ABSTRACT

In this paper, we propose a novel algorithm to detect/compensate online interference effects when integrating Global Navigation Satellite System (GNSS) and Inertial Navigation System (INS). The GNSS/INS coupling is usually performed by an Extended Kalman Filter (EKF) which yields an accurate and robust localization. However, interference cause the GNSS measurement noise to increase unexpectedly, hence degrading the positioning accuracy. In this context, our contribution is twofold. We first study the impact of the GNSS noise inflation on the covariance of the EKF outputs so as to compute a least square estimate of the potential variance jumps. Then, this estimation is used in a Bayesian test which decides whether interference are corrupting the GNSS signal or not. It allows us to estimate their times of occurrence as well. In this way, the impaired measurements can be discarded while their impact on the navigation solution can be compensated. The results show the performance of the proposed approach on simulated data.

Index Terms—Covariance analysis, Fault diagnosis, Global positioning system, Inertial navigation.

1. INTRODUCTION

The Global Navigation Satellite System (GNSS) is a generic term which encompasses all the satellite constellations that can be used for localization such as the Global Positioning System (GPS), the European project Galileo or the Russian system Glonass. Nowadays, GNSS is widely used not only for mass-market but also for strategic applications. In this case, GNSS measurements are usually coupled with inertial sensors to fulfill stringent requirements in terms of positioning accuracy and reliability. The hybridization is classically carried out by an Extended Kalman Filter (EKF). However, GNSS relies on radio frequency (RF) signals, hence is vulnerable to interference. If not taken into account, they can severely impair the positioning accuracy and even result in a GNSS signal outage. Thus, to ensure the user safety, it is crucial to detect them and to identify the degraded satellite measurements. In this paper, we focus on interference which simply result in measurement noise variance jumps [1]. More precisely, we consider that the GNSS receiver does not lose track of the satellites. The developed approach aims at jointly detecting these variance jumps, estimating their amplitudes and their times of occurrence, and finally correcting the hybridization filter estimates accordingly.

Several techniques have been introduced in the literature to identify unknown or partially unknown noise covariance matrices in a Kalman filter (KF) setting. Most of them can be found in [2]. For time-invariant linear systems, the maximum likelihood [3] and the correlation techniques [4] have been proposed. Covariance matching is another solution to this problem [5]. Finally, Béänger [6] has extended the correlation technique to time-varying linear systems. However, the above-mentioned approaches rely on simplifying assumptions on the form of the covariance matrices. Moreover, they only apply when the unknown covariances are time-invariant. As far as variance jumps are concerned, detection algorithms using a priori information like the CUSUM [7] test can be used. Although easy to implement, this test requires to know approximately the jump amplitudes so as to tune some thresholds. As an alternative, particle filtering strategies have been investigated to jointly detect/estimate variance jumps in a time-varying system [1]. However, the main drawback of particle filtering remains the heavy computational cost. In parallel, Willsky [8] has developed the well-known Generalized Likelihood Ratio (GLR) algorithm to detect mean jumps affecting the state or the measurement vector of a KF. This approach has the advantage of estimating both the amplitude and the time of occurrence of the jumps, thereby allowing for their compensation. Finally, in [9], the GLR is adapted to deal with both mean and variance jumps. Nevertheless, so as to obtain closed-form estimates of the amplitudes of the variance jumps, the authors neglect their propagation in the KF equations. In this paper, following the work of Béanger [6], we propose to compute least square (LS) estimates of the potential measurement variance jumps which take into account the gradual error induced in the KF outputs. Then, these estimates serve as input for a sequential Bayesian test which decides whether interference are degrading the GNSS measurements or not. It also provides their most likely times of appearance. At last, we consider two strategies. The first one consists in compensating for the variance jump effects on the navigation solution. The second one proceeds to the exclusion of the degraded measurements and can be used when a large number of measurements is available.

The remainder of the paper is organized as follows. In Section 2, we briefly formulate the problem of navigation using GNSS and inertial sensors. The associated state space representation in the presence of interference is also given. Section 3 presents the proposed sequential Bayesian test for interference detection. Moreover, the analytical computation of the measurement noise variance jumps in a time-varying system is developed. Simulation results are provided in Section 4. Finally, conclusions and perspectives are drawn in Section 5.

