ORTHOGONAL WAVELET DIVISION MULTIPLEXING FOR WIDEBAND TIME-VARYING CHANNELS

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ABSTRACT
Block transmission of multi-scale orthogonal wavelet division multiplexing (OWDM) is proposed for signaling over wideband linear time-varying channels (LTV). Such channels are best modeled by multi-scale, multi-lag (MSML) models and the proposed OWDM designs are tailored to such channels. Given this signaling, the effective channel matrix for the received signal is banded, allowing for the modification of prior methods of equalization for orthogonal frequency division multiplexing over narrowband LTV channels. Performance of such equalizers and signaling is provided via simulation and shown to offer good performance coupled with high spectral efficiency over previously proposed designs.

Index Terms— Wideband channels, wavelets, OFDM, OWDM.

1. INTRODUCTION
Wideband linear time-varying (LTV) channels are of interest in a variety of wireless communication scenarios including underwater acoustic systems and wideband terrestrial radio frequency systems such as spread-spectrum or ultrawideband. Due to the nature of wideband propagation, such channels exhibit some fundamental differences relative to so-called narrowband channels. In particular, it has been shown that multi-scale, multi-lag (MSML) channel descriptions offer improved modeling of LTV wideband channels over multi-Doppler-shift, multi-lag models [1, 2]. Orthogonal frequency-division multiplexing (OFDM) has been examined for wideband channels. Approaches include splitting the wideband LTV channel into parallel narrowband LTV channels [3] or assuming a simplified model which reduces the wideband LTV channel to a narrowband LTV channel with a carrier frequency offset [4].

 Receivers for single-scaled wavelet-based pulses for wideband MSML channels are presented in [1, 2], and a similar waveform is adopted in spread-spectrum systems [5] over wideband channels modeled by wavelet transforms; while [6] considers equalizers for block transmissions in wideband MSML channels. In order to achieve better realistic channel matching, single-scaled rational wavelet modulation was designed in [7]. However these schemes all employ single-scale modulation and thus do not maximize the spectral efficiency. In particular, herein, we shall focus on a form of orthogonal wavelet division multiplexing (OWDM) which has been previously examined in [8] but for additive white Gaussian noise channels. While we shall employ such a modulation over wideband MSML channels, new equalizer designs are necessary.

The main contributions of this work is a particularization of OWDM to a MSML channel model using multi-scale block transmission, identifying proper wavelets for the wideband LTV channel and providing equalizer designs for our system. In particular, we observe that our signaling and the channel yield a received signal with an effective channel matrix which is banded. As such, equalization methods designed for combating inter-carrier interference due to the use of OFDM in narrowband LTV channels (e.g. [9, 10]) can be exploited as we do so herein.

Notation: Upper (lower) bold-face letters stand for matrices (vectors); Superscript H denotes Hermitian, ∗ conjugate, T transpose, and † matrix pseudo-inverse. We reserve j for the imaginary unit, [·] for integer ceiling, ⌊·⌋ for integer flooring, [A]k,m for the (k, m)th entry of the matrix A, diag(x) for a diagonal matrix with x on its main diagonal, and nmod/k for the modulus of i divided by k. δk represents a delta function which is equal to one only if k = 0 and zero otherwise.

2. WIDEBAND CHANNEL MODEL
A wideband LTV channel can be described by a general MSML model [11],

\[ r(t) = \int_{-\infty}^{\infty} h(\alpha, \tau) \sqrt{\alpha} x(\alpha(t-\tau) + v(t), \]  

where \( x(t) \) is the transmitted signal, \( r(t) \) is the received signal, \( v(t) \) is the time domain white noise with \( \sigma^2 \) as its power spectral density, and \( h(\alpha, \tau) \) is the wideband channel spreading function [11]. This model reflects the fact that the received signal \( r(t) \) can be represented by a superposition of different delayed (by \( \tau \) and scaled (by \( \alpha \)) versions of the transmitted signal \( \sqrt{\alpha} s(t) \) is a normalization factor). Due to practical restrictions, \( \tau \) and \( \alpha \) can be limited to \( \tau \in [0, \tau_{\text{max}}] \) and \( \alpha \in [1, \alpha_{\text{max}}] \) without loss of generality by appropriately delaying and scaling the received signal. The parameters \( \tau_{\text{max}} > 0 \) and \( \alpha_{\text{max}} > 1 \) respectively represent the delay spread and Doppler spread.

