ITERATIVE FDE FOR ASYNCHRONOUS SINGLE-CARRIER MULTIUSER SYSTEMS

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ABSTRACT

When the arrival timing difference exists in the uplink MU-MIMO systems, the conventional MUD techniques, without timing control, failed under this circumstance. In this paper, we proposed a joint distortion precancellation and iterative FDE architecture, without timing control, for the asynchronous system. The proposed method derives the iterative FDE filters, via the MMSE criterion, by treating the precanceled distortion components as colored noise. The asymptomatic MSE performance is shown to approach MFB, and the achievable SER performance is evaluated in various system settings, which confirms its effectiveness.

Index Terms— multiuser detection, multiuser MIMO, iterative FDE, arrival timing difference

1. INTRODUCTION

Single-carrier frequency-domain equalization (SC-FDE) has received much attention for its pragmatic advantages [1], such as low peak-to-average ratio (PAR) and insensitivity to the carrier frequency offset (CFO). The multiuser multiple-input multiple-output (MU-MIMO) technique is an attractive candidate to provide extremely high capacity for multiuser wireless communications [2]. In this paper, we focus on uplink MU-MIMO systems with SC-FDE, where single antenna is deployed at each user terminal, and antenna array is deployed at the receiver site.

In the uplink, the arrival timing differences at the receiving array among the signal streams from multiple users caused by the differences in the geographical distance between the array and each user represent a significant problem. Furthermore, when the transmit timing control is imperfect or not used, and all the users asynchronously transmit their signals, the arrival timing differences are uniformly distributed and exceed the CP period. A asynchronous multiuser detection (MUD) method has been proposed in [3], which performed with overlapping of the fast Fourier transform (FFT) blocks. However, the linear MMSE filter settings in [3], which is for a low complexity consideration, limits the MUD performance, especially in severely frequency selective MU-MIMO scenarios.

In this paper, we proposed a joint distortion-cancelation and iterative FDE architecture to further enhance the asynchronous MUD performance. The proposed method performs in frequency domain and does not require transmit timing control and a CP. Since the interference components due to the asynchronous MU-MIMO system settings cannot be regarded as white noise, the conventional iterative FDE [4]-[5] developed for synchronous single-input single-output (SISO) systems cannot directly be applied to the proposed MUD method. Here, we treat the interference components as colored noise and derive the MUD weights via the constrained optimization of an MSE cost function.

2. SYSTEM MODEL

Assuming relative delays exist between the reference user, which is the first one arrives at the receiving array, and the other users, we index the user \( j \) with respect to its relative delay \( \tau_j \), in which \( \tau_1 = 0 \) for the reference user, w.l.o.g.

The received block of \( M \) symbols at the \( i \)-th antenna is

\[
y_{\text{k};\text{k}+M-1}^i = \sum_{j=1}^{N_f} H_{i,j}^{\text{k},\text{k}+M-1} x_{\text{k}-\nu-\tau_j;\text{k}+M-1}^j + n_{k;\text{k}+M-1}^i, \quad (1)
\]

where \( H_{i,j}^{k,M-1} \) is a Toeplitz channel matrix of size \( M \times (M + \nu + \tau_j) \), with the first row \( h_{i,j}^{\text{k},\text{k}+M-1} = [h_{0,j}^{\text{k},\text{k}+M-1}, \ldots, h_{M+j}^{\text{k},\text{k}+M-1}] \), and \( M = N_f + \nu + \tau_{\max} \). \( N_f \) and \( \nu \) denote transmitted block size and channel excessive length, respectively, and \( \tau_{\max} = \max\{\tau_1, \tau_2, \ldots, \tau_{N_f}\} \). It can be shown that the received signal block consists of three components

\[
y_{\text{k};\text{k}+M-1}^i = \sum_{j=1}^{N_f} H_{i,j}^{\text{k},\text{k}+M-1} x_{\text{k}-\nu-\tau_j;\text{k}+M-1}^j + n_{k;\text{k}+M-1}^i + \sum_{j=1}^{N_f} H_{\text{tail}}^{i,j} x_{\text{k}-\nu-\tau_j;\text{k}-1}^j + \sum_{j=1}^{N_f} H_{\text{head}}^{i,j} x_{\text{k}+N_f+\tau_j;\text{k}+M-\tau_j-1}^j.
\]

