ABSTRACT

Two proportionate affine projection sign algorithms (APSA) are proposed for system identification applications, such as network echo cancellation (NEC), where the impulse response is often real-valued with sparse coefficients and long filter length. The proposed proportionate-type algorithms can achieve fast convergence and low steady-state misalignment by adopting a proportionate regularized matrix to the APSA. Benefiting from the characteristic of $\ell_1$-norm algorithms, affine projection, and proportionate matrix, the new algorithms are robust to impulsive interferences and colored input, and achieve much faster convergence rate in sparse impulse responses than the original APSA, the normalized sign algorithm (NSA), and the proportionate least mean square (PNLMS) algorithm. The computational complexity of the new algorithms is lower than the affine projection algorithm (APA) family due to elimination of matrix inversion.

Index Terms—Adaptive filter, network echo cancellation (NEC), proportionate adaptive algorithm, affine projection algorithm, sign algorithm.

1. INTRODUCTION

In network echo cancellation (NEC) applications, modern voice communication networks impose several challenges on conventional adaptive filters. Long network impulse responses (NIRs), highly colored non-Gaussian speech signals and undetected double-talk often cause extremely slow convergence or even divergence in adaptive filters.

As the required adaptive filter lengths grow, the conventional normalized least mean squares (NLMS) exhibits a slower convergence rate with longer adaptive filters. The proportionate NLMS (PNLMS) [1] algorithm has been designed to ameliorate this situation by exploiting the sparse nature of the NIR. Several proportionate algorithms [2, 3] were also developed for the affine projection algorithm (APA) which is well known for its better convergence for colored input than PNLMS. To mitigate undetected double-talk, robust algorithms [4, 5] and variable step-size algorithms [6, 7] were proposed from different angles. Yet another strategy is to use adaptive algorithms based on $\ell_1$ rather than $\ell_2$ error norms because $\ell_1$ algorithms are especially robust to impulsive noise like speech. A conventional $\ell_1$ algorithm is the normalized sign algorithm (NSA). Unfortunately, NSA’s robustness comes at the price of slower than NLMS convergence. Recently, a new affine projection sign algorithm (APSA) [8] addresses this problem and provides good robustness and fast convergence. Indeed it has been shown [8] that PSA achieves faster convergence and lower steady-state normalized misalignment than the NLMS, APA, and NSA under impulsive interference. This is achieved without the need for a matrix inversion as in APA.

In this paper we combine the proportionate approach with APSA to obtain even faster convergence when the echo path is sparse. The resulting algorithm is called proportionate APSA. Two types of proportionate matrix, one based on the PNLMS [1] and another based on the improved PNLMS (IPNLMS) [9], are applied to the APSA for real-coefficient systems, and the resulting real-coefficient proportionate APSA (RP-APSA) and real-coefficient improved proportionate APSA (RIP-APSA) achieve fast convergence in sparse NEC applications with robustness to colored input and undetected double-talk. The computational complexity of the two proportionate APSAs is slightly higher than the original APSA but is lower than the APA family.

2. PROPORTIONATE AFFINE PROJECTION SIGN ALGORITHMS

Consider the NEC scheme shown in Fig. 1, where $x(k)$ is the far-end signal, $z(k)$ and $v(k)$ are the near-end speech and background noise signal, respectively. The NIR of the true echo path is denoted by a length $L$ coefficients vector $\mathbf{h}$. The level of the sparseness in the NIR may vary according to the
changing network environment, which is measured by [10],
\[ \xi = \frac{L}{L - \sqrt{L}} \left( 1 - \frac{\|h\|_1}{\sqrt{L\|h\|_2}} \right) \]  
where 0 ≤ ξ ≤ 1. In the extreme case, if h is a Dirac delta function, then ξ = 1. On the other hand, if all the elements of h have equal value, then ξ = 0. In other words, a larger ξ corresponds to a sparser impulse response, while a smaller ξ corresponds to a more dispersive impulse response.

