ROBUST FREQUENCY SYNCHRONIZATION FOR AN OFDMA UPLINK SYSTEM DISTURBED BY A COGNITIVE RADIO SYSTEM INTERFERENCE

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ABSTRACT
In Cognitive Radio (CR) systems, spectrum sensing plays a key role to determine the free frequency bands. However, when the primary-user (PU) signal spectrum exhibits localized fading, PU detection cannot be guaranteed. In addition, as the CR may use the PU faded frequencies, the PU spectrum can be disturbed by a narrow-band interference (NBI) and synchronization algorithms used for the PU carrier frequency offset (CFO) estimation suffer degradations. In this paper, we propose a new scheme that jointly allows the CR-NBI to be detected and the PU-CFOs and the channels to be estimated in an orthogonal frequency division multiple access (OFDMA) system. It combines a sigma point Kalman filter and a test aiming at detecting a variation of the measurement-noise covariance matrix. Simulation results confirm that the proposed algorithm can accurately detect the CR-NBI and estimate the PU-CFOs.

Index Terms— OFDMA, frequency offset estimation, cognitive radio, interference detection, sigma point Kalman filter.

1. INTRODUCTION
Cognitive Radio (CR) systems [1] have been proposed as a solution to optimize the spectrum allocation. Among the parameters (power, constellation size, etc.) to be adjusted, they dynamically change the transmission and reception bandwidths by using spectrum sensing to find out which frequency bands are not used. Due to the spectral diversity to allocate the communication resources over different subchannels, the physical layer modulation for CR is usually based on multicarrier modulation, such as filter bank multicarrier (FBMC) and orthogonal frequency-division multiplexing (OFDM). However, when CR systems are used, the primary-user (PU) has to be detected. When dealing with multicarrier transmissions, energy detection can be chosen as a spectrum sensing technique [2], [3].

It is optimal if the cognitive devices have no a priori information about the PU signal. In case of shadowing or fading, PU detection can no longer be guaranteed. Then the PU signal may be disturbed by an undesirable narrow-band interference (NBI) caused by the CR. This may lead to an inaccurate estimation of the carrier frequency offset (CFO) and hence increases the bit error rate. Therefore, NBI has to be detected and taken into account to deduce the CFO.

Concerning NBI detection, Lau et al. [4] propose an approach that consists of a repetitive pilot block in the time domain to detect an NBI localized in time. Since the two duplicated parts remain identical after passing through the channel, their subtraction at the receiver provides the contribution of the noise-plus-interference, which is then easily detected. Nevertheless, this method cannot be used in the presence of CFO.

Concerning the CFO, their estimation and compensation in OFDM systems are required to maintain the orthogonality between subcarriers. In [5], Morelli et al. illustrate various schemes to estimate the CFO among different subcarrier allocation strategies. In [6], we suggest using a Sigma Point Kalman filter to perform the CFO estimation. The algorithms proposed in [5] and [6] provide accurate estimation as long as the received signal is only disturbed by the propagation channel and an additive white Gaussian noise (AWGN), whereas significant degradations are expected in the presence of an NBI. In [7], Morelli et al. propose a novel scheme that jointly estimates CFO and the interference power on each subcarrier. They take advantage of an inverse fast Fourier transform (IFFT) property to estimate the fractional part of the CFO, by transmitting pilot symbols over some specific subcarriers while setting the others to zero. By performing the IFFT step, a block composed of several identical parts is obtained and used to estimate the CFO. However, an exhaustive grid search is considered over the possible range of the CFO, leading to a high computational cost. In addition, the CFO estimation is very sensitive to the number of subcarriers disturbed by the NBI.

In this paper, we propose to jointly estimate the PU-CFOs and detect the CR-NBI in an orthogonal frequency-division multiple access (OFDMA) PU-system, damaged by a CR-NBI localized in time. The approach is based on sigma point Kalman Filter (SPKF) [8] with an additional step based on the analysis of the variation of the measurement noise covariance matrix [9]. Knowing the innovation distribution, two algorithms are then proposed to detect the CR-NBI. The first one is based on a classic binary hypothesis test (BHT), whereas the second one consists in comparing the derivate of a cumulative sum (CUSUM) test to a threshold.

The paper is organized as follows. System and signal models are presented in section 2. Section 3 shows how to jointly detect the CR-NBI and estimate the PU-CFOs. Simulation protocol and results are presented in section 4 and finally conclusions are given in section 5.

In the following, \( Re \{ \cdot \} \) and \( Im \{ \cdot \} \) denote the real and the imaginary part of \( \{ \cdot \} \), respectively; \( I_L \) is the identity matrix of size \( L \) and \( || \cdot || \) is the Euclidean norm.

