Global Emergent Behaviors in Clouds of Agents

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Abstract—Networks of biological agents (for example, ants, bees, fish, birds) and complex man-made cyberphysical infrastructures (for example, the power grid, transportation networks) exhibit one thing in common – the emergence of collective global phenomena from apparently random local interactions. This paper proposes a distributed graphical model of interacting agents (a stochastic network type model) and studies its appropriate asymptotics. We show that metastability may occur – i.e., under certain conditions, the agents act in synchrony and may exhibit collectively possibly different stable equilibria – these are the global emergent behaviors of the cloud of interacting agents. We characterize these global behaviors as synchronous fixed points determined from ordinary differential equations that arise as mean field limits of the adopted stochastic model.

Index Terms—Emergent behavior, Multiagent modeling, Event-based phenomena, Metastability, Synchronization

I. INTRODUCTION
A. Background and Motivation
This paper proposes a stochastic network model to explain emergent behaviors that arise in large scale networks of loosely coupled agents. Each agent acts locally and senses locally but, as observed in many cyberphysical systems (e.g., the power grid, [1]), herds or colonies of biological agents (e.g., ants or bee colonies, [2], [3]) or in cyber networks (e.g., botnets, [4], [5]), their combined, distributed interactions lead to complex, coordinated behaviors. Ours is a stochastic network model that extends the work in [6]. We describe the network of agents by two layers: a sparsely connected network of (super) nodes, where each node is a dense interconnect of agents. Agents react to exogenous (external) factors and may influence or be influenced by internal interactions from other agents. We adopt an event-based approach where the external factors or the internal interactions correspond to events of possibly \( K \) classes, \( K \) being a finite (but, possibly large) integer. These events enter the network of agents at a certain rate, the individual behavior of each agent is affected for a certain average time, or agents may impact other agents at a certain rate. The paper shows how the coordinated global behaviors can be explained as the equilibria of dynamical equations (difference or differential) that arise as limiting behaviors of suitable renormalizations of the stochastic network model as the colony size expands to infinity.

The organization of the rest of the paper is briefly described as follows. Section III introduces the network model. Section IV explains the event model and the desired renormalization to pass to the limit of large agent size. Section V addresses the asymptotic dynamics as the size of the network grows. Finally, Section VI concludes the paper.

Due to space limitations, the proofs of the results are omitted and will appear in a future publication.

II. COLONIES OF AGENTS
Insect colonies like ants or bees perform collective tasks and achieve collective decisions, from moving the colony, distributing specialized tasks for defending the colony, foraging and finding new sources of food. They perform these tasks without a clear hierarchical structure or a declared leader. Each individual in the society, ant or bee, has very narrow spatial sensing, inadequate cognitive ability, access to restricted information, and limited ability to motivate or compel other members of the colony to perform. Still, beyond the apparent random behavior of the individuals in the colony, coordinated complex behaviors arise that exhibit order, possibly inducing the colony to operate in synchrony. This synchrony is, however, very different from the synchrony of the heart rhythms, see Peskin’s model of the cardiac Pacemaker, [7], fireflies flashing in synchrony, [8], synchronization of pulse-coupled biological oscillators, [9]. In these applications, synchrony is usually described by coupled oscillators, for which there are several models, including Kuramoto’s, [10], [11], [12]. We are concerned with a different type of synchrony – a synchrony that becomes apparent when one abstracts from the behavior of the individuals and takes long term limits or averages. These synchronous behaviors that we will simply refer to as global or emergent behaviors are akin to ergodic limits or to functional limits à la strong or weak laws of large numbers (sample averages converging to ensemble averages with probability 1, or in probability.) To capture such behaviors, we consider stochastic network models [13], renormalize them, and then consider thermodynamic (mean field) limits. For certain classes of such models, we study the empirical distributions (or sample histograms) of the states of the networked system. Upon appropriate normalization of the system and under appropriate conditions, these limiting behaviors are described by the equilibria of ordinary differential (or difference) equations. We will introduce a new stochastic model based on queuing theory and then show how in the mean field limit synchronous behavior arises.

