ITERATIVE WATER FILLING BASED ON SLNR WITH 1-SHOT 1-BIT FEEDBACK

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ABSTRACT

The paper proposes a subcarrier power allocation method for downlink OFDMA systems. The proposed method utilizes an iterative water-filling (IWF) algorithm with signal-to-leakage-plus-noise ratio (SLNR) rather than conventional signal-to-interference-plus-noise ratio (SINR), which enables us to considerably reduce the overhead. Computer simulation results show that the proposed IWF based on SLNR with 1-shot 1-bit feedback per subcarrier can achieve very close sum-rate performance to the conventional IWF based on SINR, while only replacing SINR to SLNR in the IWF algorithm results in poor performance.

Index Terms—OFDMA, power allocation, iterative water-filling, SLNR

1. INTRODUCTION

Co-channel interference is one of the serious limiting factors for cellular mobile communications systems, especially when frequency reuse factor is set to be 1. OFDMA (Orthogonal Frequency Division Multiple Access) scheme can reduce the impact of the interference by adequately allocating subcarriers (and/or power of them) to each user depending on channel conditions. Iterative water-filling (WF) algorithm is one of the major power allocation methods for downlink OFDMA systems[1],[2], which is based on the idea of water-filling (WF) theorem for parallel Gaussian channels[3]. IWF requires the information on signal-to-interference-plus-noise ratio (SINR) at the desired mobile terminal (MT) for each subcarrier in order to regard the co-channel interference as noise in the WF algorithm. The iterative process is essential because changing power allocation at some base station (BS) results in the change of interference power at the MTs in the other cells. Although it is heuristically known that the IWF converges to a good solution when the interference power is small[1], the actual signal transmission from BS and feedback of SINR from MT, which are required for each iteration, can be a huge overhead for practical mobile communications systems.

Recently, signal-to-leakage-plus-noise ratio (SLNR) has been proposed for the transmit beamforming[4], which is basically the same idea as virtual uplink in [5], and it is proved that, in the transmit beamforming, Nash equilibrium of the non-cooperative game based on SLNR corresponds to Pareto optimal point of the non-cooperative game with SINR[6]. The virtue of SLNR metric in downlink scenario is that it can be calculated only by locally available information, if we can assume a symmetric channel for up- and downlink.

In this paper, we propose a novel power allocation method for downlink OFDMA systems by introducing SLNR to the IWF algorithm in order to reduce the overhead. Since the simple replacement of SINR to SLNR in the IWF algorithm results in poor performance, as we will see in the numerical results, we propose to use 1-shot 1-bit feedback per subcarrier from the MT based on the received signal from the BS. Computer simulation results show that the proposed IWF based on SLNR with 1-shot 1-bit feedback can achieve very close performance to the conventional IWF based on SINR, while reducing the overhead significantly.

2. SYSTEM MODEL AND CONVENTIONAL POWER ALLOCATION METHOD

2.1. System model

Consider a downlink of OFDMA system with $N$ pairs of BS and MT. Assuming one MT for each cell, the same index $i$ is assigned to the BS and the MT in the same cell. For simplicity, we assume each BS communicates with a desired MT using all subcarriers, therefore, for each MT, one BS out of $N$ BSs is a desired BS, while the signals from the rest of $N-1$ BSs are considered as interference.

Let $s^m_i$, $m \in \{0, \ldots, M-1\}$ denote the signal transmitted from the $i$-th BS to the $i$-th MT on the $m$-th subcarrier, where $M$ is the number of subcarriers. Assuming the length of the guard interval is greater than or equal to the channel order, the received signal at the $j$-th MT on the $m$-th subcarrier is
written as

\[ t_j^n = \sum_{i=1}^{N} \sqrt{P_i^m} \lambda_{ji}^m s_i^n + n_j^m, \]  

(1)

where \( n_j^m \) denotes a zero mean additive white Gaussian noise (AWGN) at the \( j \)-th MT on the \( m \)-th subcarrier with variance \( \sigma_n^2 \) and \( \lambda_{ji}^m \) is a frequency response on the \( m \)-th subcarrier between the \( i \)-th BS and the \( j \)-th MT. \( P_i^m \) denotes the transmitted signal power of the \( i \)-th BS on the \( m \)-th subcarrier, and we consider the total power constraint as

\[ P = \sum_{m=0}^{M-1} P_j^m. \]  

