TARGET TRACKING AND LOCALIZATION WITH AMBIGUOUS PHASE MEASUREMENTS OF SENSOR NETWORKS

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ABSTRACT

When tracking a target using phase-only signal returns, range ambiguities are a major issue. In this work, a look-up table between phase measurement space and target location space is constructed for phase measurement mapping. Solving such problems via solutions to Diophantine equations has been used to locate candidate locations. Here we show how such target location ambiguity can also be resolved over time when the underlying target is in motion and where issues of clutter are treated via a phase distribution discrimination method. That is, a probability density function of the ambiguous phase-only measurement that takes both sensor noise and target motion distributions into account is derived based on directional statistics. This approach to solving phase ambiguity under significant clutter conditions has promising results.

Index Terms— Phase-only measurement, data association based filtering, target tracking, range ambiguity, distributed sensor localisation

1. INTRODUCTION

Many measurement methods in Physics and Engineering result in a set of ambiguous phase signals from which the unknown quantity (for example, a distance or angle) has to be calculated by a combination of phase measurements [1].

In sensor networks high frequency continuous wave returns from a target are often available from sensors at different locations. When they are used to measure the distances between the sensors and target, multiple values can be identified as a result of the received signal phase shifts modulo $2\pi$. The measurement model can be ascribed to a class of Diophantine Equations [2]. A Diophantine equation is an indeterminate polynomial equation that allows the variables to be integers only. Diophantine problems usually have fewer equations than unknown variables and involve finding integers that satisfy all equations [3, 4].

There are several Diophantine-based approaches to resolving phase ambiguity. For example, Wenzler [1] proposed a nonlinear processing approach based on the nonius principle in order to achieve an optimized determination of the unknown quantity from ambiguous measurements. Using this method we can find all possible solutions to the Diophantine equations. However, in the context of tracking or localising a target, solutions to the Diophantine equations can not remove the ambiguity of target locations.

Other approaches to solving ambiguous phase measurement is phase unwrapping such as [5] and measurement density modeling methods for identifying source trajectories as discussed in [6]. This latter technique has similarities to the proposed approach for moving targets.

The main focus of this paper is to deal with the ambiguities caused phase measurements and to elucidate how to identify and remove these ambiguities in tracking and localization. Here we combine the above approaches in terms of creating mappings between Target Location and Phase Measurement spaces so that the nonlinear and indeterminate Diophantine problem reduces to the acquisition of a collection of possible target locations. We then present two tracking/localisation methods which are able to resolve the ambiguity of target location in the context of (1) tracking a target with significant motion; (2) localising a target of random vibration or micro-motion.

2. PROBLEM

Consider the problem of tracking a target using phase-only measurements as shown in Fig. 1. Denoted by $\theta_k$, the target state at time $k$ will typically consist of position and velocity components. The target dynamics are assumed to follow a
Markov process subject to a random fluctuation $v_k$, i.e.

$$\theta_{k+1} = g(\theta_k) + \nu_k, \quad \nu_k \sim \mathcal{N}(0, Q_k)$$  \hspace{1cm} (1)

where $g$ is the system transition (dynamical) model and $\nu_k$ represents process noise, which can often be usefully approximated by a Gaussian distribution with zero mean and covariance $Q_k$. The measurement of the system at time $k$ is modeled as

$$\varphi_k = h(\theta_k) + w_k, \quad w_k \sim \mathcal{N}(0, C_k)$$  \hspace{1cm} (2)

where $\varphi_k = [\varphi_{1,k}, \varphi_{2,k}, \varphi_{3,k}]^T$ ($\varphi_i \in [0, 2\pi]$ $i = 1, 2, 3$), and

$$h(\theta_k) = 4\pi f_c \frac{\sin(\sqrt{(x_k - \eta_1)^2 + (y_k - \xi_1)^2})}{c} \Bmod 2\pi$$  \hspace{1cm} (3)

where $h(\theta_k)$ describes the relationship between the measurement $\varphi_k$ and the target state $\theta_k$, and $w_k = [w_{1,k}, w_{2,k}, w_{3,k}]^T$ is the measurement noise, which is a zero mean wrapped Gaussian distribution with covariance $C_k$. $[\eta_i, \xi_i]^T, i = 1, 2, 3$ denotes the location of three sensors and $\Bmod$ is the modulo operation. $f_c$ is the frequency of the continuous waves transmitted by sensors. $c$ is the speed of propagation. Without loss of generality, the frequencies of the three sensors are assumed to be the same.

