BIO-INSPIRED SWARMING MODELS FOR DECENTRALIZED RADIO ACCESS INCORPORATING RANDOM LINKS AND QUANTIZED COMMUNICATIONS

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ABSTRACT

This paper proposes a distributed resource allocation strategy for cognitive radio networks based on a swarming model that incorporates random link failures and quantized communications. The swarming mechanism is used to minimize the interference produced by the cognitive users, take advantage of cooperative sensing, avoid collisions among the users and limit the spread of resources in the time-frequency domain. The mechanism assumes only local exchange of data among nearby nodes. The communications among the nodes are assumed to be affected by noise and random fading. Packet drops are taken into account as inducing a random topology, where a link is on or off depending on the decision errors. Using classical results from stochastic approximation theory, we prove that the swarm always converges in probability to a final allocation even in the presence of non ideal communications among the nodes. Numerical results show how the convergence rate of the algorithm is affected by the probability of link failures. The proposed procedure is applied to a bi-dimensional allocation in the time-frequency plane where the primary users’ activity is modeled as a set of continuous-time Markov processes.

Index Terms— Social Foraging Swarming, Distributed Resource Allocation, Cognitive Networks, Stochastic Approximation

1. INTRODUCTION

The need to improve the efficiency of conventional rigid spectrum access techniques has motivated an intensive research activity on networks able to sense the environment and adapt their access strategy dynamically, in order to make the best use of the available radio resources. Cognitive networks (CR) has become a general framework encompassing this kind of strategy, even though in some cases CR tends to be restricted to the idea of using or sharing spectrum holes. Femtocell networks represent a potential field of application of the cognitive paradigm. Femtocell networks are composed of femto-access points (FAP), small base stations to be installed at home or in companies, covering cells of radius in the order of ten meters. FAP’s are fully compatible with existing cellular standards and can exploit the available wired links to communicate with the macro base stations. The major advantages of FAP’s is the improvement of indoor coverage, by avoiding the wall penetration losses, and better spatial reuse of radio resources. The most critical aspect of femtocells is that a potentially massive deployment of FAP’s makes a centralized resource control harder to implement, thus making the whole system prone to larger interference values if proper counteractions are not properly taken into account. For this reason, interference management is one of the most critical issues in femtocell networks.

In this context, devising a decentralized radio access strategy, possibly requiring only rather simple and robust interaction mechanisms among the nodes has a clear importance. In this scenario, it can be of interest to translate some bio-inspired models, governing for example the swarm of populations of simple birds in search for common goods. Biological systems offer a variety of examples where each individual has relatively little intelligence and yet the collaborative behavior of the population leads to a global intelligence, which permits to solve complex tasks. Being able to translate these good features in an engineering project methodology can have a great impact on future networks. For example, in [1] the authors proposed a bio-inspired spectrum sharing algorithm based on the adaptive task allocation model in insect colonies. In [2], we proposed a decentralized radio access strategy mimicking a swarm foraging model, where the spatial distribution of food was intended to be the interference power distribution in the time-frequency domain. Our approach built on the social foraging model proposed in [3] and generalized it by allowing for local, rather than full, interaction among the individuals composing the swarm. This approach was considered assuming ideal communications among the network nodes and then considering a fixed topology. In [4], a swarm model affected by a switching topology has been thoroughly studied and it was proved convergence to a common velocity vector and stabilization of inter-agent distances, regardless of switching, as long as the network remains connected at all times. In a realistic communication scenario, the wireless channel introduces fading and noise, which induce errors in the received packets. To avoid the retransmission of erroneous packets, it is of interest to study what happens if erroneous packets are simply discarded. This situation can be studied by modeling the graph describing the interaction among the nodes as a random graph, where an edge (link) can be on or off depending if the packet is received correctly or not. This approach has been followed, for example, in [5], where it was studied a consensus protocol affected by both additive noise and random graph topology. The resulting algorithm made use of stochastic approximation tools to devise appropriate iterative algorithms and prove their convergence. In this work, we also use stochastic approximation theory to prove the convergence of the swarming algorithm, proposed in [2], in the presence of random link failures and quantization noise.

The contribution of this paper is twofold: (a) we extend the swarming algorithm of [2] to the case of inter-nodes communications affected by random link failures and quantization noise and (b) we apply the proposed procedure to the distributed resource allocation on the time-frequency plane, where the activity of the primary users is modeled as set of continuous-time Markov processes.