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where $H_{\text{circ}}^{i,j}$ is a circulant channel matrix of size $M \times M$, with the first column $h_{i,j}^{0} = [h_{i,j}^{0}, \ldots, h_{i,j}^{N-1}]^T$, $x_{k}^j$ of size $M \times 1$ denotes the augmented signal vector transmitted from user $j$ at the $k$-th block, and $n_{k}^{i:k+M-1}$ of size $M \times 1$ denotes the noise vector at the $k$-th block of the $i$-th receive antenna, respectively.

$$x_{k}^j = \left[ 0_{T_{1} \times 1}^T x_{k:k+N}^{j:j-1}^T 0_{T_{N_{\text{max}}-T_{j}} \times 1}^T \right]^T. \tag{3}$$

$H_{\text{tail}}^{i,j}$ and $x_{k}^{j:k-\nu-\tau_{j}-1}^j$ are the channel matrix and signal vector of the interference components from preceding (succeeding) symbols, between user $j$ and the $i$-th receive antenna, respectively.

$$H_{\text{tail}}^{i,j} = \begin{bmatrix} h_{i,j}^{0} & \ldots & h_{i,j}^{N-1} & 0 & \ldots & 0 \\ 0 & \ldots & h_{i,j}^{0} & 0 & \ldots & 0 \\ \vdots & \ddots & \vdots & \ddots & \ddots & \ddots \\ 0 & \ldots & 0 & h_{i,j}^{0} & \ldots & 0 \end{bmatrix} \tag{4}$$

and

$$H_{\text{head}}^{i,j} = \begin{bmatrix} h_{i,j}^{0} & 0 & \ldots & \ldots & \ldots & 0 \\ h_{i,j}^{1} & h_{i,j}^{0} & 0 & \ldots & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\ \vdots & \vdots & \ddots & 0 & \ddots & \ddots \\ 0 & \ldots & 0 & h_{i,j}^{0} & \ldots & h_{i,j}^{N-1} \\ 0 & \ldots & h_{i,j}^{0} & \ldots & h_{i,j}^{N-1} & h_{i,j}^{0} \end{bmatrix} \tag{5}$$

where $o_{\text{tail}}$ is of size $(N_f + \tau_{\text{max}} - \tau_{j}) \times (\nu + \tau_{j})$, and $o_{\text{head}}$ is of size $(N_f + \nu + \tau_{j}) \times (\nu + \tau_{\text{max}} - \tau_{j})$.

Since $\{H_{\text{circ}}^{i,j}\}$ are circulant matrices, we convert the received signals at $N_R$ antennas into frequency-domain using $M$-point FFT operations. The frequency response of the $N_R \times 1$ received vector signal at the $m$-th tone is

$$\mathbf{y}_m = \mathbf{H}_m \mathbf{x}_m + \mathbf{N}_m + \mathbf{H}_{\text{tail}}^m \mathbf{x}_{\text{tail}} + \mathbf{H}_{\text{head}}^m \mathbf{x}_{\text{head}}, \tag{6}$$

where

$$\mathbf{H}_m = \begin{bmatrix} f(m)^T \mathbf{H}_{\text{tail}}^1 & \ldots & f(m)^T \mathbf{H}_{\text{tail}}^{N_f} \\ \vdots & \ddots & \vdots \\ f(m)^T \mathbf{H}_{\text{head}}^1 & \ldots & f(m)^T \mathbf{H}_{\text{head}}^{N_f} \end{bmatrix} \tag{7}$$

$$\mathbf{H}_{\text{tail}}^m = \begin{bmatrix} f(m)^T \mathbf{H}_{\text{tail}}^1 & \ldots & f(m)^T \mathbf{H}_{\text{tail}}^{N_f} \\ \vdots & \ddots & \vdots \\ f(m)^T \mathbf{H}_{\text{tail}}^1 & \ldots & f(m)^T \mathbf{H}_{\text{tail}}^{N_f} \end{bmatrix} \tag{8}$$

$$\mathbf{H}_{\text{head}}^m = \begin{bmatrix} f(m)^T \mathbf{H}_{\text{head}}^1 & \ldots & f(m)^T \mathbf{H}_{\text{head}}^{N_f} \\ \vdots & \ddots & \vdots \\ f(m)^T \mathbf{H}_{\text{head}}^1 & \ldots & f(m)^T \mathbf{H}_{\text{head}}^{N_f} \end{bmatrix} \tag{9}$$

and

$$\mathbf{x}_{\text{tail}} = \left[ x_{k}:^{(1)}_{k-\nu-\tau_{j}+1} \cdots x_{k}^{(N_f)} \right]^T \tag{10}$$

$$\mathbf{x}_{\text{head}} = \left[ x_{k+1}^{(1)} \cdots x_{k+N_f-1}^{(1)} \right]^T \tag{11}$$

