ADAPTIVE COMPRESSIVE SENSING AND PROCESSING FOR RADAR TRACKING

Ioannis Kyriakides
Department of Electrical and Computer Engineering
University of Nicosia, Nicosia, Cyprus
kyriakides.i@unic.ac.cy

ABSTRACT

Compressive sensing and processing of radar waveforms enables high-resolution tracking while using low sampling rates and inexpensive processing. Compressive processing, however, introduces an additional estimation error, especially when the compressive sensing process is agnostic to the received waveform characteristics. In this work, an adaptive compressive sensing and processing scheme is applied to the radar tracking problem. The adaptive scheme naturally incorporates sequentially updated information on target state that is readily available from a particle filter based tracker. The proposed method is shown to improve tracking performance compared to a non-adaptive scheme, while maintaining a low-sampling rate and a computationally inexpensive operation.

Index Terms— Compressive sensing and processing, particle filtering, radar tracking.

1. INTRODUCTION

Radar tracking performance depends on the delay-Doppler resolution, the signal-to-noise ratio (SNR), and the sidelobe levels of the ambiguity function (AF) [1] that appear at the output of the matched filter (MF). High delay-Doppler resolution requires a radar waveform of long discrete-time length and high bandwidth. Moreover, when sampling the radar return waveform at the Nyquist rate, followed by matched filtering, high delay-Doppler resolution is achieved, SNR is maximized, and AF sidelobe levels only depend on the waveform used. Nyquist sensing and processing (NSP), however, requires both a very fast analog-to-digital (A/D) converter, and intensive processing of long sequences.

Compressive sensing and processing (CSP) [2, 3] can, instead, be used at a sub-Nyquist sampling rate and with low computational expense. Compressive sensing losslessly projects a high-dimensional waveform, that is sparse in some basis, onto an acquisition matrix, that is incoherent with the sparsifying basis, resulting to a low-dimensional compressively sensed (CS) sequence [4]. The use of a non-adaptive random acquisition matrix simplifies the sensing process while preserving information and high delay-Doppler resolution [2, 3]. On the other hand, random CSP increases the compressed AF sidelobes and significantly decreases the SNR at the output of the compressed MF [3].

In this work, an adaptive CSP method is proposed for radar tracking that embeds information, which is sequentially acquired by the tracker, into the compressive sampling process. Compressive sampling is performed in a single step after having considered all available information. This approach differs from existing adaptive compressive sensing methods, such as [5], where the sampling process is adapted during the acquisition process. The proposed adaptive scheme uses readily available tracking information and reduces the number of CS measurements required for a given tracking performance compared to the random CSP case. Therefore, it adds little complexity to the compressive sensing receiver compared to random CSP. Theoretical and numerical results demonstrate the improved SNR at the output of the compressed MF when using adaptive versus random CSP. Moreover, a single target tracking simulation based study demonstrates that, when using adaptive CSP, tracking performance is improved compared to when using random CSP.

2. NYQUIST SENSING AND PROCESSING

Based on sequentially acquired information, a particle filter based tracker identifies, at each discrete time step \( k \) of the tracking scenario and for each sensor \( u \), a finite set of delay-Doppler pairs that are likely to correspond to the true target state. This set is limited by the kinematic model and the posterior distribution on the target state at time step \( k - 1 \), as described in [6] in detail. The elements of this set are indexed as \( l_{u,k} = 0, \ldots, L_{u,k} - 1 \), where \( k = 1, \ldots, K \) and \( u = 1, \ldots, U \). \( L_{u,k} \) is the cardinality of the set which increases as the uncertainty on target state increases. Each index \( l_{u,k} \) is then associated with a delay-Doppler pair \( \{ \tau_{l_{u,k}}, \nu_{l_{u,k}} \} \).