2. SYSTEM DESCRIPTION
On the one hand, let us consider a multicarrier CR system network with an available bandwidth \( W_{cr} \) divided among \( K_{cr} \) subcarriers. On the other hand, the uplink PU-OFDMA system consists of a single base station (BS) and \( U \) simultaneously independent users. Then, the available bandwidth \( W_{pu} = \alpha W_{cr} \) of the PU-OFDMA system is divided among \( K_{pu} \leq K_{cr} \) subcarriers, with \( 0 < \alpha < \frac{K_{pu}}{K_{cr}} \). See figure 1.

In the following, the subscript \( u \) denotes the information associated to the \( u \)th user in the PU-OFDMA system, with \( u \in \{ 1, \ldots, U \} \).

\( S_u^{pu} = [S_u^{pu}(0), S_u^{pu}(1), \ldots, S_u^{pu}(K_{pu} - 1)]^T \) is the emitted symbol vector, with the symbol time \( T_{pu} \) and where \( S_u^{pu}(k_{pu}) = 0 \) if the \( k_{pu} \)th subcarrier is not assigned to the \( u \)th user,
with $k_{pu} \in \{0, \ldots, L_{pu} - 1 \}$. In addition, the channel impulse response is $h_{pu}^n = [h_{pu}^n(0), h_{pu}^n(1), \ldots, h_{pu}^n(L_{pu} - 1)]$, where $L_{pu}$ is the length of the maximum channel delay spread. At the receiver, due to the propagation conditions, time offset and CFO are induced in the signal. The PU-OFDMA received signal after the cyclic prefix removal can be written as follows:

$$ R_u(n, \epsilon_u, h_{pu}^n) = e^{j2\pi f_{pu}u \epsilon_u} \sum_{k_{pu}=0}^{K_{pu}-1} h_{pu}^n(k_{pu}) h_{pu}^n(k_{pu}) e^{j2\pi k_{pu}u} $$

(1)

where $H_{pu}^n(k_{pu}) = \sum_{l=0}^{L_{pu}-1} h_{pu}(l)e^{-j2\pi k_{pu}l}$ is the frequency response associated to the $k_{pu}$th subcarrier, $\epsilon_u$ is the normalized PU-CFO and $n \in \{0, \ldots, K_{pu} - 1 \}$ is the PU sample number. Let us now define the row vectors that contain the normalized PU-CFOs and the channel impulse responses of each user, respectively:

$$ \epsilon = [\epsilon_1, \epsilon_2, \ldots, \epsilon_u, \ldots, \epsilon_U] $$

(2)

$$ h_{pu} = [h_{pu,1}^n, h_{pu,2}^n, \ldots, h_{pu,U}^n, \ldots, h_{pu,U}^n] $$

Concerning the CR-signal emitted, the unknown CR-NBI produced by a fault of spectrum sensing detection is written:

$$ I(n_c) = \sum_{k_c=0}^{K_c-1} S_c(k_c) e^{j2\pi k_c n_c} $$

(3)

where $S_c = [S_c(0), \ldots, S_c(k_c), \ldots, S_c(K_c - 1)]^T$ is the CR emitted symbol vector, with the symbol time $T_c$, $S_c(k_c) = 0$ if the spectrum sensing decides that the $k_c$th subcarrier is busy, with $k_c \in \{0, \ldots, K_c - 1 \}$ and $n_c \in \{0, \ldots, K_c - 1 \}$ is the CR sample number.

At the PU-BS, the unknown downsampled CR-NBI is written as follows:

$$ Z(n) = \sum_{k_c=0}^{K_c-1} S_c(k_c) H_c(k_c) e^{j2\pi k_c n_c} $$

(4)

where $H_c(k_c)$ represents the unknown channel frequency response associated to the $k_c$th subcarrier, $n' \in \{n_1, n_1 + 1, \ldots, n_2 \}$ are the samples disturbed by the CR-NBI, where $n_1 \in \{0, 1, \ldots, K_{pu}(1 - \alpha) + 1 \}$ and $n_2 = n_1 + \alpha K_{pu} - 1$. Then, (4) can be rewritten as follows:

$$ Z(n') = \sum_{k_c=0}^{K_c-1} S_c(k_c) H_c(k_c) e^{j2\pi k_c n'} $$

(5)

Thus, the PU-OFDMA received signal at the BS satisfies:

$$ R(n) = \begin{cases} f(n, e, h_{pu}^n) + Z(n) + B(n) & \text{if } n_1 \leq n \leq n_2 \\ f(n, e, h_{pu}^n) + B(n) & \text{otherwise} \end{cases} $$

(6)

where $f(n, e, h_{pu}^n) = \sum_{l=0}^{U} R_l(n, \epsilon_u, h_{pu}^n)$ and $B(n)$ is a zero-mean complex AWGN with variance $\sigma_B^2$.