One of the most critical problems in tracking and localization applications in such a measurement system is the ambiguity of the measurement due to the modulo of $2\pi$, which results in a collection of possible target locations.

3. LOOK-UP TABLE

Eq. (3) indicates that for a given target state, a unique vector valued phase measurement can be found while a single phase measurement may correspond to a finite set of target locations in the region of interest. In this work, a mapping between target location space $\theta \in \mathbb{R}^2$ and phase measurement space $\varphi \in \mathbb{R}^3$ for a given “resolution grid” in the region of interest is proposed in look-up table form.

In general, a vector valued phase measurement $\varphi_k$ corresponds to a set of possible target locations $\theta_i, i = 1, \cdots, n$ and they can be identified from the look-up table. For example, a target located at $A$ in Fig. 2 corresponds to a phase measurement $\varphi_A$. Multiple locations at which the target yields the same phase measurement are found by using the look-up table so providing an alternative solution to the phase Diophantine approach.

4. MOVING TARGET TRACKING

When the underlying target motion (with known dynamics) is significant large variations in “possible target location” measurements occur from the observed phase sequence over time. In this case the target trajectory can be estimated via target tracking in clutter techniques such as Probabilistic Data Association Filter [7]. An example of tracking a target of constant acceleration is given below. In this case, the location of the target $\theta_k = [x_k, y_k]^T$ at $k$ is described by

$$\begin{align*}
x_k &= x_{k-1} + v_x T + \frac{1}{2}(2k-1)\alpha_x T^2 + \nu_{x,k} \\
y_k &= y_{k-1} + v_y T + \frac{1}{2}(2k-1)\alpha_y T^2 + \nu_{y,k}
\end{align*}$$  \hspace{1cm} (4)

where the initial state of the target is $[x_0, y_0]^T = [-5, 6]^T$, the velocity and acceleration of the target are $[v_x, v_y]^T = \ldots.
\[ [4, 0]^T \text{ and } [a_x, a_y]^T = [0, 0.6]^T, \text{ respectively.} \]

In this example the standard deviation of the system process noise in (4) is \( \sigma_x = \sigma_y = 0.2 \) m and the standard deviations of measurement noise in (3) is \( \sigma_{\phi_1} = \sigma_{\phi_2} = \sigma_{\phi_3} = 0.02 \times 2\pi \) rad. A sequence of phase measurements in 50 scans is mapped into the target location space via the look-up table, as illustrated in Fig. 3 and the underlying target trajectory can be clearly identified. Therefore, once phase data is mapped into target location space in terms of virtual location data, conventional multi-target tracking techniques can be used to resolve the phase measurement ambiguity and estimate the target trajectories.

5. LOCALISATION OF FLUCTUATING TARGETS

When the underlying target is in micro-motion, the ambiguous target locations mapped from a sequence of phase measurements can no longer be resolved via a target tracking technique because the yield set of possible target locations essentially have no significant change over time.

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Fig. 3. Demonstration of resolving phase measurement ambiguity via filtering technique with data association in the target location domain, where in (b) the possible target locations due to the first five phase measurements are marked with symbols “+,” “x,” “□,” “∗,” “>” respectively.

Fig. 4. Demonstration of resolving target location ambiguity in phase measurement distribution discrimination.

However, in principle we can still resolve the phase measurement ambiguities if the target is in micro-motion. We demonstrate this using a target of motion type described by a Gaussian distribution. Fig. 4(b) shows three possible target locations which have the same phase measurement. When the target motion at these locations follows identical Gaussian distributions, as illustrated in Fig. 4(a), the phase measurement distributions are different and distinguishable.

