Loudspeakers in portable consumer electronic devices are frequently small in size. Due to the low sensitivity of their drive units, they are pushed to their power handling and mechanical limits by powerful amplifiers in an attempt to reach high volumes. To protect against excessive diaphragm excursions, a model based algorithm is proposed which regulates the voltage input signal to the loudspeaker while minimizing unnecessary system interventions.

Index Terms—loudspeakers, excursion protection
A continuous-time model for the electrical behavior of a loudspeaker is [2]:

$$v_c(t) = R_{eb}i_c(t) + \phi_0x_d(t), \quad (1)$$

where \(v_c(t)\) is the voltage input across the terminals of the voice coil, \(i_c(t)\) is the voice coil current, \(R_{eb}\) is the blocked electrical resistance, \(x_d(t)\) and \(\dot{x}_d(t)\) are the diaphragm excursion and velocity, \(\phi_0\) is the transduction coefficient at the equilibrium state \(x_d(0) = 0\).

The mechanical dynamics of a loudspeaker can be modeled as a single-degree-of-freedom mechanical oscillator:

$$m_d\ddot{x}_d(t) + c_d\dot{x}_d(t) + k_dx_d(t) = f_c(t), \quad (2)$$

where \(m_d\) is the mass of the diaphragm, \(c_d\) and \(k_d\) are the mechanical resistance and stiffness due to diaphragm suspension and \(f_c(t)\) is the EMF exerted on the voice coil. At the equilibrium state where \(x_d(t) = 0\),

$$f_c(t) = \phi_0i_c(t). \quad (3)$$

Combining the electrical and mechanical loudspeaker models, we can write the s-domain transfer function of excursion versus voltage input at the equilibrium state as

$$\frac{x_d(s)}{v_c(s)} = \frac{1}{m_ds^2 + (c_d + \phi_0^2/R_{eb})s + k_d}. \quad (4)$$

A z-domain transfer function can be obtained by applying either a bilinear transformation or the impulse invariance method to (4).

However, in order to yield a more precise model several additional nonlinear factors need to be taken into consideration [4]. Mechanical nonlinearities are caused by the fact that the transduction coefficient \(\phi\) and suspension stiffness \(k_d\) vary as quadratic functions of the excursion \(x_d(t)\). As such, a more precise expression for \(f_c(t)\) is given by

$$f_c(t) = \phi(x_d(t))i_c(t) - k_1(x_d(t))x_d(t), \quad (5)$$

where \(k_1(x_d[n])\) is the variation of the suspension stiffness as a function of excursion

$$k_1(x_d(t)) = k_d(x_d(t)) - k_d(0). \quad (6)$$

Likewise, electrical nonlinearities are caused by \(R_{eb}\) depending on temperature \(T\)

$$R_{eb}(T) = R_{eb}(T_0)(1 + \alpha(T - T_0)), \quad (7)$$

where \(\alpha\) is the temperature coefficient \((\alpha_{copper} = 0.004K^{-1})\) and \(T_0\) is the ambient temperature. Therefore, (1) should be rewritten as

$$v_c(t) = R_{eb}(t)i_c(t) + \phi(x_d(t))\dot{x}_d(t). \quad (8)$$

Equations (2,5,8) complete the continuous-time nonlinear loudspeaker model. Examples of the measurement and tracking of these parameters can be found in [5] and [6].

A discrete-time model for the electrical behavior of a loudspeaker is [2]:

$$v_c[n] = R_{eb}i_c(n) + \phi_0x_d[n], \quad (1)$$

where \(v_c[n]\) is the voltage input across the terminals of the voice coil, \(i_c[n]\) is the voice coil current, \(R_{eb}\) is the blocked electrical resistance, \(x_d[n]\) and \(\dot{x}_d[n]\) are the diaphragm excursion and velocity, \(\phi_0\) is the transduction coefficient at the equilibrium state \(x_d[0] = 0\).

The mechanical dynamics of a loudspeaker can be modeled as a single-degree-of-freedom mechanical oscillator:

$$m_d\ddot{x}_d[n] + c_d\dot{x}_d[n] + k_dx_d[n] = f_c[n], \quad (2)$$

where \(m_d\) is the mass of the diaphragm, \(c_d\) and \(k_d\) are the mechanical resistance and stiffness due to diaphragm suspension and \(f_c[n]\) is the EMF exerted on the voice coil. At the equilibrium state where \(x_d[n] = 0\),

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Combining the electrical and mechanical loudspeaker models, we can write the s-domain transfer function of excursion versus voltage input at the equilibrium state as

$$\frac{x_d[s]}{v_c[s]} = \frac{1}{m_ds^2 + (c_d + \phi_0^2/R_{eb})s + k_d}. \quad (4)$$

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$$v_c[n] = R_{eb}(n)i_c[n] + \phi(x_d[n])\dot{x}_d[n]. \quad (8)$$