We assume that the wideband transmitted signal has a bandwidth of \( W_c \) and a Mellin support of \( M_c \). Then (1) can be approximated by the following finite-dimensional discrete MSML model ([1, 2]):

\[ r(t) = \sum_{r=0}^{R_c} \sum_{l=0}^{L_c} h_{r,l} a_r^{\sigma/2} x(a_r^{\alpha} t - l T_s) + v(t), \]  

where \( T_s = 1/W_c \) is the arithmetic time resolution and \( a_r = e^{1/M_c} > 1 \) is the geometric scale resolution of the model. Further, the channel coefficients are given by \( h_{r,l} = h(a_r^{\sigma}, l T_s) / a_r^{\alpha} \) with \( h(\alpha, \tau) \) defined in [1] as the doubly-smoothed version of \( h(\alpha, \tau) \). Finally, the scale order \( R_c \) is given by \( R_c = \lfloor \log_{a_{\text{max}}} (\alpha_{\text{max}}) \rfloor \), and the delay order for the \( r \)th scale \( L_r(r) \) is given by \( L_r(r) = \left\lfloor \log_{a_{\text{max}}} (\alpha_{\text{max}}) \right\rfloor \).

This work is supported in part by NWO-STW under the VICI program (project 10382).

1The Mellin support is the scale analogy of the Doppler spread for narrowband LTV channels – further details and definitions can be found in [12].
As in [1] that the discrete MSML model is well-matched and thus there is negligible difference between (1) and (2).

3. SCALE LAYERED TRANSMISSION

As previously noted, most prior work on wideband LTV systems employed transmitted signals at a single scale (or a single delay); herein we generalize it to multiple scales and delays to increase spectral efficiency by using an OWDM scheme. By further imposing interscale and inter-delay orthogonality via wavelet signaling, we can simplify receiver processing. However, it should be observed that just like in the narrowband time-varying case, we cannot perfectly jointly diagonalize across delay and scale.

3.1. Signaling Scheme

We start from a unit-energy orthogonal mother wavelet \( \psi(t) \) with unity orthogonality time shift (we call this base time from now on) and a base scale of \( a \), i.e., \( \int_{-\infty}^{\infty} \psi(a^k t - n) \psi^* (a^k t - n') dt = \delta_{k-k'} \delta_{n-n'} \). A symbol is modulated onto the \( k \)th scale and \( n \)th delay via the unit-energy pulse

\[
\psi_{k,n}(t) = \sqrt{\frac{a^k}{T}} \psi(a^k t / T - n),
\]

which has a base time of \( T / a^k \) and the base scale \( a > 1 \). It is easy to show that \( \int_{-\infty}^{\infty} \psi_{k,n}(t) \psi^*_{k',n'}(t) dt = \delta_{k-k'} \delta_{n-n'} \).

A critical element of our system is the assumption that we can properly match the scales and delays of our signaling to that of the channel, that is, the base time \( T / a^k \) and base scale \( a \) of \( \psi_{k,n}(t) \) are matched to the time resolution \( \tau \) and scale resolution \( \delta_a \) of the channel, which on its turn is determined by the bandwidth and the Mellin support of the transmitted signal, respectively. Thus, \( \psi_{k,n}(t) \) should have a bandwidth of \( a^k / T \) and a Mellin support of \( 1 / \ln a \) (equivalently \( \psi(t) \) should have a bandwidth of \( 1 \) and a Mellin support of \( 1 / \ln a \)). Not all wavelet families satisfy these constraints; in particular, band-pass natured wavelets often violate our constraints. However, some orthogonal wavelet families possess good frequency and Mellin localization (e.g., Shannon wavelets) and as such approximately satisfy our constraints and thus are candidates for our signaling system. We assume for simplicity that the desired matching can be achieved without any errors.