2. THE INS/GNSS NAVIGATION PROBLEM

In this section, we first outline the state space representation when integrating INS with GNSS. In a second part, the EKF as hybridization filter is described.

2.1. The State Space Representation

Inertial navigation systems are composed of three accelerometers and three gyrometers, each of them along the three mobile axes. By integrating the acceleration measurements twice, the mobile position...
can be obtained. The gyrometer information is then used to express the data in an adequate system of coordinates. However, due to inertial sensor biases, the precision of the INS estimation decreases with time. To overcome this difficulty, GNSS measurements are usually used to estimate the INS errors.

We consider herein a strapdown INS in a tightly INS/GNSS hybridization context. The state model describes the INS error evolution over time. The corresponding state equation is:

\[ X_t = A_t X_{t-1} + B_t w_t, \]

where the state vector is \( X_t = [\delta v_t, \delta v_t, b_{x,t}, b_{y,t}, b_{z,t}]^T \) at time \( t \), with \( \delta v_t \) the INS velocity, attitude and position estimation errors, respectively. The vectors \( b_{x,t} \) and \( b_{y,t} \) denote the accelerometer biases and the gyrometer biases, respectively. The last components, \( b_{x,t} \) and \( b_{z,t} \), are a GNSS clock offset and its derivative which must be estimated jointly with the INS states as presented later. The vector \( w_t \) is the state noise vector, each component of which is assumed Gaussian distributed. For the sake of brevity, we do not present explicitly the state matrix \( A_t \) and the noise matrix \( B_t \). The reader can refer to [10] for more details.

The observation equation relates the GNSS measurements to the INS positioning error \( \delta p_t \). GNSS receivers determine their own positions by measuring the propagation delays of signals broadcast by in-view GNSS satellites. Distance measurements are then obtained at each time step by multiplying the propagation delays by the speed of light. At least four measurements, namely four in-view GNSS satellites, are required to solve the positioning problem because the receiver clock offset with respect to the GNSS reference time has to be estimated as well.

Let \( Z_t \) be the observation vector of size \( N \) at time \( t \), which is composed of the \( N \) measurements associated to the in-view GNSS satellites tracked by the receiver. The observation equation can be written as follows:

\[ Z_t = h_t(X_t) + \varepsilon_t, \]

where \( \varepsilon_t \) is the noise vector at time \( t \). The \( i \)th component of \( h_t \), for \( i \in [1, N] \), satisfies:

\[ h_{i,t}(X_t) = \|p_{i,0}^{\text{INS}} + \delta p_t - p_{i,t}^{\text{INS}}\| + b_{i,t}, \]

where \( p_{i,0}^{\text{INS}} = [x_{i,0}^{\text{INS}}, y_{i,0}^{\text{INS}}, z_{i,0}^{\text{INS}}]^T \) represents the position coordinate vector of the receiver computed by the INS in the system of coordinates chosen as a reference for the motion, \( p_{i,t}^{\text{INS}} = [x_{i,t}, y_{i,t}, z_{i,t}]^T \) is the position coordinate vector of the \( i \)th satellite and \( \|\cdot\| \) denotes the euclidian distance.

In the absence of interference, \( \varepsilon_t \) is Gaussian distributed centered of known covariance matrix, denoted \( R_t \). It can be approximately determined as a function of the class of the GNSS receiver and the constellation geometry with respect to the mobile such as the satellite elevation angles [11]. In the case of a measurement noise variance jump, the covariance matrix of the noise vector is equal to \( R_t + \Delta R \), where \( \Delta R \) is an unknown diagonal semi-definite positive matrix.

2.2. The Hybridization Filter

As the GNSS measurement equation (2) is not linear, the EKF is used to integrate INS and GNSS information. The EKF operates in two steps.