Equations (2,5,8) complete the continuous-time nonlinear loudspeaker model. Examples of the measurement and tracking of these parameters can be found in [5] and [6].
Note that due to the point-wise nonlinearity of $k_d$ and $\phi$ with regard to $x_d(t)$, the excursion to voltage relationship can also be inverted. According to (11),

$$f_c[n] = \frac{1}{b_1} \left( x_d[n] + a_1 x_d[n] + a_2 x_d[n-1] \right)$$ \hspace{0.5cm} (14)

According to (8) and (5),

$$v_c[n] = R_{eb}[n] \left\{ \frac{1}{\phi(x_d[n])} \left( f_c[n] + k_1(x_d[n] x_d[n]) \right) \right\} + \phi(x_d[n]) \dot{x}_d[n]$$ \hspace{0.5cm} (15)

4. MODEL BASED EXCURSION PROTECTION

The easiest way to protect a loudspeaker from unsafe excursion is to lower the gain. A disadvantage of reducing the gain is the obvious reduction of perceived loudness, which is a key issue for small loudspeakers and drivers. To overcome this drawback, the characteristics of the excursion to voltage transfer function in (4) are taken advantage of.

Figure 3(a) shows an example of the frequency response of the transfer function in (4). The lowpass characteristic suggests that large excursions are caused by low frequency components in the input signal. Therefore, loudspeakers can be protected against excessive excursion by reducing the gain in the low frequency signal components, i.e., simply highpass filtering the input signal.

Figure 3(b) shows an example of the frequency response of an example highpass filter. The disadvantage associated with the highpass filtering technique is the loss of low frequency content, which is potentially more than necessary when there are only a few excessive excursions.

As shown in Figure 2, this paper proposes using the voltage to excursion model to predict excursions, and addresses excessive excursions directly using an excursion compression algorithm, then uses the excursion to voltage model to determine the required modification (if any) to the voltage signal. The benefit of this formulation is that a minimal magnitude and number of modifications are made to the voltage signal to address excessive excursion.

Figure 4 shows a gain mapping characteristic of the excursion compression module. Instead of hard clipping the excursion at the safe limit, the compression is only triggered when the input excursion exceeds a pre-defined threshold. Different compression ratios can be applied with the ratio $= \infty$ corresponding to hard clipping. Logical modifications such as soft knees can be introduced to allow for smoother compression and less perceived distortion.

Figure 5 shows an example of the distortion and excursion protection effects when playing a clip of music with both the traditional highpass filtering protection method and the proposed excursion protection method. The frequency response of the highpass filter is the same as the one shown in Figure 3(b). The sampling frequency is $F_s = 16$ kHz and the safe excursion limit is $x_{\text{max}} = \pm 0.5$ mm. The gain mapping curve applied in the proposed algorithm has a threshold of $0.9x_{\text{max}}$ and a compression ratio of 2.

As shown in Figure 5(b), without protection the excursions occasionally exceed the safe limits as indicated by the green horizontal lines. Highpass filtering the input voltage signal is able to reduce the excursions at the cost of constantly suppressing the bass. As a result, the input voltage signal to distortion ratio (SDR), where the distortion is defined as $\sum_n (v_c[n] - v_c^*[n])^2$, is degraded to -0.93 dB. With the proposed protection algorithm, interventions to the input voltages are minimized and justified by the underlying loudspeaker model. The distortions introduced by the proposed protection algorithm are minimal when comparing the spectrograms of the original and protected voltage input signal. Consequently, the input voltage SDR increases to 18.97 dB with the proposed protection algorithm.

Fig. 3. (a) A linearized micro-loudspeaker excursion to voltage frequency response with $m_d = 0.014g$, $c_d = 0.039$ Ns/m, $k_d = 284$ N/m, $\phi_0 = 0.3$ N/A and $R_{eb} = 7.5\Omega$. (b) The frequency response of a corresponding digital Butterworth highpass filter with $f_{\text{cutoff}} = 200$ Hz and $F_s = 16$ kHz.

Fig. 4. The input-output relationship of the excursion compression module.

5. CONCLUSIONS

This paper proposed a model based excursion protection algorithm to protect loudspeakers from excessive diaphragm excursion, one of the key causes of loudspeaker failure. A
Fig. 5. A comparison of distortion and excursion protection effects when playing a music clip. (a) The original input voltage spectrogram and (b) diaphragm excursion without protection; (c) a spectrogram of the highpass filtered input voltage and (d) diaphragm excursion with the highpass filtering protection algorithm; and (e) a spectrogram of the input voltage processed by the proposed algorithm and (f) diaphragm excursion with the proposed algorithm.

digital loudspeaker model was established to predict the diaphragm excursion based on the input voltage signal. The predicted excursion is controlled using dynamic range compression in the excursion domain. The corresponding input voltage to the loudspeaker is then determined by an inverse loudspeaker model. Compared with a conventional highpass filtering algorithm, the proposed algorithm provides a more accurate and timely control of the loudspeaker diaphragm excursion while minimizing unnecessary interventions to the system.

6. REFERENCES


[2] Andrew Bright, Active control of loudspeakers: An inves-