### 3. JOINT CR-NBI DETECTION AND PU-CFO ESTIMATION

In this section, we suggest jointly detecting the CR-NBI and estimating PU-CFOs and channels. A preamble of one OFDMA symbol is used to perform the channel/CFOs estimation. We assume that the system is synchronized in time and that the PU-channels and the PU-CFOs do not vary over the OFDMA symbols of the OFDMA frame. First of all, let us define the following state vector:

$$ x(n) = \begin{bmatrix} \epsilon(n) \ Re \{h_{pu}^n(n)\} \ Im \{h_{pu}^n(n)\} \end{bmatrix}^T $$

(7)

that satisfies the following state equation:

$$ x(n) = x(n-1) + v(n) $$

(8)

Secondly, let $Y(n)$ be the observation vector that stores the real and the imaginary parts of the PU-OFDMA received signal $R(n)$:

$$ Y(n) = \begin{bmatrix} A(n, x(n)) + \Gamma(n) + V(n) \end{bmatrix} \begin{bmatrix} \text{if } n_1 \leq n \leq n_2 \end{bmatrix} $$

(9)

where $A(n, x(n)) = \begin{bmatrix} \epsilon(n) \ Re \{f(n, e, h_{pu}^n)\} \ Im \{f(n, e, h_{pu}^n)\} \end{bmatrix}^T$, $V(n) = \begin{bmatrix} \epsilon(n) \ Re \{B(n)\} \ Im \{B(n)\} \end{bmatrix}^T$ is a white Gaussian noise vector with covariance matrix $(\sigma_B^2/2)I_2$ and $\Gamma(n) = \begin{bmatrix} \epsilon(n) \ Re \{Z(n)\} \ Im \{Z(n)\} \end{bmatrix}^T$ is the zero-mean Gaussian CR-NBI vector with covariance matrix $(\sigma_z^2/2)I_2$.

When dealing with the non-linear state-space representation (8)-(9) of the system, local methods such as the extended Kalman filter (EKF) and the SPKF, namely the unscented Kalman filter (UKF) or the central difference Kalman filter (CDKF), can be considered [8]. The advantages of the SPKF over the EKF is that it does not require calculations of Jacobians or Hessians and that the EKF is more likely to diverge. The SPKF provide "accurate" estimations as long as the received signal is only disturbed by the AWGN, whereas significant degradations are expected in the presence of the CR-NBI.

To maintain the estimation performances, we propose to jointly detect the CR-NBI and estimate the PU-CFOs.

Our approach operates for the $r$th sample in three steps: 1/ estimating the state vector by means of SPKF, 2/ calculating the innovation energy $||\hat{Y}(n)||^2$ where $\hat{Y}(n) = Y(n) - Y(n)$ = $\begin{bmatrix} \epsilon(n) \ Re \{\hat{R}(n)\} \ Im \{\hat{R}(n)\} \end{bmatrix}^T$ and $Y(n) = A(n, x(n))$ is the estimation of $Y(n)$, 3/ testing whether there is CR-NBI or not by using the innovation energy.

**Remark 1:** If the CR-NBI is assumed to appear at the $r$th sample, the estimated state vector at time $n - 1$ and its covariance matrix are stored: $\hat{x}_{n-1} = x(n-1)$ and $P_{\hat{x}}^n = P_{x}(n-1)$. They will be used as a new starting point for the channel and CFO estimation once there is no CR-NBI. During the CR-NBI perturbation, SPKF provides an estimation of the state vector that is not reliable, but useful to detect the presence of the CR-NBI.

**Remark 2:** $||\hat{Y}(n)||^2$ increases much when there is CR-NBI. This gap is due to the received signal variance that jumps from $\sigma_{\hat{x}}^2$ to $\sigma_{\hat{x}}^2 + \sigma_z^2$.

In the next subsections two tests are presented.

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1. Timing errors between the incoming signal and the receiver can be avoided by using a sufficiently long cyclic prefix between two adjacent OFDM symbols.
2. The CR-NBI signal can be considered as a zero-mean gaussian vector due to the large number of subcarriers $K_{pu}$. 