First, according to (1), the Kalman state vector prediction is:

\[ \hat{X}_{t-1}^+ = A_t \hat{X}_{t-1}, \]

where \( \hat{X}_{t-1}^+ \) is the state vector estimate at time \( t-1 \). Then, after a Taylor expansion around the prediction \( \hat{X}_{t-1}^+ \), the measurement equation can be expressed in the linear form:

\[ Y_t = H_t X_t + \varepsilon_t, \]

with \( Y_t = Z_t - h_t(\hat{X}_{t-1}^+) + H_t \hat{X}_{t-1}^+ = \text{H}\text{Jacobian matrix composed of the partial derivatives of } h_t \text{ around the prediction.} \]

The Kalman update equation is thus:

\[ \hat{X}_t^+ = \hat{X}_{t-1}^+ + K_t \hat{z}_t, \]

where, at time \( t \), \( K_t \) is the Kalman gain and \( \hat{z}_t = Y_t - H_t \hat{X}_{t-1}^+ \) is called the Kalman innovation vector.

3. THE PROPOSED ALGORITHM

In this section, we first present the sequential Bayesian test used to detect RF interference. Then, we study the propagation of the measurement noise variance jumps in the GNSS/INS time-varying system so as to derive a LS estimate of their amplitudes.

3.1. The sequential Bayesian test

We propose to use the same architecture as the GLR algorithm initially introduced by Willsky [8] to detect mean jumps:

- for each potential time of occurrence \( k \), LS estimation of the jump amplitude ;
- selection of the most likely time of occurrence \( \hat{k} \);
- statistical test to decide whether the jump has occurred or not.

We consider RF interference which result in an increase of the measurement noise variances. Let \( \{\alpha_i\}_{i \in [1,N]} \), with \( \alpha_i > 0 \), denote the variance jump affecting the \( i \)th GNSS measurement. The GNSS noise covariance matrix becomes:

\[ R_t[k] = R_t + \Delta R, \]

where \( R_t[k] \) is the GNSS noise covariance matrix assuming a variance jump at time \( k \). \( R_t \) is the nominal value of this covariance matrix. \( \Delta R = \text{diag}(\alpha) \) with \( \alpha = [\alpha_1, \ldots, \alpha_N]^T \) and \( \text{diag}(x) \) is a diagonal matrix the elements of which are the components of \( x \). In the sequel, \( \alpha \) is referred to as the jump amplitude vector.

At each time \( t \), we perform a multiple hypothesis test which compares pairwise the following hypotheses:

- \( H_{0,i} \): no variance jump up to time \( t \).
- \( \{H_t(k, \alpha)\}_{k \geq 0} \): variance jump of amplitude \( \alpha \) at time \( k \).

To avoid an exponential increase of the computational complexity with time, we limit the search to a finite-length window, i.e. we assume \( k \in [t-L+1, t] \) with \( L > 0 \).

In a Bayesian setting, the decision in favor of a variance jump is based on the following rule:

\[ \max_{k \in [t-L+1, t]} \frac{p[H_t(k, \alpha)|Y_{k-1}]}{p[H_{0,t}]} > 1. \]

However, the main difficulty is that the jump amplitude vector \( \alpha \) is unknown. Thus, it is replaced by its LS estimate denoted \( \hat{\alpha}[k] \) when assuming a time of occurrence \( k \). The computation of \( \hat{\alpha}[k] \) is detailed in the next section.
Finally, applying Bayes’rule, equation (8) becomes:

$$\max_{k \in [t-L+1,t]} T_t[k] > 2 \log \frac{p(\mathcal{H}_t)}{p(\mathcal{H}_t(k, \widehat{\alpha}[k]))},$$

(9)

where $p(\mathcal{H}_t)$ and $p(\mathcal{H}_t(k, \alpha))$ are the a priori probabilities of the hypotheses $\mathcal{H}_t$ and $\mathcal{H}_t(k, \alpha)$, respectively. The log-likelihood ratio $T_t[k]$ is expressed as follows:

$$T_t[k] = 2 \log \frac{p(Y_{t:k} \mid \mathcal{H}_t(k, \widehat{\alpha}[k]))}{p(Y_{t:k} \mid \mathcal{H}_t)}$$

$$= \sum_{m=k}^t \log |S_m| - \log |S_m[k]| + \sum_{i=1}^{|S_m[k]|} - \bar{z}_m[k] S_m^{-1} \bar{z}_m[k],$$

(11)

where $Y_{t:k} = [Y_{t}, \ldots, Y_{t-k}]^T$, $\bar{z}_m[k]$ and $S_m[k]$ are the innovation vector and its covariance matrix given a variance jump at time $k$. Note that the factor 2 is introduced for notational convenience.