III. NETWORK MODEL
To model how patterns of behavior arise in insect societies like ants or bees, we rely on stochastic networks, [13],[6]; these models, as we discuss below, may exhibit metastability depending on the values of the network parameters. When ants forage for food sources they explore the space by wandering around apparently in random directions. Trails may develop, when ants find sources of food and run back to the nest, depositing pheromone. Their trails are then followed by other nestmates that reinforce the pheromone trail. Different behaviors may arise. With few scouts or smaller colonies, several trails will survive; with larger number of scouts and larger colonies, a single of just a few stronger trails emerge that correspond to richer food sources. In technical terms, we may say that these different behaviors correspond to distinct equilibria. Ants behavior may exhibit bifurcation when suddenly a trail is reinforced at the expense of all other trails. To understand how these different patterns of global behavior emerge, we model the foraging ants by stochastic networks. A simplistic version of the model is a two level (hierarchy) network, see Figure 1. At the higher level, we have \( M \) supernodes that are sparsely connected. The \( M \) supernodes model the paths from nest to food sources. Each supernode is an aggregate of a large number, say \( N \), of nodes. In contrast to the sparseness interconnects of the \( M \) supernodes, we assume that the \( N \) nodes in each supernode are fully connected – this may be simplistic and unrealistic, but as first order approximation leads to interesting observations. Also, this model can be relaxed, for example, assuming regular degree nodes. The \( N \) nodes are place holders for individual ants that may traverse the path corresponding to the supernode (food source.) We model ants engaging in a particular path (supernode) \( k \) as a point process (say, Poisson) with rate parameter \( \lambda_k \). Ants involved in supernode \( k \) may leave the supernode as another point process with mean \( \mu_k \). This model describes ants that leave the trail, either because they lost the trail or they got tired and stay at the nest. Also, ants in...
a supernode $k$ may transfer to another supernode $\ell$ (i.e., engage in another trail to another food source) according to a point process with parameter $\gamma_{k\ell}$. The graphical model representing the ants foraging behavior is then a sparsely connected undirected simple supergraph where each supernode is itself an undirected complete simple graph. This model generalizes [6] where there are no supernodes.

![Event based network model: Supergraph of supernodes, each supernode being a dense interconnect of agents.](image)

The above model can be generalized to several types of behavior that can correspond to different tasks performed by ants. We consider briefly this more general model. As mentioned before, we adopt an event based approach. The events (e.g., ants’ tasks) may be of different classes $C_1, \ldots, C_K$, $K \geq 1$. Each class may reflect a different local behavior of an agent or may capture local interaction of the agents in different supernodes (inter) or agents inside a supernode (intra) or with the external world. An agent may be affected by an event for a random time. The inward arrows in Figure 1 directed towards the nodes represent random external perturbations triggered at random times. These local perturbations are strongly coupled at the intra-supernode level manifesting in rapidly changing short-term behavior. The interactions at the inter-supernode level are weaker and manifest at relatively larger time scales.

We emphasize that the research questions we pose are primarily concerned with the understanding of the effect of random local phenomena on qualitative global behavior. We develop an event-based analytical framework for modeling and studying these systems. An event-based approach, as opposed to a traditional model involving a large set of coupled (stochastic) evolution equations, has two key advantages: (1) in most cases a detailed modeling of the network dynamics at the individual node-level may not be feasible, due to the large size of the network and the uncertain information regarding the parameters at each node; and (2) more importantly, in the study of critical global system phenomenon, the object of interest is a particular event or a class of such events, occurring at a random time and not, in particular, the continuous-time behavior of a large set of coupled stochastic evolution equations.

Accordingly, we model the local events of interest as random point processes, i.e., instead of considering the detailed state-space evolution of a node, we set to characterize the random temporal occurrence of local events (may be local faults or deviations from some nominal behavior) which may trigger an alteration in the global network state. The state of the system is then modeled by a multidimensional Markov point process. The propagation of local events across the network is described by rate parameters of point processes. As will be pointed out later, this form of analysis not only provides insight into global aspects of the system, but also offers a nice framework to study the asymptotic behavior in the limit of increasing network size or large parameter deviations.