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2. SWARM MODEL

We consider a set of $M$ secondary users aimed at allocating power in an $n$-dimensional Euclidean space. A typical setting is the one where the resource space is the time-frequency domain (i.e., $n = 2$) and every secondary user is trying to access time and/or frequency slots where there is small interfering power. To keep the notation as general as possible, the single resource selected by agent $i$ is described by a vector $x_i \in \mathbb{R}^n$, denoting, for example a frequency subchannel and a time slot. The interaction between the cognitive nodes can be modeled as an undirected graph $G = (V, E)$, where $V \equiv \{1, 2, ..., M\}$ denotes the set of nodes and $E \subseteq V \times V$ is the edge set. Typically, there is a link (edge) between two nodes if the received power exceeds a minimum value and this depends on the channel properties. The graph describing the network topology can be described by the adjacency matrix $A := \{a_{ij}\}$, composed of nonnegative entries $a_{ij} \geq 0$, the degree diagonal matrix $D$, whose diagonal entries are $d_{ii} := \sum_{j=1}^{M} a_{ij}$, and the Laplacian $L$, defined as $L = D - A$. We denote by $N_i$ the set of neighbors of agent $i$, defined as $N_i = \{j \in V : a_{ij} \neq 0\}$.

The resource allocation problem was formulated in [2] as the minimization of the following potential function

$$J(x) = \sum_{i=1}^{M} \sigma(x_i) + \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} a_{ij} [J_a(||x_j - x_i||) - J_r(||x_j - x_i||)] ,$$

(1)

where $\sigma \in C^1: \mathbb{R}^n \to \mathbb{R}$ represents the interference power over the optimization domain (e.g., the time-frequency plane) and $x := (x_1, ..., x_M)^T$. The swarm model (1) is borrowed from the social foraging model proposed in [3], with the important distinction that the coefficients $a_{ij}$ in our model need not to be all different from zero, thus allowing for local, rather than global, connectivity. The minimization of the first term of (1) leads every node to find a position $x_i$, such that the overall interference power is minimal. The second term of (1) incorporates a short range repulsion term $J_r(||x_j - x_i||)$, whose effect is to avoid collisions among the cognitive nodes, and a long range attraction $J_a(||x_j - x_i||)$, whose goal is to induce a swarm cohesion, e.g., to avoid an excessive spread of the selected radio resources in the time-frequency domain. Hence, in our model, minimizing (1) leads each node to allocate its own resources in time-frequency regions where there is less interference, trying to avoid conflicts among users and limit the spread of the occupied domain. Furthermore, there is a unique distance at which the attraction and repulsion forces balance: the so-called equilibrium distance in the biological literature.

A possible way to achieve the distributed minimization of (1) is to use a gradient based optimization, so that every node starts with an initial guess, let us say $x_{0i}$, and then it updates its own resource allocation vector $x_i$, in time according to the following dynamical system:

$$\dot{x}_i = -\nabla_{x_i} J(x) = -\nabla_{x_i} \sigma(x_i) + \sum_{j=1}^{M} a_{ij} g(x_j - x_i),$$

(2)

$i = 1, ..., M$, with $x(0) = x_0$ and $g(\cdot)$ denoting a vector function defined as

$$g(y) = [g_a(||y||) - g_r(||y||)] y,$$

(3)

where $g_a(r)$ and $g_r(r)$ are the derivatives of $J_a(r)$ and $J_r(r)$ with respect to $r$, respectively. In this paper we consider attraction/repulsion functions having a linear attraction term $g_a(||y||) = c_A$ and bounded repulsion $g_r(||y||) = c_B \exp(-||y||^2)$. The equilibrium distance between attraction and repulsion forces is properly adjusted acting on the parameters $c_A$ and $c_B$. In our setting, this equilibrium distance is chosen proportional to the bandwidth of the frequency subchannel, in the frequency domain, or to the duration of the elementary time slot. The updating rule (2) is amenable for distributed implementation because each node needs to know only the local gradient of the interference profile and it has to interact only with a small subset of neighbors. In the ideal case, the iterative procedure (2) converges to a local minimum of the global potential function (1). However, in an imperfect communication scenario, where the links fail randomly and the data to be communicated have to be quantized first, the nodes have access only to a random subset of neighboring states and the data is corrupted by quantization noise. In these operative conditions, the convergence is not assured and the swarming algorithm needs to be adjusted to accommodate for such an imperfect communication scenario. In this paper, we make the following assumptions on the stochastic processes affecting the algorithm:

1) Random Link Failure: At any time $k$, the graph Laplacians can be written as

$$L[k] = \bar{L} + \tilde{L}[k]$$

(4)

where $\bar{L} = E[L[k]]$ and $\tilde{L}[k]$ is a sequence of zero mean, independent identically distributed (i.i.d.) Laplacian matrices. During the same iteration, the link failures may be spatially correlated but independent across iterations.