We underscore that the base time (and thus bandwidth) of each \( \psi_{k,n}(t) \) is different per scale. We equivalently refer to scale or layer in the context of the signaling scheme. Hence, for the \( k \)th layer, the system model of (2), ignoring the additive noise, is adapted to the appropriate time and scale resolutions, i.e., we select \( a_s = a \) and \( T_s = T / a^k \) in (2), which leads to

\[
r_k(t) = \sum_{r=0}^{R} \sum_{l=0}^{L(r+k)} h_{r,l}(k) a_r^{r/2} x_{k,n}(a^r t - lT/a^k),
\]

where \( x_{k,n}(t) \) is the transmitted signal at the \( k \)th layer (scale), \( r_k(t) \) is the received signal at the \( k \)th layer, and \( h_{r,l}(k) = h(a^r, lT/a^k) \). We further have \( R = \log_a (\alpha_{max}) \) and \( L(r) = \lceil a^{r \tau_{max}} / T \rceil \).

We generate the transmitted signal \( x_{k,n}(t) \) for the \( k \)th layer as

\[
x_{k,n}(t) = \sum_{n} s_{k,n} \psi_{k,n}(t),
\]

where the symbol sequence \( s_{k,n} \) modulates the shaping pulses \( \psi_{k,n}(t) \) at a symbol rate of \( a^k / T \). Substituting (4) into (3), we then obtain

\[
r_k(t) = \sum_{r=0}^{R} \sum_{l=0}^{L(r+k)} h_{r,l}(k) a_r^{r/2} \sum_{n} s_{k,n} \psi_{k,n}(a^r t - lT/a^k)
\]

\[
= \sum_{r=0}^{R} \sum_{l=0}^{L(r+k)} h_{r,l}(k) \sum_{n} s_{k,n} \sqrt{\frac{a^{r+k} T}{T}} \psi(a^{r+k} t / T - l - n)
\]

\[
= \sum_{r=0}^{R} \sum_{l=0}^{L(r+k)} h_{r,l}(k) \sum_{n} s_{k,n} \psi_{r+k,l+n}(t).
\]

Transmitting data on multiple layers, our OWDM (multi-layer) waveform can finally be described as

\[
x(t) = \sum_{k} \sum_{n} s_{k,n} \psi_{k,n}(t),
\]

Its corresponding received signal \( r(t) \) can be expressed as

\[
r(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} s_{k,n} \psi_{r+k,l+n}(t).
\]

3.2. Block Transmission

We limit the number of layers to \( K \), where \( K \) is related to the overall available transmission bandwidth, i.e., \( K \) and \( T \) are selected such that \( a^{K-1} / T \) matches the overall bandwidth. Further, the data on every layer in blocks of length \( N \) are separated by a cyclic prefix (CP) of length \( Z \), thus facilitating block processing. Focusing on the first block of data, the transmitted signal can be written as

\[
x(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} s_{k,n} \psi_{k,n}(t),
\]

and its related received signal can be written as

\[
r(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} \sum_{l=0}^{L(r+k)} h_{r,l}(k) \sum_{n=0}^{N-1} s_{k,n} \psi_{r+k,l+n}(t),
\]

where

\[
s_{k,n} = \begin{cases} b_{k,n-Z}, & \text{for } Z \leq n < N \\ b_{k,N+n-Z}, & \text{for } 0 \leq n < Z \end{cases}
\]

At the receiver, we will only consider the received data on the layers for \( k \in \{0, 1, \ldots, K-1\} \). To avoid interblock interference (IBI) on these layers, we need \( Z \geq L(k) \), for all \( k \in \{0, 1, \ldots, K-1\} \), or in other words \( Z \geq \lceil a^{K-1} \tau_{max} / T \rceil = \lceil a^{K-1} / (a-1)T \rceil \). In contrast, the single-layered approach has a maximal efficiency of \( a^{K-1} / T \) (on the \( K-1 \)th layer) using the same overall bandwidth. Note that for \( a > 1 \), it is clear that the OWDM approach achieves an increased efficiency.