**Network parameters**
- **Event Classes-(N.1):** Events may be of different nature, namely, of classes $C_1, \ldots, C_K$.
- **External Influence-(N.2):** Events of a given class $k$ enter the network from the external world as Poisson point processes. In particular, we assume that the entry of class $k$ events at each node of supernode $i$ follows a Poisson process with rate $\lambda_{ik}$. Also, the entry processes are mutually independent across the agents and the classes.
- **Event Influence Time-(N.3):** A class $k$ event at an agent at supernode $i$ affects the agent for a random exponentially distributed time with mean $1/\mu_{ik}$ after which it leaves the network. The event may actually end up affecting the agent for a smaller time by being transferred to another agent due to agent interactions according to (N.4)-(N.5) as follows.
- **Intra-supernode transfers/interactions-(N.4):** A class $k$ event at an agent inside supernode $i$ is transferred to another agent in the same supernode with rate $\gamma_{ik}$, the destination node being chosen uniformly with probability $1/2$. This transfer of an event at an agent to another agent in the same supernode models the dense intra-supernode interaction.
- **Inter-supernode transfers/interactions-(N.5):** A class $k$ event at supernode $i$ is transferred to a node at supernode $j$ with rate $\gamma_{kj}$, the destination node at supernode $j$ being chosen uniformly with probability $\frac{1}{2}$. Since, the inter-supernode graph is sparsely connected, for most pairs $(i,j)$, the transfer rate $\gamma_{ik} = 0$, $\forall k$, modeling the sparse inter-supernode interactions.

Each node has the same event-handling capacity $C$, and in all these transitions, we assume that, if an event enters a saturated node, it is rejected from the network. Also, a class $k$ event, if accepted at node $i$, occupies $A_k$ units of capacity till it leaves the node. The interpretation of the node capacity $C$ is different for different applications.

**Remark 1** We point out the major differences of our model with the one adopted in [6]. The model in [6] consists of a single supernode $(M = 1)$ with a dense network of agents interacting according to (N.4). In this context, we note that (N.4), which allows each agent in a supernode to interact with the others in the same supernode, is necessary to obtain a meaningful mean-field (or thermodynamic) type scaling in the limit of large agent size. This is a common assumption in statistical mechanics based models, where the network is assumed to consist of homogeneous agents with each agent being able to interact with the $N - 1$ other agents, the interaction probability decreasing proportionately as the network size $N$ increases. While a single supernode with such a thermodynamic type agent interaction is applicable in some scenarios, it often fails to capture the sparse graph phenomena observed in some practical systems of interest. In our model, we capture the sparse coupling between agents located in (possibly) different geographical regions by incorporating multiple supernodes $(M > 1)$. The (super) graph induced by inter-supernode coupling could be sparse, (N.5), thus allowing us to capture the spatial decay of coupling strengths observed in large distributed socio-economic networks, whereas, the dense intra-supernode interactions model the commonly observed fast, random interactions occurring in a local cluster.

**Remark 2** We note that the model above with assumptions (N.1)-(N.5) may be generalized in different ways depending on the network phenomena to be modeled. For example, when modeling dynamics of viruses spread in a network of botnets or fault propagation in large power grids, it seems more appropriate to assume that an event when transferred from one agent to the other (N.4) or (N.5)) may leave its trace on the original node. In other words, an event transfer between agents may result in a copy of the event being propagated, the original event still residing in the originating agent. These modifications will, in general, lead to different qualitative equilibrium behavior of the model, although the current analysis methodology (Section IV) will mostly be applicable.

### IV. Empirical State Distribution

We start by introducing notation and stating a general assumption.

**A. Notation and Generic Assumption**

We denote by $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{R}^m$ the non-negative integers, reals, non-negative reals, and the $m$-dimensional Euclidean space respectively. The indicator function of a generic set $Y$ is denoted by $\mathbb{1}_Y$.

Upper case letters are used to denote stochastic processes. Because of an abundance of indexing, we simplify our notation. Stochastic processes are simply referred to as $Z(t)$, without explicitly mentioning the domain of the index, $t \in \mathbb{Z}_+$. Similarly, referring to random sequences of arbitrary quantities $(z^{(i)})$, where each $z^{(i)}$ may represent a stochastic process, we omit the reference index domain $N \subset \mathbb{N}$. “

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Any continuous time stochastic processes, \( Z(t) \), in this paper is assumed to have cadlag (right continuous with limits from the left) sample paths. The weak convergence (or convergence in distribution) of a sequence \( \{Z^n(t)\} \), \( N \in \mathbb{N} \), of stochastic processes to a limit stochastic process \( Z(t) \) as \( N \to \infty \) is denoted by \( Z^n(t) \Rightarrow Z(t) \). Finally, the topology for weak convergence of cadlag stochastic processes is assumed to be the usual Skorokhod topology (see [14]).