The $M$-point Fourier transform vector $f(m)$ is defined as

$$f(m) = \frac{1}{\sqrt{M}} \left[ 1 \exp(-j2\pi m/M) \cdots \exp(-j2\pi (M-1)m/M) \right]^T.$$  

\section{3. ASYNCHRONOUS ITERATIVE MUD FRAMEWORK}

In this section, we introduce a distortion pre-cancelation technique, which suppresses the interference component with its raw estimate in an iterative manner. The frequency response of the corresponding pre-canceled vector signal at the $n$-th pre-cancelation stage is

$$\tilde{\mathbf{y}}_n = \mathbf{H}_m \mathbf{x}_m + \mathbf{N}_m + \mathbf{H}_{\text{tail}}^n \Delta_{\text{tail}}^n + \mathbf{H}_{\text{head}}^n \Delta_{\text{head}}^{n-1}, \tag{11}$$

where the residual distortions from interference cancelation are defined as

$$\Delta_{\text{tail}}^n = \mathbf{x}_{\text{tail}} - \tilde{\mathbf{x}}_{\text{tail}}^n \tag{12}$$

$$\Delta_{\text{head}}^{n-1} = \mathbf{x}_{\text{head}} - \tilde{\mathbf{x}}_{\text{head}}^{n-1} \tag{13}$$

$\tilde{\mathbf{x}}_{\text{tail}}^n$ and $\tilde{\mathbf{x}}_{\text{head}}^{n-1}$ denote the raw estimate of the preceding and succeeding interference components, respectively.

We further integrate iterative FDE with the distortion pre-cancelation front-end, and develop a joint cancelation and equalization architecture for asynchronous MUD. Denoting the $m$-th frequency response of feedforward and feedback vector filters corresponding to the $j$-th user, at the $l$-th pass, as $\mathbf{F}_m^{(j)}$ and $\mathbf{B}_m^{(j)}$, the $m$-th frequency response of user $j$ at the iterative FDE output prior to the decision element is

$$\tilde{\mathbf{x}}_{\text{m}}^{(j)} = \mathbf{F}_m^{(j)} \tilde{\mathbf{y}}_n - \mathbf{B}_m^{(j)} \tilde{\mathbf{x}}_{\text{m}}^{(j)-1}, \tag{14}$$

where $\tilde{\mathbf{x}}_{\text{m}}^{(j)-1}$ is the $m$-th frequency response of the decision output from all users at previous pass.

The iterative FDE filters in (14) are jointly designed via constrained minimization of the MSE cost function, which stated as

$$\text{minimize} \quad \text{MSE}_{m}^{(j)}(\mathbf{F}_m^{(j)}, \mathbf{B}_m^{(j)}) \tag{15}$$

subject to $\frac{1}{M} \sum_{m=0}^{M-1} \mathbf{B}_{m,j}^{(j)} = 0 \tag{16}$

where $\text{MSE}_{m}^{(j)}$ denotes the tone-wise MSE corresponding to user $j$

$$\text{MSE}_{m}^{(j)}(\mathbf{F}_m^{(j)}, \mathbf{B}_m^{(j)}) = \mathbb{E}[|\tilde{\mathbf{x}}_{\text{m}}^{(j)} - \mathbf{x}_{\text{m}}^{(j)}|^2], \tag{17}$$

and the constraint on $\mathbf{B}_{m,j}^{(j)}$ ensures joint ISI and MAI suppression with respect to user $j$.

