How much feedback overhead is required for base station cooperative transmission to outperform non-cooperative transmission?

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1. INTRODUCTION

Base station (BS) cooperative transmission, which is also known as coordinated multi-point transmission (CoMP) in Long Term Evolution Advanced (LTE-A), is an effective way to avoid inter-cell interference in universal frequency reuse cellular systems [1].

Coherent cooperative transmission provides the full benefit of CoMP systems, when both data and channel state information (CSI) can be perfectly obtained at a central unit (CU). However, various overhead is introduced to frequency division duplexing (FDD) systems in order to achieve the promising performance gain, which includes the training overhead for estimating the downlink multi-cell CSI [2] and the feedback overhead for reporting the CSI from users to coordinated BSs. Non-perfect channel estimation, channel quantization and outdated CSI all lead to imperfect CSI, which will reduce the performance gain of cooperative transmission dramatically [3].

In this paper, we study the impact of CSI quantization error on downlink multi-user multiple-antenna CoMP systems. We will compare the performance of CoMP and NonCoMP systems when zero-forcing beamforming (ZFBF) is employed with quantized CSI. To quantify how many feedback bits are required for CoMP transmission to outperform NonCoMP transmission, we derive an optimal bit allocation among local and cross channels of each user. The quantization of multi-cell channels is based on per-cell codebook [4] and independent codeword selection [5], which is not only flexible but also of low complexity for practical application. Different from the bit allocation for distributed cooperative transmission [6], which only depends on the large scale fading gains of local and cross channels of the desired user, our bit allocation depends on the large scale fading gains of the co-scheduled users as well.

2. SYSTEM MODEL

Consider a cellular system with $N$ cells. Each cell contains only one BS with $n_k$ antennas and serves one single-antenna mobile station (MS). We assume that the channel information from the coordinated $N$ BSs to the $N$ MSs are forwarded to the CU via ideal backhaul links, and multi-user precoding is applied for coherent cooperative transmission.

An example of the CoMP system is shown in Fig. 1. The global channel vector from all cooperative BSs to MS$_k$ is represented by

$$g_k = [\alpha_{k1} h_{k1}, \ldots, \alpha_{kN} h_{kN}],$$

(1)

where $\alpha_{kb}$ and $h_{kb} \in \mathbb{C}^{1 \times n_t}$ are respectively the large scale fading coefficient and the small scale fading channel vector of channel between BS$_b$ and MS$_k$, the entries of $h_{kb}$ are assumed to be independent and identically distributed (i.i.d.) unit variance complex Gaussian variables, $\alpha_{kb} h_{kb}$ is the composite local channel of MS$_k$. The ${\mathbf{h}_{kb}}$ for $b \neq k$ are its composite cross channels.

The data to be transmitted to all $N$ MSs is $d = [d_1, \ldots, d_N]^T$. Without loss of generality, we assume that $E(\mathbf{d} \mathbf{d}^H) = I$ and power are equally allocated among MSs. The receive signal at MS$_k$ is

$$y_k = \sqrt{P} g_k v_k d_k + \sqrt{P} \sum_{j=1, j \neq k}^{N} g_k v_j d_j + n_k,$$

(2)

where $P$ is the transmit power for each MS, $v_j \in \mathbb{C}^{N_t \times 1}$ represents the precoding vector from all cooperative BSs for MS$_j$, and $n_k$ is additive white Gaussian noise with zero mean and variance $\sigma^2$. 

2.1. Finite Rate Feedback Model

We assume that MS$_k$ has perfect knowledge of the global channel vector $g_k$. Considering that the number of cooperative BSs in CoMP systems may be dynamic [4], we employ per-cell codebook to quantize $g_k$. The MS$_k$ separately quantizes the channel vector $h_{kb}$, $b = 1, \ldots, N$, then feeds back these quantized per-cell channels BS$_b$. BS$_k$ forward the information to the CU, who finally re-
constructs the global channel of MS\(k\) [5].