Inspired by the above observation, in this work we derive the phase measurement noise distribution by taking target “noise” into account, which serves as “ground truth” to compare with the one summarised from the received phase measurement sequence. The target location ambiguity problem can then be resolved using one of the density based distance measures such as the Kullback Leibler divergence (KLD) by
identifying the location of minimum density distance to its “ground truth” from the location candidate collection.

Let \( \mathbf{n} = [n_x, n_y]^T \) be the target fluctuation noise such that the underlying target state at \( k \) is given by

\[
\Theta_k = \Theta + n_k, \quad n_k \sim \mathcal{N}(0, \sigma^2 I_2)
\]

where \( I_2 \) is 2 \( \times \) 2 identity matrix. The phase measurement model (2) is then written as

\[
\begin{bmatrix}
\varphi_1 \\
\varphi_2 \\
\varphi_3 \\
\end{bmatrix} = \kappa \begin{bmatrix}
\sqrt{(x_k - \eta_1 - n_x)^2 + (y_k - \xi_1 - n_y)^2} \\
\sqrt{(x_k - \eta_2 - n_x)^2 + (y_k - \xi_2 - n_y)^2} \\
\sqrt{(x_k - \eta_3 - n_x)^2 + (y_k - \xi_3 - n_y)^2}
\end{bmatrix} - 2\pi \begin{bmatrix}
m_1 \\
m_2 \\
m_3
\end{bmatrix} + \begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}
\]

where \( m_i, i = 1, 2, 3 \) take values of integer due to the modulo operator and \( \kappa = \frac{2\pi}{\lambda L} \).

Due to space limitation, we can only present the derived results without proofs.

1. The distribution of the phase measurement (6) can be approximated by a wrapped multivariate Gaussian distribution of the form

\[
\varphi \sim \mathcal{N}_w\left( \mu(\Theta), \Sigma(\Theta) \right)
\]

where

\[
\Sigma(\Theta) = \kappa^2 \sigma^2 \begin{bmatrix}
1 & \frac{\sigma_2^2}{\kappa^2\sigma^2} & \rho_{12}(\Theta) & \rho_{13}(\Theta) \\
\rho_{21}(\Theta) & 1 & \frac{\sigma_3^2}{\kappa^2\sigma^2} & \rho_{23}(\Theta) \\
\rho_{31}(\Theta) & \rho_{32}(\Theta) & 1 & \frac{\sigma_1^2}{\kappa^2\sigma^2}
\end{bmatrix}
\]

\( \rho_{ij}(\Theta), i, j = 1, 2, 3 \) are coefficients of the correlation between the three phase measurements and they are target state dependent.

2. The knowledge of target noise \( \sigma \) can be estimated from data, i.e.,

\[
\sigma^2 = \frac{1}{N \kappa^2} \left[ \frac{1}{3} \text{tr}^2(\Sigma) - \sigma_w^2 \right]
\]

where \( \Sigma \) is the sample covariance of phase measurement.

Therefore, the procedure of the proposed method for estimating the location of a target with fluctuated motion from a sequence of phase measurements can be established as follows.

**Step 1:** Estimate sample mean and covariance from the received measurement sequence.

**Step 2:** Estimate the target noise variance from the measurement sequence.

**Step 3:** Find the set of possible target locations from the look-up table.

**Step 4:** Calculate the theoretical phase measurement distributions for each of the set of possible target locations.

**Step 5:** Calculate KLD between each of the theoretical phase measurement distributions and the sample distribution and the correct target location is found by selecting the one with minimum KLD.

6. CONCLUSIONS

In this paper, estimating the trajectory/location of a target using phase-only measurements observed by three sensors in different locations is considered, where phase ambiguity caused by phase wrapping is a challenging issue. Methods for resolving the phase ambiguities are proposed. Firstly, the phase wrapped measurement is mapped into target location space via a look-up table. We then resolve the target location ambiguity in two cases. For targets in significant motion, the technique of stochastic filtering with data association is suggested. For micro-motion targets, a distribution discrimination method is proposed and the probability density function of the phase measurement is derived. The effectiveness of these methods are demonstrated.

7. REFERENCES


