, the way of estimating the spectrum is the only difference between these techniques. Thus, Real cepstrum consists in using the periodogram obtained directly from the windowed signal, and corresponds to our baseline. The technique called LPC cepstrum is based on the use of a standard all-pole model with order $p = 25$, as described in our previous work [11]. Finally, the techniques named AP1 cepstrum and AP2 cepstrum are based on the first and second approaches proposed in the previous sections using 18 and 16 parameters, respectively. In these latter cases, $H(z)$ is estimated with the Prony’s method. Although the proposed models could be zero-pole, AP1 and AP2 also correspond to all-pole models since some preliminary experiments have shown that zeros, although could be helpful to obtain a minimum modeling error, do not provide useful information for discrimination. Finally, it is necessary to note that $M$ and $m$ were fixed to 12 and 9 samples, respectively, in order to model the total width of the plate and the position of the virtual interface.

Minimal weighted error (3.06 %) is obtained with the second approach for signals that have been previously preprocessed with a Hamming window. First, these results confirm that the use of a Hamming window improves the classification. More interesting is the fact that a modeling that includes mechanical information and a reduced number of parameters has a better discriminative capability than classical cepstrum-based approaches. Finally, it is noteworthy that adding a single coefficient in the middle-term sum of Equation 5 reduces the weighted error to its half part.

### 5. CONCLUSIONS

This study shows the capability of a mechanical-based signal modeling to discriminate the damage level of a CFRP plate subjected to different impact energies. The discriminative performance of the proposed parameterization has been evaluated by a system based on cepstral distances that recognizes the specific damage level corresponding to a given test signal, leading to the following conclusions: (1) It has been demonstrated that modeling the complex wave propagation pattern using a virtual interface provides better results than other classical cepstrum-based techniques. (2) It has been confirmed that the use of a temporal windowing improves the classification, and thus the importance of considering the echoes for discriminating between different damage levels. Ongoing works may include a further study and incorporation of the underlying mechanical concepts, in order to provide higher sensitivity on the damage parameters.

### 6. REFERENCES