![Fig. 1. Structure of a network echo canceller (NEC).](image)

The estimated impulse response of the NEC is denoted by \( w(k) = [w_0(k), w_1(k), \ldots, w_{L-1}(k)]^T \), where k is the time index and the superscript T denotes transposition. The signal y(k) contains the near-end signal and the echo. That is, \( y(k) = x^T(k)h + z(k) + v(k) \), where \( x(k) = [x(k), x(k-1), \ldots, x(k-L+1)]^T \) is the far-end signal vector. Generating the replica echo \( \tilde{y}(k) = x^T(k)w(k) \), the NEC tries to minimize the difference between y(k) and \( \tilde{y}(k) \) with an adaptive \( w(k) \).

Different from the l_2-norm algorithms [1, 7], the original APSA algorithm [8] is obtained by minimizing the l_1-norm of the a posteriori error vector with a constraint on the filter coefficients,

\[
\min_{w(k+1)} \|y(k) - X^T(k)w(k+1)\|_1 \quad \text{subject to} \quad \|w(k+1) - w(k)\|_2^2 \leq \mu^2 \tag{2}
\]

where \( \mu^2 \) is a parameter to ensure the weight coefficient vector does not change too much in one iteration, \( X(k) = [x(k), x(k-1), \ldots, x(k-M+1)] \), \( y(k) = [y(k), y(k-1), \ldots, y(k-M+1)]^T \), and M is the projection order. Using the method of Lagrange multipliers, we get

\[
w(k+1) = w(k) + \frac{1}{2\lambda} X(k)\text{sgn}[e(k)] \tag{4}
\]

where \( \lambda \) is a Lagrange multiplier, the error vector \( e(k) = y(k) - X^T(k)w(k) \), and \( \text{sgn}[\cdot] \) is the signum function. For sparse h, we would like to adapt the coefficients of \( w(k) \) proportionately by pre-multiplying the update vector with a proportionate matrix. Then, (4) can be rewritten as

\[
w(k+1) = w(k) + \frac{1}{2\lambda} G(k)X(k)\text{sgn}[e(k)] \tag{5}
\]

where \( G(k) = \text{diag}\{g_0(k), \ldots, g_{L-1}(k)\} \) is a diagonal proportionate matrix whose element may be selected according to [1, 9]. Using (3) and (5), we obtain

\[
\frac{1}{2\lambda} = \frac{\mu}{\sqrt{x_{gs}(k)x_{gs}(k)}} \tag{6}
\]

where \( x_{gs}(k) = G(k)X(k)\text{sgn}[e(k)] \). Substituting (6) into (5) and adding a small positive regularization parameter \( \delta \), the weight updating equation for the proportionate APSAs is

\[
w(k+1) = w(k) + \frac{\mu x_{gs}(k)}{\delta + x_{gs}^T(k)x_{gs}(k)} \tag{7}
\]

where \( \mu \) is regarded as the step size. In this paper, we choose the proportionate matrix \( G(k) \) according to [1] and [9], respectively. For real-valued system, we call the resulting proportionate-type algorithms RP-APSA and RIP-APSA, respectively, which are summarized in Table 1.

### 3. Simulation Results

The proposed algorithms were evaluated via computer simulations. The echo path had 512 coefficients with some significant (active) coefficients and many near zero (inactive) coefficients. Throughout our simulations, algorithms were tested using an AR(1) with a pole at 0.8 as input while white Gaussian noise (WGN) was added to give a signal-to-noise ratio (SNR) of 30 dB. Near-end speech was also added to the system with a signal-to-interference ratio (SIR) of −10 dB as a strong impulsive interference modeled by a Bernoulli-Gaussian (BG) signal [8, 11]. The BG distribution was generated as the product of a Bernoulli process and a Gaussian process, i.e., \( z(k) = \omega(k)n(k) \), where \( n(k) \) was WGN with zero mean and variance \( \sigma_n^2 \), and \( \omega(k) \) was a Bernoulli process with the probability mass function given as \( P(\omega) = 1 - Pr \) for \( \omega = 0 \), and \( P(\omega) = Pr \) for \( \omega = 1 \). The average power of the BG process was \( Pr \cdot \sigma_n^2 \). Keeping the average power constant, the BG process was spikier when \( Pr \) was smaller. It reduced to a Gaussian process when \( Pr = 1 \). We chose \( Pr = 0.001 \) for our simulations. The performance of the algorithms was mainly measured by the normalized misalignment, which is defined by \( \eta(k) = 10\log_{10} \left[ \|h - w(k)\|_2^2 / \|h\|_2^2 \right] \). In addition, the regularization parameter \( \delta = 0.01 \) for all the following algorithms except the improved proportionate-type algorithms [9], where \( \delta_{1P} = \delta(1 - \alpha) / 2L \). The simulation results shown were obtained by ensemble averaging 10 independent trials.