Denote the instantaneous channel direction information (CDI) of channel between MS\(k\) and BSs as \(\hat{h}_{k,b} \equiv \text{Re}(\hat{h}_{k,b})/|\hat{h}_{k,b}|\). Denote the codebook for quantizing the CDI as \(C_{k,b}\), which consists of unit norm row vectors \(c_{k,b} = \{1, \cdots, 2^{b_{k,b}}, B_{k,b}\}\) is the number of bits used to quantize \(\hat{h}_{k,b}\). Then the quantized CDI is \(\tilde{h}_{k,b} = c_{k,b}^{\text{arg} \max_{1 \leq j \leq 2^{b_{k,b}}} |\hat{h}_{k,b}c_{k,b}^{H}|^2}\).

Define \(\sin^2 \theta_{k,b} = 1 - |\hat{h}_{k,b}c_{k,b}^{H}|^2\) and \(\hat{\sigma}^2_{k,b} = \mathbb{E}[\sin^2 \theta_{k,b}]\) to reflect the instantaneous and average per-cell channel quantization errors. We consider random vector quantization (RVQ) to quantize the per-cell CDI, then \(\hat{\sigma}^2_{k,b} < 2 \pi^{-1}\) [7].

After MS\(k\) quantizes the CDI for both local channels, it feeds back the indices \(\{i_{k,1}, \cdots, i_{k,N}\}\) to its serving BS, i.e., BS\(k\), which requires \(B_{k,\text{sum}} = \sum_{i=1}^{N} B_{k,i}\) bits in total. We assume that the large scale fading gains are perfectly fed back to BSs and the instantaneous norm of per-cell channel are not fed back.\(^2\) Then all BS\(s\) send the channel information to the CU, and the CU reconstructs the global channel of MS\(k\) as:

\[
\tilde{g}_{k} = [\hat{g}_{k,1}, \tilde{h}_{k,1}, \cdots, \hat{g}_{k,N}, \tilde{h}_{k,N}].
\]

2.2. Multicell Zero-forcing Beamforming

For CoMP transmission, the problem of zero-forcing beamforming subject to a per-BS power constraint is non-trivial, whose performance is hard to analyze. In this paper, we consider a suboptimal but more tractable precoder, which is obtained by normalizing the columns of the pseudo-inverse of the channel matrix. Denote \(G = [\hat{g}_{1}, \cdots, \hat{g}_{N}]\) and \(V = \hat{G}^{H} (\hat{G}^{H} \hat{G})^{-1}\), then the beamforming vector of all cooperative BSs for MS\(k\) is \(v_{k} = V(:,k)/\|V(:,k)\|\).

We consider per-user power constraint (PUC) for both CoMP and NonCoMP transmission. This indicates that the signal power transmitted from BS\(k\) to MS\(k\) under NonCoMP transmission is the same as the sum power of signals transmitted from all coordinated BSs to MS\(k\) under CoMP transmission.

3. PERFORMANCE COMPARISON BETWEEN COMP AND NONCOMP SYSTEMS

In this section, we will compare the performance of CoMP system with NonCoMP system when quantized CSI is applied for ZFBF. Although average throughput is the most convincing metric, its explicit expression is hard to derive for CoMP system when channel quantization errors are taken into account.

For mathematical tractability, we assume that the selected MSs are mutually orthogonal in terms of their quantized global channels as in [8]. In practice, this assumption will hold when the number of MSs is large enough. We will verify through simulation that the following analysis is applicable for more realistic cellular scenarios without the assumption. Under the orthogonal scheduling assumption, the beamforming vector for MS\(k\) reduces to \(v_{k} = \tilde{g}_{k}^{H}/\|\tilde{g}_{k}\|\).

To understand the behavior of CoMP and NonCoMP transmission under limited feedback, we will separately analyze the average signal power and interference power in the following.