Local Agent State: Broadly speaking, in accordance to the event based dynamics, the local state of a network agent at a given instant of time is the number of (active) events of different classes being handled by the agent at that time. To formalize, following [6], define the set

\[
X = \left\{ n = (n_1, \cdots, n_K) : \sum_A k_n k_l \leq C \right\}
\]

(1)
of allowable node configurations to be a \( K \) tuple representing the number of each event being handled by an agent. The local state, \( X_{i,n}(t) \), of the \( l \)-th agent at superno\( de \) \( i \) is a stochastic process\(^5\) taking values in the set of allowable configurations \( X \), the dynamics of which are determined by the transition laws \([\text{N.2]}-\text{[N.5]}\). The collection \( \{X_{i,n}(t)\}_{t \leq i \leq M} \) and \( \{X_{i,n}(t)\}_{t \leq i \leq N} \) of the local agent states may be shown to be a jump Markov process, [15], taking values in the product set \( X^{MN} \). Clearly, the dimensionality of the collection of local agent states increases with \( N \), and, in fact, other than describing the detailed dynamics at the microscopic agent level, does not convey information on the global qualitative network state.

Global Network State: Following [6] (with obvious modifications due to the presence of multiple supernodes), for superno\( de \) \( i \), we define the empirical distribution process, \( Y_{i,n}(t) \in \mathbb{R}^N \), as

\[
Y_{i,n}(t) = \left( Y_{i,n}^1(t), \ldots, Y_{i,n}^N(t) \right)
\]

(2)
the proportion of nodes in superno\( de \) \( i \) with configuration \( n \) at time \( t \). Define the process \( Y \) as \( Y(t) = \left( Y_{1,n}(t), \ldots, Y_{M,n}(t) \right) \in \mathbb{R}^{MX} \) to be the collection of the superno\( de \) empirical distribution processes. In the following we use the empirical distribution process \( Y(t) \) to characterize the global (macroscopic) properties of the network. The interpretation of \( Y(t) \) as the global state process is due to the fact, that, it represents the macroscopic (averaged or normalized, (2)) behavior of the superno\( de \)s subjected to fast intra and slower inter transitions. The significance of the empirical process varies between applications, for example, in a power grid, it represents the proportion of nodes with a particular faulty configuration at any time instant. In such networks of large agents, the dynamics at the individual agent level is not so important, what we are interested is in estimating the global patterns that emerge due to the agent interactions, and the probability measure valued process \( Y(t) \) provides a macroscopic network characterization in terms of the properties of the distribution (or histogram) of local states accumulated over all the network agents. For example, the set of allowable agent configurations \( X \) may represent the different operating conditions of a network agent, and the empirical process \( Y(t) \) indicates the proportion of agents with a particular configuration at a time instant, thus focusing on the global network trend and not on the microscopic local dynamics. We note that the use of the global empirical distribution process for characterizing global network properties is common in socio-economic modeling, see, for example, [16] for a modeling of large financial applications. As (2) shows, the process \( Y(t) \) is a functional of the more detailed collection of local agent processes \( \{X_{i,n}(t)\}_i \), the latter being a jump Markov process. In fact, under the assumptions, especially the symmetricity of the intra-superno\( de \) transitions, it can be shown that,

\[\text{Proposition 3}\] Under assumptions \([\text{N.2]}-\text{[N.5]}\), the process \( Y(t) \) of superno\( de \) empirical distributions is a jump Markov process in itself taking values in a discrete subset (depending on \( N \)) of \( \mathbb{R}^{MX} \).

In the following we will address the limiting properties of the sequence of empirical distribution processes \( \{Y(t)\}_t \), as the number of agents \( N \) in each superno\( de \) goes to \( \infty \), while maintaining a constant number \( M \) of superno\( de \)s.