4. WIDEBAND RECEIVER DESIGN

In this section, we develop the receiver for the proposed OWDM transmission. The first part of the receiver is an extension of the RAKE receiver concept proposed in [1] and consists of a matched filter bank for the layers of \( k \in \{0, 1, \ldots, K-1\} \) as shown in Fig. 2. One may compare the discarding of the layers for \( k \in \{K, \ldots, K+\}

$\psi_{a,0}(t)$ here with the removal of the OFDM edge null subcarriers at an OFDM receiver.

More specifically, the matched filter bank related to the first block computes the sufficient statistics

$$y_{k,n} = \int \sum_{k'=0}^{K-1} \sum_{l=0}^{N-2} h_{r,l}(k') \delta_{r+k'-k,n'-n} \delta_{l+n'-n-2} r(t) \psi_{k',n'}^{*}(t) dt,$$

for $k \in \{0, 1, \ldots, K - 1\}$ and $n \in \{0, 1, \ldots, N - 1\}$. Note that in (8) the CP is implicitly removed. Substituting (6) into the matched filter output (8), ignoring the noise, we have

$$y_{k,n} = \sum_{k'=0}^{K-1} \sum_{l=0}^{N-2} h_{r,l}(k') \delta_{r+k'-k,n'} \delta_{l+n'-n-2} r(t) \psi_{k',n'}^{*}(t) dt,$$

where we have used that $s_{k,n} = b_{k,n}$ for $k \in \{0, 1, \ldots, K - 1\}$ and $n \in \{0, 1, \ldots, N - 1\}$. Due to the orthogonal properties of the wavelet family, the additive noise $\nu(t)$ passing through the filter bank, results in white Gaussian noise with zero mean and variance $\sigma^2$.

The output of the matched filter bank (9) yields the input of a subsequent equalization step. If we define $y_k = F[y_k, 0, \ldots, y_k, N - 1]^T$, $b_k = F[b_k, 0, \ldots, b_k, N - 1]^T$, $v_k = F[v_k, 0, \ldots, v_k, N - 1]^T$, and $F[n, m] = \frac{1}{N} 2^{\delta_{n,0}} \delta_{m,0}$ is the discrete Fourier Transform (DFT) matrix, we can write the relationship between $y_k$ and $b_k$ as

$$y_k = \sum_{r=0}^{R} H_r(k-r) b_k + v_k,$$

where $H_r(k-r) = \text{diag} \{h_r, 0, k-r, \ldots, h_r, N-1, k-r\}$ is an $N \times N$ diagonal matrix for $k \in \{0, 1, \ldots, K - 1\}$ and $r \in \{0, 1, \ldots, R\}$, and $\tilde{h}_r, p(k-r) = \sum_{l=0}^{K-1} h_r, l(k-r) e^{j 2 \pi \frac{rl}{M}}$ for $p \in \{0, 1, \ldots, N - 1\}$. Thus, within a single layer, we can orthogonalize the channel as occurs in OFDM and linear time-invariant channels; however, there is inter-scale interference, as will be seen in the sequel. If we stack all the $y_k$'s into $y = [y_0^T, \ldots, y_{K-1}^T]^T$, all the $b_k$'s into $b = [b_0^T, \ldots, b_{K-1}^T]^T$ and $v = [v_0^T, \ldots, v_{K-1}^T]^T$, we obtain the following relationship