3.1. Average Signal Power

The average signal power received by MS\(k\) under NonCoMP transmission can be derived as

\[
\mathbb{E}[S_{k}^{\text{NC}}] = \mathbb{P}\left(\left|\hat{h}_{k,b}v_{k,b}\right|^{2}\right) = \mathbb{P}_{\beta_{k,b}} \mathbb{E}\left(\left|\tilde{h}_{k,b}\tilde{h}_{k,b}^{H}\right|^{2}\right)
\]

\[
= P_{\beta_{k,b}} \alpha_{k,b}^{2} (1 - \hat{\sigma}_{k,b}^{2}),
\]

where \((\alpha)\) is obtained by considering that each BS serves only one MS using ZFBF,\(^3\) which results in \(V_{k}^{\text{NC}} = \sqrt{\mathbb{P}_{\beta_{k,b}} P_{\beta_{k,b}} \mathbb{E}\left(\left|\tilde{h}_{k,b}\tilde{h}_{k,b}^{H}\right|^{2}\right)}\), \(\hat{\sigma}_{k,b}^{2}\) is the average per-cell channel quantization error of NonCoMP system.

By applying orthogonality assumption and averaging over both small scale fading channels and quantization errors, we can derive the averaged signal power of MS\(k\) under CoMP transmission as

\[
\mathbb{E}[S_{k}^{\text{CoMP}}] = \mathbb{E}[\left|\hat{h}_{k,b}v_{k,b}\right|^{2}] = \mathbb{E}[\left|\tilde{h}_{k,b}\tilde{h}_{k,b}^{H}\right|^{2}]
\]

\[
= P_{\beta_{k,b}} \sum_{b=1}^{B} \frac{\alpha_{k,b}^{2}}{\sum_{j=1}^{N} \alpha_{k,j}^{2}} (1 - \hat{\sigma}_{k,b}^{2}).
\]

We assume that the bits used for quantizing the local channel of CoMP and NonCoMP transmission are identical. When CoMP transmission is considered, here we further assume that the number of bits used to quantize the per-cell CDI of both local channel and cross channels are also identical. Then we have \(\hat{\sigma}_{k,1}^{2} = \cdots = \hat{\sigma}_{k,B}^{2}\). Considering that in general the large scale fading gain of local channel, \(\alpha_{k,k}^{2}\), exceeds that of any cross channels, \(\alpha_{k,j}^{2}\), \(j \neq k\), we can easily obtain that \(\mathbb{E}[S_{k}^{\text{NC}}] < \mathbb{E}[S_{k}^{\text{CoMP}}]\).

Observation 1: The average receive signal power of CoMP transmission is less than that of NonCoMP transmission when the two schemes are working on the following assumptions, 1) the bits for quantizing local channel under CoMP transmission equals to that under NonCoMP transmission, 2) more bits are used to quantize the cross channels for CoMP transmission, and 3) the power transmitted to the MS are identical of CoMP and NonCoMP transmission.

3.2. Average Interference Power

The averaged interference power at MS\(k\) under NonCoMP transmission can be derived as

\[
\mathbb{E}[I_{k}^{\text{NC}}] = \sum_{b=1}^{B} \mathbb{P}_{\beta_{k,b}} \mathbb{E}\left(\left|\hat{h}_{k,b}v_{k,b}\right|^{2}\right) = P_{\beta_{k,b}} \sum_{b=1}^{B} \alpha_{k,b}^{2}.
\]