2) Quantization Noise: we assume that each inter-node communication channel uses a uniform quantizer, whose input-output relation may be modeled by the quantizing function, $q(\cdot): \mathbb{R} \to \mathbb{Q}$, i.e. $q(y) = b_{\Delta} \\
\left( b_{-1/2} \leq y \leq b_{1/2} \right)$, where $y$ is the channel input and $\Delta > 0$ is the quantization step. Now, adding to $y[k]$ a dither sequence $\{d[k]\}_{k \geq 0}$ of i.i.d. uniformly distributed random variables on $[-\Delta/2, \Delta/2]$ independent of the input sequence, the resultant error sequence $\{e[k]\}_{k \geq 0}$ can be written as

$$e[k] = q(y[k] + d[k]) - (y[k] + d[k]).$$

(5)

3. STOCHASTIC CONVERGENCE

In this section, we prove the condition for the convergence in probability of the proposed swarming mechanism, in the presence of random link failure and quantization noise. To this end, we recall here below a basic theorem of stochastic approximation theory [7].

Theorem 1: Let $\{x[k]\}_{k \geq 0}$ the Markov process defined by the difference equation

$$x[k+1] = x[k] + \alpha[k] [R(x[k]) + \Gamma(k, x[k], \omega)]$$

(6)

with initial condition $x[0] = x_0$, where $R(\cdot): \mathbb{R}^m \to \mathbb{R}^m$ is Borel-measurable, $\Gamma(k, x[k], \omega)$ is a family of zero-mean random vectors in $\mathbb{R}^m$, defined on some probability space $(\Omega, F, P)$, and $\omega \in \Omega$ is a canonical element of $\Omega$. Assume that there exists a nonnegative function $V(x) \in C_2$ with bounded second order partial derivatives and a constant $K > 0$ satisfying the conditions

$$\lim_{||x|| \to \infty} V(x) = \infty,$$

(7)

$$\sup_{x \in U \cup U(\epsilon)} \langle R(x), \nabla V(x) \rangle < 0 \ \ \text{for} \ \ \epsilon > 0,$$

(8)

$$||R(x)||^2 + E||\Gamma(k, x)||^2 \leq K(1 + V(x)),$$

(9)
where \((\cdot,\cdot)\) denotes the inner product operator and \(U_{\epsilon,1/\epsilon}(B) = \{ x \in \mathbb{R}^M : \epsilon < \| x - x_s \| < 1/\epsilon, x_s \in B, \epsilon > 0 \} \). Then, the process \(\{x[k]\}_{k \geq 0}\) converges almost surely (a.s.), as \(k \to \infty\), either to a point of the solution set \(B = \{ x : R(x) = 0 \}\), or to the boundary of one of its connected components, provided that

\[
\alpha[k] > 0, \quad \sum_{k=0}^{\infty} \alpha[k] = \infty, \quad \sum_{k=0}^{\infty} \alpha^2[k] < \infty. \quad (10)
\]

Proof: The proof can be derived directly from [7].

Let us consider now the discrete time version of the swarming algorithm in (2) that, in presence of random link failures and dithered quantization noise, can be written as

\[
x_i[k+1] = x_i[k] + \alpha[k] \left[ - \nabla_{x_i[k]} \sigma(x_i[k]) + \sum_{j=1}^{M} a_{ij}[k] g(x_j[k] - x_i[k] + d[k] + e[k]) \right]
\]

(11)

where \(\alpha[k]\) is a positive iteration dependent step size. In the presence of a small quantization noise, we can take a first order Taylor series’ expansion of the vector function \(g(\cdot)\), thus approximating the updating rule (11) as

\[
x_i[k+1] \simeq x_i[k] + \alpha[k] \left[ - \nabla_{x_i[k]} \sigma(x_i[k]) + \sum_{j=1}^{M} a_{ij}[k] g(x_j[k] - x_i[k]) + \sum_{j=1}^{M} a_{ij}[k] J g(x_j[k] - x_i[k])(d[k] + e[k]) \right]
\]