From the set described above, a dictionary of \( L_{u,k} \) delay-Doppler shifted versions of the transmitted waveform \( s(m), m = 0, \ldots, M - 1 \) is built at every time step \( k \) and for each radar sensor \( u \). The dictionary is expressed as an \( M_d \times L_{u,k} \) matrix \( S_{u,k} \) with columns \( s_{u,k}(l,m) = s(m - \tau_{l_{u,k}} + \nu_{l_{u,k}}) e^{j2 \pi m \nu_{l_{u,k}} / M_d}, m = 0, \ldots, M_d - 1, l = 0, \ldots, L_{u,k} - 1 \). The length \( M_d > M \) is chosen to accommodate the maximum delay of a radar return waveform.
At time step \( k \) of a tracking scenario, \( U \) radar sensors receive delay-Doppler shifted versions of the transmitted waveform due to the target's range and range-rate. The discrete-time received sequence after sampling at the Nyquist rate is

\[
d_{u,k}(l, m) = A_k s_{u,k}(l, m) + v_{u,k}(m)
\]

with \( m = 0, \ldots, M_d - 1 \). \( A_k \) is assumed to be zero-mean complex Gaussian with variance \( 2\sigma^2 \) according to the Swerling I model [7]. \( v_{u,k}(m) \) is a zero-mean complex Gaussian noise with variance \( 2N_0 \).

The received waveform is filtered using a bank of \( L_{u,k} \) template waveforms at each sensor \( u \) contained in \( S_{u,k} \) described above. Matched filtering is then limited to the set in \( S_{u,k} \) as this set includes all delay-Doppler shifts expected in the return waveform according to the sequentially updated target state information. The MF operation is equivalent to projecting the received waveform \( d_{u,k}(l, m) \) onto \( S_{u,k}^* \) where \( \cdot^* \) denotes conjugate transpose. The MF statistic with template \( s_{u,k}(l, m) \) is then

\[
\tilde{y}_{l,u,k} = \sum_{m=0}^{M_d-1} d_{u,k}(l, m) s_{u,k}^*(\bar{l}, m)
\]

which using (1) yields

\[
\tilde{y}_{l,u,k} = A_k \xi_l A_{s,u,k}(l, \bar{l}) \sum_{m=0}^{M_d-1} v_{u,k}(m) s_{u,k}^*(\bar{l}, m)
\]

where, the AF of the transmitted waveform is given by

\[
A_{s,u,k}(l, \bar{l}) = \frac{1}{\xi_s} s_{u,k}(l, m) s_{u,k}^*(\bar{l}, m)
\]

which also represents projections of \( s_{u,k}(l, m) \) onto the dictionary \( S_{u,k}^* \) and \( \xi_s = \sum_{m=0}^{M_d-1} s(m)s^*(m) \) is the energy of \( s(m) \). In this work, the radar waveform \( s(m) \) is a multicarrier phase-coded (MCPC) [1] sequence consisting of Björck constant amplitude autocorrelation (CAZAC) sequences [6]. MCPC waveforms can be adaptively configured at each time step of the tracking scenario, based on the possible delay-Doppler shifts in \( S_{u,k} \), to have zero AF sidelobes [8]. Therefore, throughout this work, it is assumed that \( A_{s,u,k}(l, \bar{l}) = 1 \) for \( \bar{l} = l \) while \( A_{s,u,k}(l, \bar{l}) = 0 \) for \( \bar{l} \neq l \).

Moreover, the MF statistic \( \tilde{y}_{l,u,k} \) is a zero mean complex Gaussian with variance (see \([3, 6]\) for proof): \( \sigma^2 = 2\sigma^2 \xi_s^2 |A_{\tau \nu,k}(l, l)|^2 + 2N_0 \xi_s \). Using \( \sigma^2 \) above the SNR at the output of the MF is defined as \( SNR_0 = \frac{\sigma_C^2}{N_0} \).