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3.1. Combining SPKF and BHT (SPKF-BHT)

To decide whether there is CR-NBI in the OFDMA symbol or not, we test the following binary hypothesis:

\[
\begin{align*}
H_0 & : A(n, x(n)) + \Gamma(n) + V(n) \\
H_1 & : A(n, x(n)) + V(n)
\end{align*}
\]

The complex residual \( \hat{R}(n) \) is a zero-mean Gaussian process with variance \( tr(\hat{P}_{YY}(n)) \) where \( \hat{P}_{YY}(n) \) is the innovation covariance matrix obtained by the SPKF at the nth recursion. Then, the probability of false alarm can be defined as:

\[
P_{fa} = P(|\hat{R}(n)|^2 > \lambda(n)|H_1) = P(||\hat{Y}(n)||^2 > \lambda(n)|H_1) = 2P(||\hat{Y}(n)|| > \sqrt{\lambda(n)}) \quad (11)
\]

where \( \lambda(n) \) is a threshold at the \( n \)th recursion set by the practitioner. Using \( ||\hat{Y}(n)|| \) properties and the cumulative density function of the Rayleigh distribution, we obtain:

\[
\lambda(n) = tr(\hat{P}_{YY}(n))\ln\left(\frac{2}{P_{fa}}\right) \quad (12)
\]

If \( ||\hat{Y}(n)||^2 \leq \lambda(n) \) no CR-NBI is assumed to be present. When \( ||\hat{Y}(n)||^2 > \lambda(n) \), the CR-NBI is supposed to be at the \( n \)th sample.

3.2. Combining SPKF and CUSUM (SPKF-CT)

Only the measurement noise variance is available. We now suggest using the upper control limit of the CUSUM test:

\[
C^+(n) = \max(0, C^+(n-1)) + ||\hat{Y}(n)||^2 - tr(\hat{P}_{YY}(n)) \quad (13)
\]

Given (13), the CUSUM test makes it possible to detect when the CR-NBI begins, but does not give information when it ends. Thus, we propose to compute \( \frac{\Delta C^+(n)}{\sqrt{\lambda(n)}} \) when the PU-OFDMA symbol is disturbed by the CR-NBI and hence \( \frac{\Delta C^+(n)}{\sqrt{\lambda(n)}} \approx ||\hat{Y}(n)||^2 \). Otherwise, \( tr(\hat{P}_{YY}(n)) \ll ||\hat{Y}(n)||^2 \) or \( \frac{\Delta C^+(n)}{\sqrt{\lambda(n)}} \approx 0 \). If \( C^+(n-1) \leq 0 \), \( \frac{\Delta C^+(n)}{\sqrt{\lambda(n)}} > ||\hat{Y}(n)||^2 \) if there is CR-NBI. Otherwise, it is very small.

Therefore, one has to compare \( \frac{\Delta C^+(n)}{\sqrt{\lambda(n)}} \) to a threshold in order to detect the end of the CR-NBI in the PU-OFDMA symbol. When \( \frac{\Delta C^+(n)}{\sqrt{\lambda(n)}} \) is higher than the threshold, the CR-NBI is assumed to be present. Otherwise, no CR-NBI is assumed to be present. Concerning the choice of the threshold, we suggest the one defined in subsection 3.1.

3.3. Improving the CR-NBI detection

Due to the distribution of the CR-NBI, \( ||\hat{Y}(n)||^2 \) is not always higher than the threshold when the CR-NBI disturbs the PU-OFDMA symbol. To solve this issue and improve the CR-NBI detection when using the BHT, we propose to take into account the evolution of \( ||\hat{Y}(n)||^2 \) over several recursions. We suggest defining a new residual value \( ||\hat{Y}_{mean}(p)||^2 \) as follows:

\[
||\hat{Y}_{mean}(p)||^2 = \frac{1}{\beta} \sum_{q=\beta}^{\beta(p+1)-1} ||\hat{Y}(q)||^2 \quad (14)
\]

If the value of \( \beta \) samples of \( ||\hat{Y}_{mean}(p)||^2 \) exceeds the value of the threshold, the algorithm decides that the CR-NBI is present in the \( \beta \) recursions. Based on the same idea to improve the CR-NBI detection when using the CUSUM test, we propose to keep only \( \frac{\Delta C_{mean}}{\beta} \) samples as follows:

\[
C_{mean}^+(p+1) = C^+(\beta \times (p+1) - 1) \quad (15)
\]

Therefore, one has to compare \( \frac{\Delta C_{mean}^+(p)}{\beta} \) to a threshold in order to detect the end of the CR-NBI in the PU-OFDMA symbol. When \( \frac{\Delta C_{mean}^+(p)}{\beta} \) is higher than the threshold, the CR-NBI is assumed to be present. Otherwise, no CR-NBI is assumed to be present. Concerning the choice of the threshold, we suggest the one defined in subsection 3.1.