(12)

where \(J g(x_j[k] - x_i[k])\) is the Jacobian of \(g(\cdot)\) evaluated at \((x_j[k] - x_i[k])\). Now, exploiting the structure of the function \(g(\cdot)\) in (3), the dynamic of the overall system can be expressed in compact form as

\[
x[k+1] = x[k] + \alpha[k] \left[ - \Sigma^\nabla(x[k]) - (\mathbf{L}_x[k] \otimes I_n) x[k] + \mathbf{Y}_x[k] + \Psi_x[x[k]] \right]
\]

(13)

where \(\Sigma^\nabla(x[k]) = \text{col}([\nabla_{x_i[k]} \sigma(x_i[k])]_{i=1}^{M})\), \(\mathbf{Y}_x[k]\) and \(\Psi_x[x[k]]\) are the state-dependent aggregated contribution of quantization noise and \(\mathbf{L}_x[k] = \mathbf{D}_x[k] - \mathbf{A}_x[k]\), where \([A_{x_i[k]}]_{ij} = \{a_{ij}(c_A - c_R e^{-\|A_i[k] - x_i[k]\|^2})\}\) is a symmetric state dependent adjacency matrix. The Laplacian matrix \(\mathbf{L}_x[k]\) can be decomposed as the sum of a mean value plus a random part as in (4), allowing us to write

\[
x[k+1] = x[k] + \alpha[k] \left[ - \Sigma^\nabla(x[k]) - (\mathbf{L}_x[k] \otimes I_n) x[k] + \mathbf{Y}_x[k] + \Psi_x[x[k]] \right].
\]

(14)

In the notation of Theorem 1, (14) can be written as in (6), where

\[
R(x[k]) = -\Sigma^\nabla(x[k]) - (\mathbf{L}_x[k] \otimes I_n) x[k], \quad (15)
\]

\[
\Gamma(k, x[k], \omega) = - (\mathbf{L}_x[k] \otimes I_n) x[k] + \mathbf{Y}_x[k] + \Psi_x[x[k]]. \quad (16)
\]

The original swarming problem has been converted in the search of the zeros of a deterministic function \(R(x)\) whose value, measurable at each time instant, is corrupted by an additive random disturbance \(\Gamma(x, \omega)\). This analogy is useful to prove the following theorem.

**Theorem 2**: Let us consider the discrete swarming algorithm in (11) with arbitrary initial state \(x_0\). Under the hypothesis of a small additive quantization noise and of continuous differentiability of the interference profile \(\sigma(\cdot)\), and using a step size \(\alpha[k]\) as in (10), the algorithm converges a.s., as \(k \to \infty\), to one of the zeroes of the function \(R(x)\) in (15) or equivalently to a local minimum of the function \(J(x)\) in (1) evaluated for the mean graph. Then

\[
P \lim_{k \to \infty} \rho(x[k], B) = 0 = 1
\]

(17)

where \(\rho(\cdot)\) is the standard Euclidean metric norm and \(B\) is the solution set.

Proof: The detailed proof can be found in [6] and is based on Theorem 1. We omit it here due to lack of space.

**Numerical Example : Swarming in Frequency Domain in the Presence of Channel Imperfections**

As a first example, we consider an interference profile where the spectrum is completely filled, except for a single idle band in the middle of the spectrum. We assume the presence of 15 resources to be allocated from as many cognitive users. The resources are initially scattered randomly across the frequency spectrum. At the \(k\)-th iteration of the updating rule (11), each node communicates to its neighbors the position it intends to occupy, i.e. the scalar \(x_i[k]\) representing a frequency subchannel. Because of fading and additive noise, a communication link among two neighbors has a certain probability \(p\) to be established correctly. The values to be exchanged are also affected by quantization noise, supposed to be small with respect to the equilibrium distance between two agents. In Fig. 1, we report the average behavior of the evolution of the system potential function, e.g., (1), normalized with respect to the maximum and the minimum value, averaged over 500 independent realizations, vs. the iteration index. The ideal case corresponds to \(p = 1\) and it is shown as a benchmark. We can notice that, after a sufficient time, in all the cases the network reaches an equilibrium that coincides with a swarm cohesion in the low interference region of the spectrum. Interestingly, for interference profiles such that there is only one global minimum, the only effect of the random link failures is to slow down the convergence, but not the final value of the global potential function. This proves the robustness of the proposed algorithm.