(2)

For the \( j \)-th MT, only the signal from the \( j \)-th BS is desired, and the signals from the other BSs result in the interference, therefore, the SINR at the \( j \)-th MT on the \( m \)-th subcarrier is given by

\[ \Gamma_j^m = \frac{P_j^m |\lambda_{jj}^m|^2}{\sum_{i \neq j}^{N} P_i^m |\lambda_{ji}^m|^2 + \sigma_n^2}. \]  

(3)

\subsection{2.2. Conventional power allocation method}

IWF algorithm [1],[2] has been proposed in order to determine the transmit power of each subcarrier \( P_i^m \) in a distributed manner. IWF is based on the idea of well-known WF theorem [3], which gives optimum power allocation in a parallel Gaussian channel. By replacing noise power in WF with interference plus noise power, the idea of WF can be applied for the downlink OFDMA scenario as shown in Fig. 1, where

\[ X_j^m(n) = \frac{\sum_{i \neq j}^{N} P_i^m(n) |\lambda_{ji}^m|^2 + \sigma_n^2}{|\lambda_{jj}^m|^2}, \]  

(4)

is the interference plus noise power normalized by the frequency response on the subcarrier in the \( n \)-th iteration. Note that, unlike the WF in a parallel Gaussian channel, iterative approach will be required for the case of downlink OFDMA system, because the interference plus noise power (i.e. \( X_j^m(n) \)) changes, if some BS changes the power allocation. Moreover, it requires actual signal transmission and feedback of observed SINR (or \( X_j^m(n) \)) at the MT for each iteration. The allocated power to the \( m \)-th subcarrier at the \( n \)-th iteration is given by \( R(n) - X_j^m(n) \), where \( R(n) \) is called water level and is determined by the total power budget \( P \). If \( R(n) - X_j^m(n) \) is negative, no power will be allocated to the subcarrier. More specifically, denoting the number of subcarriers having greater \( X_j^m(n) \) than \( R(n) \) as \( K \), the algorithm in the \( n \)-th iteration of the conventional IWF based on SINR is given as follows:

\begin{itemize}
  \item Set \( K = 0 \), \( R(n) = \frac{P + \sum_{m} X_j^m(n)}{M} \).
  \item If \( R(n) > \max_{m} X_j^m(n) \) then \( \forall m, P_j^m(n+1) = R(n) - X_j^m(n) \) and break, otherwise go to 3.
  \item Define a set \( A = \{ m | R(n) \leq X_j^m(n) \} \) and \( K = \text{Card}(A) \), where Card denotes the number of elements in the set. Modify \( R(n) \) as \( R(n) = \frac{P + \sum_{m \notin A} X_j^m(n)}{M - K} \), and if \( R(n) > \max_{m} X_j^m(n) \) then go to 4, otherwise go to 3.
  \item \( P_j^m(n+1) = \begin{cases} R(n) - X_j^m(n) & (m \notin A) \\ 0 & (m \in A) \end{cases} \)
\end{itemize}

\section{PROPOSED POWER ALLOCATION METHOD}

Here, we consider to introduce the SLNR metric into the conventional IWF algorithm.

\subsection{3.1. SLNR}

The SLNR of the \( j \)-th BS is defined as the ratio of the received signal power from the BS at the desired MT (the \( j \)-th MT) to the received signal power at all the MTs in the other cells plus noise power. To be specific, the SLNR of the \( j \)-th BS on the \( m \)-th subcarrier is defined as

\[ \tilde{\Gamma}_j^m = \frac{P_j^m |\lambda_{jj}^m|^2}{\sum_{i \neq j}^{N} P_i^m |\lambda_{ji}^m|^2 + \sigma_n^2}. \]  

(5)

It should be noted that SLNR is defined for the transmitter side, while SINR is defined at the receiving side in general.