An upper bound of the average interference power at MS\(k\) under CoMP transmission can be derived as

\[
\mathbb{E}[I_{k}^{\text{CoMP}}] = \mathbb{E}[\sum_{j=1,j \neq k}^{N} \left|\hat{h}_{k,b}v_{j,b}\right|^{2}] = \mathbb{E}[\sum_{j=1,j \neq k}^{N} \frac{1}{\|\tilde{g}_{j}\|^{2}} \left|\tilde{g}_{k}^{H}\right|^{2}]
\]

\[
\leq P_{\beta_{k,b}} \sum_{b=1}^{B} \alpha_{k,b}^{2} \mathbb{E}\left(\left|\tilde{h}_{k,b}\tilde{h}_{k,b}^{H}\right|^{2}\right)
\]

\[
\leq P_{\beta_{k,b}} \sum_{b=1}^{B} \alpha_{k,b}^{2} \mathbb{E}\left(\left|\tilde{h}_{k,b}\tilde{h}_{k,b}^{H}\right|^{2}\right)
\]

\[
\leq P_{\beta_{k,b}} \sum_{b=1}^{B} \alpha_{k,b}^{2} \mathbb{E}\left(\left|\tilde{h}_{k,b}\tilde{h}_{k,b}^{H}\right|^{2}\right)
\]

\[
= P_{\beta_{k,b}} \sum_{b=1}^{B} \alpha_{k,b}^{2} \mathbb{E}\left(\left|\tilde{h}_{k,b}\tilde{h}_{k,b}^{H}\right|^{2}\right),
\]

where \((\beta)\) is obtained by expressing \(\tilde{h}_{k,b}\) as \(\tilde{h}_{k,b} = \cos \theta_{k,b} \hat{h}_{k,b} + \sin \theta_{k,b} s_{k,b}\), where \(s_{k,b}\) is isotropically distributed in the nullspace of \(\hat{h}_{k,b}\), (\(a\)) comes because \(\cos \theta_{k,b}^{2} \leq 1\) and the scheduled MSs are assumed to be mutually orthogonal in terms of the quantized global channel, and (\(c\)) comes by \(\mathbb{E}[\left|s_{k,b}\tilde{h}_{k,b}^{H}\right|^{2}] = \frac{1}{\pi r_{k,b}^{2}}\) [7].

\(^3\)When single user is served by one BS, ZFBF is equivalent to maximum ratio transmission.
Observation 2: The interference power under CoMP transmission not only includes the interference from the cooperative BSs, $I^\text{inter}_k$, but also comprises the interference from the serving BS, $I^\text{intra}_k$, which is caused by signals transmitted from BSs to MSs in the coordinated cells. By contrast, the interference seen by MS$_k$ under NonCoMP transmission only comes from the non-serving BSs. When MS$_k$ moves from cell edge to cell center, its interference under NonCoMP transmission will decrease, and the $I^\text{inter}$ part of CoMP transmission will also reduce if both $\beta_{k,b}$ and $\delta_k^2$ are fixed. However, $I^\text{intra}_k$ will increase, which limits the performance of CoMP transmission.

Note that the value of $\mathbb{E}\{I_k^\text{NC}\}$ not only depends on the large scale fading gain of channels from all BSs to MS$_k$, but also relies on the value of $\beta_{k,b}$, which is associated with the large scale fading gains of channels from all BSs to the co-scheduled MSs of MS$_k$. In the following, we will analyze how many bits are required for CoMP to perform better than NonCoMP transmission by deriving a location based optimal bit allocation that minimizes the sum bits of each MS.

4. OPTIMAL BIT ALLOCATION AMONG LOCAL AND CROSS CHANNELS

To ensure that CoMP system outperforms NonCoMP system, average per-user throughput under CoMP transmission should exceed that under NonCoMP transmission. However, it is very difficult if not impossible to obtain an explicit expression of the average throughput as well as the average signal to interference and noise ratio (SINR). For mathematical tractability, we consider high signal to noise ratio (SNR) scenarios, which is reasonable in the interference-limited CoMP systems. Therefore we can analyze signal to interference ratio (SIR) instead of SINR. Moreover, we approximate the average SIR as $\mathbb{E}\{\text{SIR}_k\} \approx \mathbb{E}\{S_k\} \equiv \mathbb{E}\{\text{SIR}_k\}^{\text{app}}$, which is widely used [9].