3. COMPRESSIVE SENSING AND PROCESSING WITH AN RANDOM MATRIX

The acquisition matrix \( \Phi \) of size \( C \times M_d \), \( C < M_d \), considered in this section, consists of random Gaussian elements. Moreover, it is non-adaptive and agnostic to the received waveform \([4]\). The detailed construction of the random matrix used in this work is provided in \([3]\). The CS measurement is given by: \( d_{c,s,u,k}(l, c) = \sum_{m=0}^{M_d-1} \phi(c, m)d_{u,k}(l, m) \) which using (1) becomes

\[
d_{c,s,u,k}(l, c) = A_k s_{c,s,u,k}(l, c) + \sum_{m=0}^{M_d-1} \phi(c, m)v_{u,k}(m)
\]

for \( c = 0, \ldots, C - 1 \) and \( l = 0, \ldots, L_{u,k} - 1 \) where \( s_{c,s,u,k}(l, c) = \sum_{m=0}^{M_d-1} \phi(c, m) s_{u,k}(l, m) \) is the CS waveform. The compressed MF statistic with template \( s_{c,s,u,k}(l, c) \) is

\[
\tilde{y}_{c,s,l,u,k} = \sum_{c=0}^{C-1} d_{c,s,u,k}(l, c) s_{c,s,u,k}^*(l, c)
\]

and

\[
\xi_c = \sum_{c=0}^{C-1} s_{c,s,u,k}(c) s_{c,s,u,k}^*(c) = \beta \frac{C}{M_d} \xi_s \text{ with } 1 - \delta \leq \beta \leq 1 + \delta \text{ and } \delta \in (0, 1) \text{ is the restricted isometry constant } [2, 4].
\]

Also, \( A_{c,s,u,k}(l, \bar{l}) = \frac{M_d}{\beta \xi_s} \sum_{c=0}^{C-1} s_{c,s,u,k}(l, c) s_{c,s,u,k}^*(\bar{l}, c) \) is the compressed AF. \( \tilde{y}_{c,s,l,u,k} \) is zero mean complex Gaussian with variance (see \([3]\) for proof):

\[
\sigma^2_c = 2\sigma^2 \frac{\beta C}{M_d X_0} |A_{\tau \nu,k}(l, l)|^2 + 2N_0 \frac{\beta C}{M_d X_0} \text{. Therefore, the SNR at the output of the compressed MF is } SNR_0 = \frac{\beta C \sigma^2_c}{X_0} \text{ and is scaled by } \frac{\beta C}{M_d} \text{ compared to the NSP case. This will have a deteriorating effect on the tracking performance.}
\]

4. COMPRESSIVE SENSING AND PROCESSING WITH AN ADAPTIVE MATRIX

The adaptive CSP Matrix: The measurement acquisition matrix \( \Phi_{u,k} \) proposed in this work contains information on the target state that is sequentially updated by a particle filter \([6]\). It also incorporates a set of almost orthogonal sequences which, when using MCPC radar waveforms, reduce the sidelobes of the compressed AF to the small sidelobes of the almost orthogonal set. Specifically, the proposed acquisition matrix generated for each sensor \( u \) at time step \( k \) of a tracking scenario is given by: \( \Phi_{u,k} = Q_{u,k} S_{u,k}^* \) where \( Q_{u,k} \), which contains prior information, is defined in Section 2.

\( Q_{u,k} \) is a \( C \times L_{u,k} \) matrix with columns the \( L_{u,k} \) delay-Doppler cyclically shifted versions of a Björck CAZAC sequence \( q(c) \), \( c = 0, \ldots, C - 1 \) of length \( C \). A Björck CAZAC that is cyclically shifted in time by \( \tau \) and Doppler shifted by \( \nu \), where the pair \( \{\tau, \nu\} \) corresponds to an index \( l \), is given by \( q(c,l) = q(c, l) e^{j2\pi c \nu / C}, c = 0, \ldots, C - 1, l = 0, \ldots, L_k - 1 \). The time index is taken modulo \( C \) denoted as \( (c - \tau)C \). The choice of CAZACs as columns in \( Q_{u,k} \), that provide the almost orthogonal set (low AF sidelobes), should not be confused with the transmitted waveform which may also be a CAZAC. The AF of \( q(c) \) having the small sidelobes of a Björck CAZAC is:

\[
A_{q}(l, \bar{l}) = \frac{1}{\xi_q} \sum_{c=0}^{C-1} q(c, l) q^*(c, \bar{l})
\]