If the test statistic exceeds the threshold, RF interference are detected. In this case, their estimated time of occurrence is:

$$\widehat{k} = \argmax_{k \in [t-L+1,t]} T_t[k].$$

(12)

Next section is dedicated to the expression of $S_t[k]$ and the computation of the LS estimate $\widehat{\alpha}[k]$.

### 3.2. Propagation of variance jumps in a time-varying system

In the presence of RF interference, the EKF tends to underestimate the covariance matrices of its outputs. Assuming a variance jump at time $k$, the actual innovation covariance matrix at time $t$ is:

$$S_t[k] = S_t + \Delta R + \sum_{p=1}^{t-k} \Phi(k + p, t) \Delta R \Phi(k + p, t)^T,$$

(13)

where $S_t$ is the theoretical covariance matrix computed by the EKF, $\Phi(n, m) = \prod_{j=n}^m \Psi_j$ and $\Psi_i = H_i A_i K_{i-1}$.

In [6], Belanger advises to write $\Delta R$ in the following manner:

$$\Delta R = \sum_{i=1}^N \alpha_i \Delta R_i,$$

(14)

with $\Delta R_i$ a matrix of size $N \times N$ all the elements of which are null except the $i^{th}$ diagonal element which is equal to one.

Then, equation (13) takes the form:

$$S_t[k] = S_t + \sum_{i=1}^N \alpha_i \Gamma_i(k, t-k),$$

(15)

with $\Gamma_i(k, t-k) = \Delta R_i + \sum_{p=1}^{t-k} \Phi(k + p, t) \Delta R_i \Phi(k + p, t)^T$.

(16)

We take advantage of this expression to obtain a LS estimation of the vector $\alpha$. The strategy is to replace the true covariance matrix $S_t[k]$ by its instantaneous estimation. It ensues:

$$\bar{z}_t^Tz_t^T - S_t = \sum_{i=1}^N \alpha_i \Gamma_i(k, t-k) + \eta_t,$$

(17)

where $\eta_t$ is an error matrix which is not central Wishart distributed.

To solve the estimation problem, we rewrite equation (17) as follows:

$$\Delta S_t = F(k, t-k) \alpha + \text{vec}(\eta_t),$$

(18)

with $\Delta S_t = \text{vec}(\bar{z}_t^T z_t^T - S_t)$.

$$F(k, t-k) = [\Omega_1(k, t-k), \ldots, \Omega_N(k, t-k)],$$

$$\Omega_i = \text{vec}(\Gamma_i(k, t-k)),$$

where vec($x$) is an operator which transforms the matrix $x$ in a column vector by removing the repeated entries.

By using a sliding window of length $L$, we can write:

$$\begin{bmatrix}
\Delta S_b \\
\Delta S_t
\end{bmatrix} = \begin{bmatrix}
F(k, k) & 0 \\
\vdots & \vdots & \alpha + \begin{bmatrix}
\text{vec}(\eta_k) \\
\vdots \\
\text{vec}(\eta_t)
\end{bmatrix}
\end{bmatrix},$$

(19)

for $k \in [t-L+1,t]$. As equation (19) is linear, we can perform a LS estimation of $\alpha$. Negative values of $\widehat{\alpha}[k]$ can occur. As these values do not make sense, we only keep the positive components of $\widehat{\alpha}$. It should be noted that, since the vectors vec($\eta_t$) are not Gaussian distributed, the estimates thus obtained are not optimal in the maximum likelihood sense. This point is commented in the sequel.

Once the detection is performed, we can use the variance jump and the time of occurrence estimations to compensate for the induced errors in the RF outputs. We then propose two strategies. The first one, referred to as strategy A, is to keep all the available GNSS measurements and to increase the measurement noise variances by a factor 2. The second one, called strategy B, consists in excluding the degraded measurements after the compensation step. For that purpose, each component $\{\widehat{\alpha}_i\}_{i \in [1, N]}$ is compared with an empirical threshold $T_e$.

### 4. SIMULATIONS AND RESULTS

Several simulations were conducted to illustrate the performance of the proposed algorithm. For that purpose, we generated a trajectory of 832 s according to the Radio Technical Commission for Aeronautics (RTCA) recommendations [11].