The proposed procedure can be applied to allocate resources on the time-frequency plane where the primary user (PU) activity on each subchannel is modeled as a homogeneous continuous-time Markov process. Let us denote by

\[
Q_i = \begin{pmatrix} -\lambda_i & \mu_i \\ \lambda_i & -\mu_i \end{pmatrix}
\]

(18)

the transition rate matrix referring to the activity on the \(i\)th frequency subchannel. At each time \(t\), each subchannel can be either idle or active. Let us denote by \(P_{0i}(t)\) and \(P_{1i}(t)\) the probabilities that subchannel \(i\), at time \(t\), is idle or active, respectively. Let us also introduce the probability vector \(P_i(t) := [P_{0i}(t), P_{1i}(t)]^T\). Given the vector \(P_i(0)\) at time 0, the vector probability \(P_i(t)\) at time \(t\) is

\[
P_i(t) = e^{Q_i t} P_i(0).
\]

(19)
Let us suppose that, on each channel, it is known, through preliminary estimation, the value of the average power $p_i$ and of the transition rates from idle to idle $\lambda_i$ and from active to active $\mu_i$. Let us denote by $p_i(t; 0)$ the expected power on channel $i$, at time $t$, conditioned to the knowledge of the channel status at time 0. Suppose now that at time 0, the channel $i$ is sensed as idle. The probability vector that, at time $t > 0$, the channel will be either idle or busy is

$$
\mathcal{P}(t) = \left( 1 - \frac{\lambda_i}{\lambda_i + \mu_i} \left( 1 - e^{-(\lambda_i + \mu_i)t} \right) \right).
$$

(20)

Hence, the expected power, at time $t$ is

$$
p_i(t; \text{idle at } 0) = \frac{\lambda_i p_i}{\lambda_i + \mu_i} \left( 1 - e^{-(\lambda_i + \mu_i)t} \right).
$$

(21)

Similarly, if, at time 0, the channel is sensed as active, the expected power, at time $t$, is

$$
p_i(t; \text{active at } 0) = p_i - \frac{\mu_i p_i}{\lambda_i + \mu_i} \left( 1 - e^{-(\lambda_i + \mu_i)t} \right).
$$

(22)

**Numerical Example: Swarming in Time-Frequency Domain with Markovian Interference**

This example shows the evolution of the swarm in the time-frequency plane, in the case where the interference activity over each frequency subchannel is modeled as a continuous time Markov process. In this case, the radio resources are allocated over the bidimensional time-frequency plane and the vector $x_i$ contains the indices of the frequency subchannel and time slot that node $i$ intends to occupy. In Fig. 2, the grey background represents the interference level as a function of frequency and time (the darker areas are the ones where interference is smaller). The time variation of the interference depicts the expected interference power, as predicted by the Markov models (21) and (22), conditioned to what has been sensed at time 0. There are 15 cognitive nodes, whose initial guess, in terms of allocation in the time-frequency plane, is represented by the blue dots. The time evolution of the swarm is represented by the dotted (red) curves, evolving from the (blue) initial values up to the final (green) values. This example shows how the swarm moves towards a region where the expected interference is smaller (the prediction is better), it tends to minimize the spread in the time-frequency domain while avoiding collisions between the members of the swarm.

**Fig. 1.** Normalized system potential function vs. time index.

**Fig. 2.** Swarming in time-frequency domain

### 4. CONCLUSION

In this paper we have studied a decentralized swarming algorithm aimed to allocate resources in cognitive radio networks in case the communications among secondary users are affected by random link failures and quantization noise. In particular, we demonstrated that the swarm converges in probability to an equilibrium configuration dependent on the mean graph of the network. The resource allocation algorithm is then robust against channel imperfections whose effect is only to slow down the convergence process. In particular, reducing the probability to establish a communication link, the network requires more time to reach the final equilibrium state. Finally, the proposed approach has been applied to allocate resources in the time-frequency domain in the case where the primary activity over each channel is modeled as a continuous-time Markov process.

### 5. REFERENCES