where \( \xi_q = \sum_{c=0}^{C-1} q(c) q^*(c) \) is the energy of \( q(c) \).
The compressed AF: The entries of $\Phi_{u,k}$ are given by
\[
\phi_{u,k}(c,m) = \sum_{l=0}^{L_k-1} q(c,l) s_{u,k}^*(l,m)
\]  
for $c = 0, \ldots, C - 1, m = 0, \ldots, M - 1$. Using (6) and (2) the CS delay-Doppler shifted waveform is
\[
s_{cs,u,k}(l,c) = \sum_{m=0}^{M-1} \phi_{u,k}(c,m) s_{u,k}(l,m) = \sum_{l'=0}^{L_k-1} q(c,l') \xi_{s} A_{s,u,k}(l',l).
\]
For the MCPC $A_{s,u,k}(l',l) = 1$ while $A_{s,u,k}(l',l') = 0$ for $l' \neq l$. Therefore,
\[
s_{cs,u,k}(l,c) = \xi_{s} q(c,l).
\]  
The normalized compressed AF is, using (7), (5), and
\[
\sum_{c=0}^{C-1} s_{cs,u,k}(c,l) s_{cs,u,k}^*(c,l) = \xi_{s}^2 \xi_{q} A_{q}(l,l)
\]
where (6), (2), and the fact that $\Phi_{u,k}$ were dropped for simplicity.

The above shows that the compressed AF reduces to the Nyquist AF of a Björck CAZAC of length $C$. In a companion paper [9] numerical results show that the sidelobes of the compressed AF using adaptive CSP are reduced compared to the AF sidelobes when using random CSP.

The adaptive compressed MF and SNR: The compressed MF statistic with template the CS waveform $s_{cs,u,k}(l,c) = \sum_{m=0}^{M-1} \phi_{u,k}(c,m) s_{u,k}(l,m)$ is using (3), (4), and (8) with $\Phi_{u,k}$
\[
\hat{y}_{cs,l} = A_{s}^2 \xi_{s} \xi_{q} A_{q}(l,l) + \sum_{c=0}^{C-1} \sum_{m=0}^{M_d-1} \phi(c,m) v(m) s_{cs,u,k}^*(l,c)
\]
where the indices $u$ and $k$ were dropped for simplicity. $\hat{y}_{cs,l}$ is a zero mean complex Gaussian random variable with variance
\[
\sigma^2_{\hat{y},cs} = 2 \sigma^2_{A_{s}} \xi_{s}^2 |A_{q}(l,l)|^2 + 2 N_0 \xi_{s}^2 \sum_{c=0}^{C-1} M_{d}-1 \sum_{m=0}^{M_d-1} \phi(c,m) \phi^*(c',m)
\]
where $E[|v(m)\phi^*(m)|] = 2 N_0$, $E[AA^*] = 2 \sigma^2_A$. Since,
\[
\sum_{m=0}^{M_d-1} \phi(c,m) \phi^*(c',m) = L_k - 1 L_k - 1 \sum_{l=0}^{L_k-1} \sum_{l'=0}^{L_k-1} q(c,l) q^*(c',l') M_{d}-1 \sum_{m=0}^{M_d-1} s^*(l,m) s(l',m)
\]
the SNR at the output of the compressed MF in the adaptive CSP case is
\[
SNR_{o} = \frac{\sigma^2_{\hat{y},cs}}{\sum_{l=0}^{L_k-1} |A_{q}(l,l)|^2} = \frac{\sigma^2_{\hat{y},cs}}{N_0 |A_{q}(l,l)|^2}
\]
where $\sum_{l=0}^{L_k-1} |A_{q}(l,l)|^2 > 1$ is the sum of the mainlobe (of magnitude 1) and the sidelobes of the AF surface of $q(c)$. $\sum_{l=0}^{L_k-1} |A_{q}(l,l)|^2$ increases with a decrease in $C$ and degrades the SNR by 10 log\left(\sum_{l=0}^{L_k-1} |A_{q}(l,l)|^2\right)$ dB compared to the NSP case. Numerical results are provided next.