\subsection{3.2. IWF based on SLNR}

It is heuristically known that IWF algorithm converges to a solution, which gives good sum-rate performance, if the interference power is smaller than a certain level. However,
the overhead of actual signal transmissions and feedbacks of
the SINR can be serious problem in practical implementation.
Here, we try to utilize SLNR instead of SINR in the IWF alg-

Defining the sum of the leakage and noise power of the j-

we replace \(X_j^m(n)\) to \(\hat{X}_j^m(n)\) in the conventional IWF as follows:

**IWF based on SLNR**

1. Set \(K = 0\), \(\hat{R}(n) = \frac{P}{M} + \frac{\sum_{m=1}^{M} \hat{X}_j^m(n)}{M} \).

2. If \(\hat{R}(n) > \max \hat{X}_j^m(n)\) then \(\forall m, P^m_m = \hat{R}(n) - \hat{X}_j^m(n)\) and exit, otherwise go to 3.

3. Define a set \(A = \{m | \hat{R}(n) \leq \hat{X}_j^m(n)\}\)

   and \(K = \text{Card}(A)\). Modify \(\hat{R}(n)\) as

   \[
   \hat{R}(n) = \frac{P + \sum_{m \notin A} \hat{X}_j^m(n)}{M - K},
   \]

   if \(\hat{R}(n) > \max_{m \notin A} \hat{X}_j^m(n)\) then go to 4, otherwise go to 3.

4. \(P^m_m(n + 1) = \begin{cases} 
\hat{R}(n) - \hat{X}_j^m(n) & (m \notin A) \\
0 & (m \in A)
\end{cases}\)  

4. NUMERICAL RESULTS

4.1. System parameters

Computer simulations are conducted to evaluate the perform-

Key issue here is that, if we assume the symmetric channel in up-

3.3. IWF based on SLNR with 1-shot 1-bit feedback

Although the IWF based on SLNR does not require any actual

Here, we consider to improve the performance of the IWF

4.2. Sum-rate performance

We have evaluated sum-rate per subcarrier achieved by the

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We have evaluated sum-rate per subcarrier achieved by the

\[
\sum_{m=1}^{M} \sum_{n=0}^{N-1} \log_2(1 + \lambda_j^m(n)),
\]

where \(\lambda_j^m\) is the observed SINR at the j-th user on the m-th

Figure 2 shows the sum-rate performance of the algo-

From the performance of SLNR+FB, SLNR+FB2 and Equal+FB,
we can see that the 1-shot 1-bit feedback can improve the

especially, the proposed
SLNR+FB and SLNR+FB2 can achieve almost the same performance as that of SINR-IWF, while the proposed methods require much lower overhead. Furthermore, we can see by comparing the performance of SLNR+FB and SLNR+FB2 that utilization of power allocation obtained by SLNR-IWF for the initial observation of the SINR instead of equal power allocation can slightly improve the performance.

4.3. Examples of allocated power

Finally, Fig. 3 shows typical examples of the achieved power allocations by SINR-IWF, SLNR-IWF, SLNR-FB and SLNR-FB2. The same channel realization is used for all cases. From the figure, we can recognize the tendency that, if one BS has large power on a certain subcarrier, the transmit power of the other BS on the subcarrier is low for all the algorithms. This means that the automatic compartmentalization is achieved not only by the conventional IWF but also proposed algorithms based on SLNR. Moreover, the achieved power allocations of SINR-IWF, SLNR+FB and SLNR+FB2 are very similar, which justifies their close sum-rate performance.

5. CONCLUSION

We have considered power allocation for downlink OFDMA system with co-channel interference. Assuming symmetric channel for the up- and downlink, the utilization of SLNR instead of SINR for the IWF algorithm is proposed, which can significantly reduce the overhead. Computer simulation results show that replacing SINR to SLNR only in the conventional IWF results in poor performance, while 1-shot 1-bit feedback per subcarrier can considerably improve the performance. Moreover, the combination of the IWF based on SLNR and the 1-shot 1-bit feedback can achieve close performance to the conventional IWF based on SINR with much reduced overhead.

6. REFERENCES