The noiseless GNSS measurements were computed using GPS and Galileo almanac files that provide the trajectories of the radio-navigation satellites. The time interval between two consecutive measurements is 1 s and the number of in-view satellites varies between 13 and 14 all along the trajectory. The measurement noise statistics were set in accordance with the RTCA recommendations [11]. It should be noted that we considered the GPS II and Galileo transmitting respectively on L1 and L5 frequency bands and E1 and E5 frequency bands. The main advantage is that bifrequency measurements will permit almost entirely correlated ionospheric noises to be removed. Hence, the proposed performance results in terms of accuracy are better than those currently achieved by standard GPS. As for the inertial measurements, the INS outputs were simulated by considering the following sensor specifications: initial accelerometer bias $b_{a0} = 5.10^{-4}$ m/s² and initial gyroeter bias $b_{g0} = 5.10^{-7}$ rad/s. They evolve according to random walks of

<table>
<thead>
<tr>
<th>$L$</th>
<th>Detection threshold</th>
<th>Exclusion threshold</th>
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<tbody>
<tr>
<td>A1</td>
<td>15 s</td>
<td>35</td>
</tr>
<tr>
<td>A2</td>
<td>15 s</td>
<td>$P_{th} = 10^{-3}$/h</td>
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Table 1. Parameter setting.
standard deviations (std) $\sigma_\alpha = 5.10^{-3}$ m/s$^2$ and $\sigma_\sigma = 5.10^{-7}$ rad/s, respectively. Finally, the variance jumps were simulated by introducing from time $t = 100$ s an additive white Gaussian noise of std equal to 10 m on the first and the second GNSS measurements.

All the results have been computed by averaging the results obtained for 50 Monte Carlo runs (MC) corresponding to different realizations of the measurement noise. We compare our algorithm with the one presented in [9]. They are denoted A and A$^2$. The tuning of the algorithm parameters is given in Table 1. For the developed approach, the detection threshold is chosen empirically. Nevertheless, the proposed value only needs to be readjusted when using different classes of inertial sensors or GNSS frequency bands.

Table 2 reports the mean detection delay (MDD), the mean square positioning error (MSE), the mean value and the std of the variance jump estimates, denoted $\{\tilde{\alpha}_{i}\}_{i\in[1,2]}$ and $\{\tilde{\sigma}_{i}\}_{i\in[1,2]}$ respectively. The false exclusion probability, denoted $P_\text{f}$ and the mean exclusion delay (MED), are also provided when strategy B is adopted. Note that no missed detection and missed exclusion have occurred over the conducted simulations.

First, in a general manner, the strategy B slightly outperforms the strategy A. Thus, we recommend to exclude the measurements corrupted by interference whenever several GNSS measurements are available. Then, we can notice that the proposed approach improves the MSE and MDD compared to the algorithm A$^2$ due to a better estimation of the variance jump amplitudes. Indeed, the estimates are not only on average closer to the actual values of the $\{\alpha_{i}\}_{i\in[1,2]}$ but also less scattered. This result stems from the algorithm A$^2$ neglecting the propagation of the variance jumps in the GNSS/INS time-varying system. Moreover, using the same exclusion threshold, no false exclusions have occurred when running our algorithm against 0.28/MC with the algorithm A$^2$. The performance in terms of positioning accuracy is further studied in Fig.1 where the square root of the MSE and the $3\sigma_\alpha$ bound are plotted, with $\sigma_\sigma$ the std of the positioning error. If both algorithms yields similar MSE, it appears that the algorithm A$^2$ tends to significantly overestimate the $3\sigma_\sigma$ bound contrary to our approach.

5. CONCLUSIONS AND PERSPECTIVES

In this paper, we propose a sequential Bayesian test for the detection/compensation of variance jumps due to RF interference in a GNSS/INS aided navigation system. The developed algorithm is based on an analytical expression of the variance jump propagation in a time-varying system. Simulation results shows the good performance of our algorithm in terms of detection delay and accuracy of the variance jump estimate. We are currently working on the use of M-estimators in place of LS to estimate the variance jump amplitudes.

6. REFERENCES