We require that $\mathbb{E}\{\text{SIR}_k^{\text{NC}}\}^{\text{app}} \leq \mathbb{E}\{\text{SIR}_k^{\text{C}}\}^{\text{app}}$, which is equivalent to $\mathbb{E}\{I_k^\text{NC}\} \mathbb{E}\{\text{SIR}_k^{\text{NC}}\}^{\text{app}} \leq \mathbb{E}\{I_k^\text{C}\}$. By substituting (5) and (4) into the inequality, moving the quantization errors in the expression of $\mathbb{E}\{S_k^\text{app}\}$ to the left hand side, and replacing the quantization errors with their upper bound, we can obtain the equivalent inequality as

$$
\sum_{b=1}^{N} (1 + \beta_{k,b}v_{k,b}) \frac{\alpha_k}{\delta_k^2} \sum_{b=1}^{N} v_{k,b} \leq \Gamma_k,
$$

where $v_{k,b} = \frac{\alpha_k^2}{\delta_k^2} \sum_{b'=1, b' \neq b}^{N} \alpha_{k,b'} \mathbb{E}\{\text{SIR}_k^{\text{NC}}\}^{\text{app}}$ and $\Gamma_k = \sum_{b=1}^{N} \alpha_k^2$. The solution of the problem can be easily obtained as

$$
B_{k,b} = (m_{t} - 1) \left( \log_2 \lambda - \log_2 \frac{1}{\alpha_{k,b}^2 (1 + \beta_{k,b}v_{k,b})} \right) + \alpha_{k,b}^2 v_{k,b}^{-1},
$$

where $\lambda$ is the Lagrangian multiplier, whose value should be chosen to satisfy (8).

It is interesting to see that the optimal bit allocation is similar to the water-filling power allocation. When the value of $\eta_{k,b} = \alpha_{k,b}^2 (1 + \beta_{k,b}v_{k,b})$ is large, more bits should be allocated to quantize the CDI of channels between MS$_k$ and BS$_b$.

The number of bits, $B_{k,b}$, required for quantizing per-cell CDI $I_{k,b}^{\text{intra}}$ depends not only on the large scale fading gain of this link, i.e., $\alpha_{k,b}^2$, but also on the value of $\beta_{k,b}$, which respectively depend on the position of the MS$_k$ and that of the co-scheduled MSs of MS$_k$. The value of $\beta_{k,b}$ depend on the position of the co-scheduled MSs of MS$_k$. When $\alpha_{j,b}^2, j = 1, \ldots, N, j \neq k$, approaches zero, which is achieved when all the co-scheduled MSs of MS$_k$ are located very far from BS$_k$. $\beta_{k,b}$ approaches 0. In this case, the value of $\eta_{k,b}$ is minimal and little bits need to be allocated for quantizing $I_{k,b}^{\text{intra}}$. On the other hand, when $\alpha_{j,b}^2 = \alpha_{k,b}^2, j = 1, \ldots, N, j \neq k$, $\beta_{k,b}$ approaches its maximal value, which is achieved when all the co-scheduled MSs are located at the edge of cell $k$. $\eta_{k,b}$ reaches its maximal value. In this case, the bits allocated to quantize $I_{k,b}^{\text{intra}}$ should be considerably high.

5. SIMULATION RESULTS

In this section, we will illustrate the minimal bits required for each MS to quantize its global channel, which ensures that CoMP outperforms NonCoMP transmission. In the derivation of the optimal bit allocation, we have assumed orthogonal scheduling and applied non-integer relaxation to the number of bits. We will show that this assumption and relaxation have negligible impact on our analysis.

Although our previous theoretical analysis is applicable for general cases with any number of cooperative BSs, we simulate a simple scenario with two BSs and two single-antenna MSs to clearly observe the results of bit allocation. Each BS is equipped with 4 antennas. The small scale fading channels between BSs and MSs are i.i.d. Rayleigh channels. We only consider path loss in the large scale fading, whose fading factor is 3.76. The codebooks used for quantizing the CDI of per-cell channels are obtained by RVQ. The bits used to quantize the CDI of local channel for NonCoMP transmission are 4 bits.