4. SIMULATION PROTOCOL AND RESULTS

Simulation protocol: We consider a PU-OFDMA WirelessMAN system, which is composed of \( U = 4 \) users sharing \( K_{pu} = 512 \) subcarriers and with cyclic prefix \( N_s = K_{pu}/32 \geq L_u \). QPSK is used to modulate the information bits. The carrier frequency is at \( f_c = 2.5 \) GHz and the channel bandwidth is set to \( W_{pu} = 5 \) MHz. A CR-NBI affects \( K_{BI} = 40 \) subcarriers of the PU-OFDMA system bandwidth. We suppose a transmission over a normalized Rayleigh quasi-static frequency selective channel composed of \( L_{pu}^c = 3 \) multipaths. In addition \( \alpha = 1/2 \) which means that there is CR-NBI during 50% of the OFDMA symbol in the time domain. We define the signal-to-noise-ratio and the signal-to-interference-ratio as \( \text{SNR} = 10 \log(\frac{p^2}{\sigma^2}) \) and \( \text{SIR} = 10 \log(\frac{p^2}{\sigma^2}) \), respectively. For comparison reasons with [7], the CR-NBI is supposed to affect only the subcarriers assigned to one user and that the multiple access interference has been corrected.

First of all, we show the detection performances of the algorithms. For high values of SIR, as \( ||\hat{Y}(n)||^2 \gg 0 \) during the CR-NBI, its detection can be easy. Therefore we study the case when \( \text{SNR} = -10 \) dB and \( \text{SIR} = 0 \) dB, which corresponds to a "difficult" environment for the proposed algorithms. The CR-NBI can begin anywhere in the PU-OFDMA received symbol.

Simulation results: Figure 3 shows the CR-NBI probability of detection \( P_{fa}^C = P(||\hat{Y}(n)||^2 > \lambda(n)|H_1) \) in the PU-OFDMA symbol, for different values of \( P_{fa} \). Better performances are obtained when the BHT is used. Figure 4 shows the PU probability of detection \( P_{fa}^P = P(||\hat{Y}(n)||^2 > \lambda(n)|H_1) \). \( P_{fa}^P \) is the probability to obtain the values that are not disturbed by the CR-NBI and that are going to be used in the estimation task. Better performances are obtained with the CUSUM test. We can see that the detection is improved when using \( \beta = 4 \). The choice of an appropriate \( P_{fa} \).

By simulations we can note that \( \beta = 4 \) is good trade-off to continue on-line and have enough samples to perform the estimation.
is a key point in the development of the algorithms. In the following we chose a value of $P_{fa} = 0.05$, considering that it is a good trade-off between $P_{pu}$ and $P_{fa}$. Figure 5 shows the CFO estimation performances in terms of minimum mean square error (MMSE). In comparison with [7], an improvement of 3 dB is obtained using the SPKF-CT and 10 dB is obtained when using the improved SPKF-BHT using $\beta = 4$. Table 1 shows the estimation performances in terms of MMSE for different values of SIR. It should be noted that the proposed algorithms are not disturbed by the power of the CR-NBI.

**Remark 3:** Due to the limit of space, channel estimation results are not shown.

**Table 1.** CFO estimation performances, SNR = 0dB, $P_{fa} = 0.05$.

<table>
<thead>
<tr>
<th>SIR</th>
<th>-30dB</th>
<th>-20dB</th>
<th>-10dB</th>
<th>0dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMKF</td>
<td>0.43520</td>
<td>0.08150</td>
<td>0.00058</td>
<td>0.00172</td>
</tr>
<tr>
<td>SMKF-CT</td>
<td>$\beta = 4$</td>
<td>0.00066</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMKF-BHT</td>
<td>$\beta = 4$</td>
<td>0.00066</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**5. CONCLUSIONS**

In this paper, two algorithms are proposed to estimate the CFOs and the channels of different users in a PU-OFDMA system disturbed by a CR-NBI. Using an SPKF and without increasing the computational complexity, we take advantage of the innovation to detect a sharp variation of the measurement noise covariance matrix when there is CR-NBI. The estimation results of our SPKF outperforms the method proposed by Morelli. In the future, we propose to study the maximum percentage of time and frequency that the CR-NBI could cover in the PU-OFDMA symbol without affecting much the estimation performances. In addition, we propose to study an estimation of the variance jump, to use the complete CUSUM test to detect the CR-NBI.

**6. REFERENCES**