V. MAIN RESULTS

The first result concerns the asymptotics of the empirical distribution processes \( \{Y(t)\}_t \) as \( N \to \infty \). Throughout we assume \([\text{N.1]}-\text{[N.5]}\) to hold.

\[\text{Theorem 4}\] As \( N \to \infty \), the processes \( \{Y(t)\}_t \) converge weakly (in distribution) to a deterministic trajectory \( Y : \mathbb{R}_+ \mapsto \mathbb{R}^{MX} \). The limiting (mean-field) trajectory \( Y(t) \) is absolutely continuous and satisfies the ODE, \( Y_{i,n}(t) = f_{i,n}(Y(t)) \), \( i \leq M, n \in X \) where \( f_{i,n}(t) \) denotes derivative w.r.t. time and the vector fields \( f_{i,n} : \mathbb{R}^{MX} \mapsto \mathbb{R} \) are given by

\[
f_{i,n}(y) = \sum_{k=1}^{K} \bigg[ \frac{k}{n} \bigg] \sum_{m \in \mathbb{X}} (\gamma_{i,k} + \gamma_{i,k}) \sum_{m \in \mathbb{X}} m_j y_{j,m} + \sum_{j=1}^{K} \sum_{m \in \mathbb{X}} \gamma_{i,k} m_j y_{j,m} \bigg] \bigg[ (n_{i} + 1) y_{i,n} + (n_{i} + 1) y_{i,n} \bigg] \bigg[ (n_{i} + 1) y_{i,n} + (n_{i} + 1) y_{i,n} \bigg] - n_j y_{j,n}
\]

(3)

for \( y = (y_{i,n}, i \leq M, n \in X) \in \mathbb{R}^{MX} \). Here \( \epsilon_k \) denotes the \( k \)-th unit vector in \( \mathbb{R}^N \), with 1 at the \( k \)-th component and zero elsewhere. Also, for a configuration \( m \in X \), the notation \( m + \epsilon_k \) denotes the configuration obtained by incrementing \( n_k \) by one.

Remark 5 The proof, based on a martingale representation of the jump Markov processes \( Y(t) \) (15)) is omitted. The trajectory \( Y(t) \) essentially is a probability measure valued process, taking values in the \( MX \) dimensional probability simplex. The dynamical equations in (3) may be viewed intuitively as a coupled collection of the limiting differential equations obtained in [6], the coupling being attributed to the presence of multiple superno\( de \)s interacting with each other according to \([\text{N.5]}\). In the statistical mechanics literature, such a deterministic limiting trajectory, is often called the mean-field or fluid limit of the network processes \( \{Y(t)\}_t \) as \( N \to \infty \). The fluid limit provides qualitative characterization of the network model, for example, the stability, attractors of the asymptotic dynamical system. It also provides insight into the prelimit processes \( Y(t) \) (finite \( N \)). In a vague sense, the prelimit processes wander around the stable equilibria\(^3\) of the fluid limit for most of the time. The formal way of obtaining prelimit properties from that of the fluid limit is based on large deviation arguments ([17]), which is beyond the scope of the current paper. To summarize, the (stable) equilibria of the fluid ODE plays an important role in providing insights not only on the behavior of the asymptotic limiting system, but also of the prelimit stochastic processes. A particularly interesting phenomenon, called metastability ([18]), occurs when the fluid ODE exhibits multiple stable equilibria, and in that case, the prelimit processes spend large amounts of time around one of these equilibria until a rare event triggers an excursion to another equilibria through a saddle point. As shown in [6] for the one superno\( de \) model (\( M = 1 \)), the limiting dynamical system may exhibit multiple equilibrium points suggesting a possibility of metastable behavior. Due to the presence of multiple superno\( de \)s with coupling, it is not hard to come up with an example (depending on

\(^{3}\)An equilibrium or fixed point \( * = (y_{1,*}, \cdots, y_{N,*}) \in \mathbb{R}^{MX} \) of the asymptotic dynamical system corresponds to a zero of the vector field \( f(c) = [f_{i,n}(c)], i \leq M, n \in X \) given in (3), i.e., \( f_{i,n}(y^*) = 0 \) for all \( i,n \).
the system parameters) in our model, where the fluid ODE \( Y(t) \) has multiple equilibrium points. As an example, in the setting of a large power grid, these different equilibria may characterize states of normal and abnormal behavior of the system. The transition from one equilibrium to another (or from a normal operating state to an abnormal in the power example) is generally triggered by a rare event possibly leading to a catastrophic system behavior. The probabilities of these rare events governing the transition between different system equilibria (regimes) requires a large deviations analysis of our model and is an interesting research direction.