Figure 2 shows the minimal total bits required by MS$_1$ to quantize its CoMP channel for CoMP outperforming NonCoMP. Both the bits achieved by optimal bit allocation shown in (10) and by the exhaustive searching are illustrated. The left figure is obtained under $\alpha_{k,1}^2/\alpha_{k,2}^2 = -20$dB, i.e., MS$_2$ is located at cell center. The right figure is obtained under $\alpha_{k,1}^2/\alpha_{k,2}^2 = 0$dB, i.e., MS$_2$ is located at the exact cell edge. We can observe that the bit allocation acquired by the two methods are almost the same, except for only 1 bit fluctuation in two cases, which are $\alpha_{k,1}^2/\alpha_{k,1}^2 = 0$dB and $\alpha_{k,2}^2/\alpha_{k,1}^2 = -4$dB under $\alpha_{k,1}^2/\alpha_{k,2}^2 = -20$dB. Comparing the left and the right figure, we can find that the location of the co-scheduled MS, MS$_2$, has large impact on both the required total bits and the bit allocation between local and cross channels of MS$_1$. If $\alpha_{k,1}^2/\alpha_{k,2}^2 = 0$dB, when MS$_1$ moves from cell edge to cell center, more and more bits should be allocated to quantize its local channel, which is necessary to reduce $I_{k}^{\text{intra}}$ in (7). If $\alpha_{k,1}^2/\alpha_{k,2}^2 = -20$dB, $I_{k}^{\text{intra}}$ is quite small because $\beta_{1,1} = \alpha_{k,1}^2/\alpha_{k,1}^2 + \alpha_{k,2}^2 \approx 0.01$. In this case, both CoMP and NonCoMP systems are corrupted only by inter-cell interference. For CoMP transmission, at least 6 bits are required by MS$_1$ to quantize the CoMP channel in order to outperform NonCoMP transmission, which only needs 4 bits for quantizing the CDI of local channel.

Figure 3 illustrates the average throughput of MS$_1$ for different $\alpha_{k,2}^2/\alpha_{k,1}^2$ and different local receive SNRs, which is defined as
In the simulation, the optimal bit allocation is used for each MS. No scheduling are employed, which means that MS$_2$ is always served simultaneously with MS$_1$, no matter whether its quantized global channel is orthogonal to that of MS$_1$, or not. We can observe that the performance of MS$_1$ under CoMP transmission is close to that under NonCoMP transmission. Note that the performance under CoMP and NonCoMP should be identical with the optimal bit allocation under the orthogonality assumption. This shows that our earlier analysis still holds without such an assumption.

Figure 4 illustrates the minimal total bits required by the two MSs for quantizing their global channels when they are respectively located at various positions. It shows that the number of bits used for CoMP transmission (i.e., from 12 bits to 26 bits) should exceed that for NonCoMP transmission (i.e., 8 bits in total) in order to ensure its performance gain. Even in the case of the best user pairing (both users are located at cell edge), CoMP needs more bits (12 bits) than NonCoMP transmission. To avoid large feedback overhead, CoMP with limited feedback does not prefer to serve cell-center and cell-edge MSs simultaneously. This is quite different from the results of distributed CoMP [10], which encourages the cell-center MS to be served together with a cell-edge MS.

### 6. CONCLUSION

In this paper, we analyzed the minimal bits necessary for quantizing the channel information under CoMP transmission to ensure its performance gain over NonCoMP transmission. Our results showed that the number of bits used by each MS to feed back its CoMP channel not only depends on the location of this MS, but also relies on that of its co-scheduled MSs. Even when the most desirable co-scheduled MSs are selected, the number of bits required for CoMP transmission still exceeds that for NonCoMP transmission. This is due to the fact that channel quantization error leads to intra-cell interference except for the inter-cell interference.

### 7. REFERENCES