$$y = Hb + v,$$

Table 1. Parameters for the adopted wideband channels

<table>
<thead>
<tr>
<th>Channel</th>
<th>$L(0)$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>III</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

where $H_r(k-r) = \text{diag} \{h_r, 0, k-r, \ldots, h_r, N-1, k-r\}$ is an $N \times N$ diagonal matrix for $k \in \{0, 1, \ldots, K - 1\}$ and $r \in \{0, 1, \ldots, R\}$, and $\tilde{h}_r, p(k-r) = \sum_{l=0}^{K-1} h_r, l(k-r) e^{j 2 \pi \frac{rl}{M}}$ for $p \in \{0, 1, \ldots, N - 1\}$. Thus, within a single layer, we can orthogonalize the channel as occurs in OFDM and linear time-invariant channels; however, there is inter-scale interference, as will be seen in the sequel. If we stack all the $y_k$'s into $y = [y_0^T, \ldots, y_{K-1}^T]^T$, all the $b_k$'s into $b = [b_0^T, \ldots, b_{K-1}^T]^T$ and $v = [v_0^T, \ldots, v_{K-1}^T]^T$, we obtain the following relationship

$$y = Hb + v,$$

where $H$ is a $KN \times KN$ matrix specified as

$$H = \begin{bmatrix} H_0(0) & \ldots & 0 \\ \vdots & \ddots & \vdots \\ H_R(0) & \ldots & H_R(K-1) \end{bmatrix}$$

Let us now define $M = KN$ and introduce the $M \times M$ permutation matrix $P$, which only contains 1's in the positions \(\{i+1, \lfloor i/M \rfloor + N_{t_{\text{med}}}/M + 1\}_{i=0}^{M-1}\) and 0's elsewhere. We can then use $P$ to convert $H$ into $H = PHP^T$, which is a compactly banded matrix with its bandwidth given by $R + 1$ — we observe that the bandwidth is determined by the time variation of the wideband LTV channel. Then, (10) can be rewritten as

$$\bar{y} = \tilde{H} \bar{b} + \bar{v},$$
support in the Mellin domain is more or less the same, i.e., $a \approx 2$. We further choose QPSK for the data constellation, $N = 128$, $K = 3$, and $Z = 8$, which satisfies the constraints $K > R$ and $N > Z \geq [a^{K-1} L(0)]$.

Fig. 3 shows the bit-error-rate (BER) performance of our system without any bit coding using the LMMSE equalizer from [9] adapted to (11). The receiver performs best for Channel I which has no Doppler effects, while a slight drop in BER performance can be witnessed when the delay or scale spread increases. A similar observation can be made for an OFDM system over three narrowband LTV channels modeled by a CE-BEM [15] (ignoring its modeling error), with the same Doppler shift order $R$ and delay order $L(0)$. One may argue that the BER performance herein is much better than that in [9] even when the same Doppler shift order $R$ is adopted. The reason for this is because we assume (6) is well-matched without any error, while [9] considers the modeling errors of the CE-BEM and thus has a performance drop compared with Fig. 3. This figure also compares the BER performance of our proposed wideband receiver with that of the matched filter without equalization. As expected, the inter-scale interference severely affects system performance. Fig. 4 shows the BER performance using the Turbo-I/II/III equalizers from [10] adapted to (11) for Channel II and Channel III (without coding). The performance curves behave similarly to those in [10], and it is known that with more iterations, the BER performance can be further improved.

6. CONCLUSIONS

A block transmission scheme based on multi-scale OWDM over wideband LTV channels is proposed. Our proposed signaling offers higher spectral efficiency over previously proposed methods for such channels. An associated equalizer which employs a matched filter bank coupled with a frequency-domain equalizer is presented. The output effective channel has strong similarities to that seen for narrowband LTV systems employing OFDM and thus, equalizers for such systems can be adapted to the current scenario resulting in good performance.

7. REFERENCES