The following result shows the existence and provides a representation of the equilibrium points of the mean field ODE in (3). The result is a generalization of the corresponding in [6] to the case of multiple supernodes with coupling. To start with we set some notation: For a vector \( \rho = (\rho_1, \ldots, \rho_K) \in \mathbb{R}^K \), consider the function \( v_{\rho} : \mathcal{X} \mapsto \mathbb{R}^+ \),

\[
v_{\rho}(n) = \frac{1}{Z(\rho)} \prod_{i=1}^{K} (\rho_i)^{n_i} n_i!, \quad n \in \mathcal{X}
\]

where \( Z(\cdot) \) is the partition function given by

\[
Z(\rho) = \sum_{n \in \mathcal{X}} \prod_{i=1}^{K} (\rho_i)^{n_i} n_i!
\]

Thus \( v_{\rho} \) represents a family of probability measures on the configuration space \( \mathcal{X} \), being parameterized by the \( K \) dimensional vector \( \rho \). The following result is a generalization of its one-supernode counterpart in [6] and presents a parametric representation of the equilibrium points of the fluid dynamical system.

**Theorem 6** (1) Let \( \gamma^* \in \mathbb{R}^M_X \) be an equilibrium of the mean field dynamical system, (3). Then, there exists vectors \( \rho^1, \ldots, \rho^K \in \mathbb{R}^K \), such that, \( \gamma_i^* = v_{\rho^j}(n), \) for all \( i \leq M \) and \( n \in \mathcal{X} \) and the parameterizations \( \rho^j = (\rho_1^j, \ldots, \rho_K^j) \) satisfy the fixed point equations,

\[
\rho^j_k = \frac{\lambda_{i,k} + \gamma_i^{*} - I_{k}, v_{\rho^j} > + \sum_{j \neq i} \gamma_{i,j}^{*} < I_{k}, v_{\rho^j} >}{\gamma_{i,k} + \mu_{i,k} + \sum_{j \neq i} \gamma_{i,j}^{*}}
\]

(2) There always exists an equilibrium point of the mean-field ODE.
(3) If, in addition, \( K = 1 \), i.e., there is only one class of events, the equilibrium of the mean-field ODE is unique.

**Remark 7** The first assertion provides a characterization of the equilibrium points of the fluid ODE in terms of solutions of the fixed point equations (6), whereas the second one shows the existence of at least one equilibrium always. The equilibria may not be unique, leading to the possibility of complicated system phenomena like metastability as explained in Remark 5. However, if the number of classes \( K = 1 \), then assertions (2) and (3) conclude the existence of a unique equilibrium of the fluid ODE. In general, existence of a unique stable equilibrium would lead to a stable steady state behavior of the limiting system, rejecting the potential switching between multiple stable global configurations possible otherwise.

The last result provides conditions on the network parameters that lead to eventual (w.r.t. time) synchronous behavior of the supernodes. The synchrony between supernodes in terms of the global equilibria (regimes) requires a large deviations analysis of our model and is an interesting research direction.

**Theorem 8** Let the network parameters satisfy the following additional conditions:
(i) The rates of events of different classes are same for all the supernodes, i.e., \( \lambda_{i,k} = \lambda_k, \forall i, k. \)
(ii) \( \gamma_{i,k} + \sum_{j \neq k} \gamma_{i,j}^{*} = x_k \geq 0, \forall i, k. \)
(iii) \( \gamma_{i,k} + \mu_{i,k} + \sum_{j \neq k} \gamma_{i,j}^{*} \geq y_k, \forall i, k. \)

Then,
(1) There exists a synchronous fixed point. In other words, there exists \( \rho \in \mathbb{R}^K_+ \), such that \( (v_{\rho^1}, \ldots, v_{\rho^K}) \) is an equilibrium distribution for the asymptotic dynamical system.
(2) If, in addition, \( K = 1 \) (only one class of events), the above synchronous fixed point is the unique equilibrium of the mean-field limit \( Y(t) \).

**Remark 9** The above result may also be viewed from a system design perspective, where the goal is to design the inter-supernode coupling parameters \( \gamma_{i,j}^{*} \) under network constraints to lead to synchronous supernode operation.

**VI. CONCLUSION**

The paper introduces a queueing stochastic network model to describe the interactions of clouds of agents and studies its asymptotic behavior after suitable normalization. The asymptotic behavior of the network is described by an ordinary differential equation (ODE) that is obtained after renormalization and as the fluid limit of the network with respect to a growth parameter. Different global behaviors may emerge as different equilibria of this ODE. Further study is needed to determine the regions of the parameters spaces that lead to metastability.

**REFERENCES**