5. SIMULATION RESULTS

In the first set of simulations the $SNR_i$ at the output of the MF in the NSP and the two CSP cases is compared. $SNR_i$ (in dB) is the input SNR to the MF which corresponds to the SNR in the return waveform. This takes values: $SNR_i = [12, 15, 18, 20, 25, 30]$ dB. The SNR is varied by setting $N_0 = 1$, $\xi_{s} = 1$, and $\sigma^2_A = N_0 \log SNR_{i}/10$. In order to obtain the numerical values of $SNR_{o}$, random samples of $A_{q}$ and $v(m), m = 0, \ldots, M - 1$ were generated over 1000 Monte Carlo trials for each $SNR_i$ value. In Figure 1, $SNR_{o}$ is provided as a function of $SNR_i$. For NSP, $SNR_{o} = SNR_i$ holds for any type of waveform. In the random and adaptive CSP cases a different number of compressive measurements $C$ is considered taking values $C = [50, 100, 200, 500, 1000, 2000]$ and $C = [49, 101, 199, 499, 997, 1999]$ as indicated in the figure. For the adaptive CSP $C$ is restricted to be a prime number [6] in order to construct the CAZAC in $Q_{u,k}$. The results show that as $C$ increases the SNR line shifts upwards for both CSP cases. Both the theoretical results in Sections 3, 4 and the numerical values closely agree that the SNR is reduced by 10 log\left(\frac{\beta C}{M_d}\right)$ with $\beta \simeq 1$ in the random CSP and by 10 log\left(\frac{\beta C}{M_d}\right)$ in the adaptive CSP case. Clearly, adaptive CSP improves SNR compared to random CSP. Theoretical values were omitted to improve clarity in the figure.

In the tracking scenario a single target moves on a two-dimensional plane in the Cartesian coordinates. The motion is completed in 160 time steps in a randomly generated trajectory. The motion is modeled by a nearly constant velocity motion model [6] with covariance matrix $Q = \text{diag}(50, 6, 50,$
The target is observed by two sensors at \( \chi_1 = 2000 \) m, \( \psi_1 = -2000 \) m and \( \chi_2 = 4000 \) m, \( \psi_2 = -2000 \) m. The SNR varies as \( SNR_i = [12, 15, 18, 20, 25, 30] \) dB. The MCPC radar waveform has length \( 106 \times 47 = 4982 \) and is composed of 106 identical Björlck CAZACs of prime number length 47. In the NSP case the return waveform is sampled at 20 MHz, collecting a total of \( M_d = 5465 \) Nyquist measurements, and the carrier frequency was set to \( f_c = 40 \) GHz.

The tracking algorithm used is the likelihood particle filter in [3, 6] using 500 particles. In Figure 2, the RMSE tracking performance, over 100 Monte Carlo runs with 95% confidence intervals, is provided for the NSP and the two CSP cases versus the \( SNR_i \). For the CSP cases the number of compressive measurements \( C \) was varied as indicated in the legend. The results show that adaptive CSP outperforms random CSP, providing lower RMSE at lower numbers of compressive measurements. Moreover, in high-SNR environments, adaptive CSP offers a low sampling rate, low computational expense alternative to NSP while preserving high-resolution and low RMSE.

For example, when collecting and processing \( C = 997 \) adaptive CS measurements versus \( M_d = 5465 \) Nyquist measurements, the sampling rate decreases from 20 MHz to about 3.65 MHz. The tracking performance results in Figure 2 agree with what is expected from the SNR results of Figure 1 where the SNR degrades as the number of CS measurements \( C \) decreases.

### 6. CONCLUSIONS

In this work, an adaptive compressive sensing and processing method is proposed that utilizes information on target state that is provided by a particle filter. Theoretical and numerical results demonstrate that both the SNR and the tracking performance are improved when using the proposed adaptive CSP method compared to when using random CSP.

### 7. REFERENCES