#### 3.1. Performance Comparison Between the Proportionate APSAs and Other Algorithms

The normalized misalignment of the l_1-norm algorithms were compared with that of the l_2-norm ones. The NIR of the echo
path was sparse with $\xi = 0.758$. The step-size $\mu$ for the $l_2$-norm algorithms was 0.1 and was adjusted to achieve the same steady-state normalized misalignment for the $l_1$-norm algorithms. The proportionate parameters $\rho$ and $\alpha$ were selected as in reference papers [1, 9], as shown in Fig. 2 and Fig. 3. Benefiting from the robustness of $l_1$-norm minimization, the APSA family and the NSA converged at different speed, while the NLMS family and the APA family diverged under the strong impulse interference.

Moreover, the steady-state normalized misalignment of the APSA family with the projection order $M = 1$ and $M = 2$ were adjusted to be $-15$ dB and $-20$ dB, respectively, as shown in Fig. 2 and Fig. 3. With no projection but sign arithmetic, the APSA performed almost the same as the NSA against impulsive interferences. Adding the affine projection arithmetic, the APSA outperformed the NSA, as shown in Fig. 3. Combining the decorrelation property of affine projection and exploiting the sparse nature of the NIR, the proportionate APSAs had a faster convergence than the APSA and the NSA with both values of $M$. Also, the RIP-APSA converged a little faster than the RP-APSA under such conditions. The advantage of the proportionate APSAs in sparse NIR is clearly demonstrated.

### 3.2. Effect of Sparseness of the Impulse Responses

Ten different NIRs with sparseness measure $0.556 \leq \xi \leq 0.938$ were used to study the effect of sparseness on the convergence of the proportionate APSAs. All step-sizes were adjusted so that the algorithms achieved almost the same steady-state normalized misalignment between $-27$ dB to $-25$ dB. The number of samples taken to achieve $-25$ dB normalized
misalignment were used as a measure for the convergence rate versus the sparseness measure, as shown in Fig. 4. The projection order $M = 2$ was employed for the APSA family. The RIP-APSA with both $\alpha = -0.5$ and $\alpha = 0$ outperformed the other two algorithms over the entire sparseness region, and the number of samples taken to converge decreased approximately linearly with the increase of sparseness. In comparison, APSA performed worst except in dispersive NIRs with $\xi < 0.64$, where RP-APSA with $\rho = 0.01$ could not work. With $\rho = 0.1$, RP-APSA behaved similarly as RIP-APSA in the entire $\xi$ range. For sparse NIRs with $\xi > 0.7$, both RP-APSA and RIP-APSA had similar high convergence rates regardless of the selection of the proportionate matrix parameters $\rho$ and $\alpha$.

![Fig. 4](image)

**Fig. 4.** Number of samples to reach the $-25$ dB normalized misalignment against different sparseness measure of ten systems for the APSA, RP-APSA and RIP-APSA. The input and the interference are the same as those in Fig. 2. $M = 2$, $\rho = 0.01$ or 0.1 for the RP-APSA and $\alpha = -0.5$ or $\alpha = 0$ for the RIP-APSA.

### 4. CONCLUSION

Two proportionate affine projection sign algorithms (APSA) have been proposed for the identification of real-coefficient, sparse systems. With a modest increase in computational complexity over the original APSA, the proportionate APSAs can achieve faster convergence rate and lower steady-state misalignment under sparse network echo path, colored input, and impulsive interference environment. The computational complexity of the two proportionate APSAs is lower than the APA family due to elimination of matrix inversion.

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### 6. REFERENCES