\subsection{3.1. Filter Design}

Consider the linear constraint on $\mathbf{B}_{m,j}^{(j)}$, we construct a Lagrangian function from (15) and (16)

$$\mathcal{J}_{j}(\mathbf{F}_m^{(j)}, \mathbf{B}_m^{(j)}, \mathbf{\lambda}_j^*) = \mathbb{E}[|\tilde{\mathbf{x}}_{\text{m}}^{(j)} - \mathbf{x}_{\text{m}}^{(j)}|^2] + \frac{\lambda_j^*}{M} \sum_{m=0}^{M-1} \mathbf{B}_{m,j}^{(j)}, \tag{18}$$
where $\lambda^l_j$ is the corresponding Lagrangian multiplier. To apply the Lagrange multiplier method, we first substituting (14) and (11) into the MSE equation (17). By setting to zero the gradient of (18) with respect to $\mathbb{E}_{m}^{(j)}$, and considering the linear constraint of (16), we derive the Lagrangian multiplier $\lambda^l_j$ as
\[
\lambda^l_j = \frac{1}{M} \sum_{m=0}^{M-1} (\mathcal{H}_m \mathbf{1}_j)\mathbb{R}_{m}^{(j)\dagger},
\]
which leads to a simplified version of $\mathbb{E}_{m}^{(j)}$
\[
\mathbb{E}_{m}^{(j)} = \frac{1}{\sigma^s_s (l-1)^2} (\mathcal{H}_m \mathbb{R}_{m}^{(j)\dagger} - \gamma^l_j \mathbf{1}_j).
\]
Substituting (21) into (17) further simplifies the MSE function as
\[
\mathbb{E}[|\hat{X}_m^{(j)} - X_m^{(j)}|^2] = \sigma^2_s + \gamma^s_j - 2\gamma^s_j \frac{\rho(l-1)^2}{\sigma^s_s (l-1)^2} \mathcal{H}_m \mathbf{1}_j - \mathbb{E}[\mathcal{H}_m \mathbf{1}_j]\mathbb{R}_{m}^{(j)\dagger} \mathcal{H}_m \mathbb{R}_{m}^{(j)\dagger}
\]
where
\[
\mathcal{L}_m^{l-1} = (\sigma^2_s - \frac{\rho(l-1)^2}{\sigma^s_s (l-1)^2}) \mathcal{H}_m \mathcal{H}_m^{\dagger} + \sigma^2_n \mathbf{I}_{N_R}
\]
(23)
\[
+ (\sigma^2_s - 2\rho_1 + \sigma^2_n) \mathcal{H}_m \mathcal{H}_m^{(\text{tail})\dagger}
\]
\[
+ (\sigma^2_s - 2\rho_1 + \sigma^2_n) \mathcal{H}_m \mathcal{H}_m^{(\text{head})\dagger}.
\]
Setting to zero the gradient of the Lagrangian function with respect to $\mathbb{E}_{m}^{(j)}$, and the feedforward filter response can be derived as
\[
\mathbb{F}_{m}^{(j)} = (1 - \frac{\rho(l-1)^2}{\sigma^s_s (l-1)^2}) \sigma^2_n \Gamma_{m}^{(j)}
\]
(24)
where
\[
\Gamma_{m}^{(j)} = \frac{\mathcal{H}_m \mathbf{1}_j}{\mathcal{L}_m^{l-1}},
\]
(25)
and the corresponding scaling factors
\[
\gamma^l_j = (1 - \frac{\rho(l-1)^2}{\sigma^s_s (l-1)^2}) \sigma^2_n \beta^l_j
\]
(26)
\[
\beta^l_j = \frac{1}{M} \sum_{m=0}^{M-1} (\mathcal{H}_m \mathbf{1}_j)\mathbb{R}_{m}^{(j)\dagger},
\]
(27)
Since (21) and (24) lead to a biased estimate of the equalizer output, we define scaled feedforward and feedback filters to remove the bias
\[
\mathbb{F}_{m}^{(j)} = \frac{\mathbb{F}_{m}^{(j)\dagger}}{\gamma^l_j} = \frac{\Gamma_{m}^{(j)} / \beta^l_j}{\gamma^l_j}
\]
(28)
\[
\mathbb{B}_{m}^{(j)} = \frac{1}{\sigma^s_s (l-1)^2} (\mathcal{H}_m \mathbb{R}_{m}^{(j)\dagger} - \mathbf{1}_j)
\]
(29)
which leads to a tone-wise MSE
\[
\text{MSE}_{m}^{(j)} = (1 - \frac{\rho(l-1)^2}{\sigma^s_s (l-1)^2}) \sigma^2_s
\]
\[
+ \frac{1}{\beta^l_j} - 2(1 - \frac{\rho(l-1)^2}{\sigma^s_s (l-1)^2}) \sigma^2_s \frac{1}{\beta^l_j} \mathbb{R}_{m}^{(j)\dagger} (\mathcal{H}_m \mathbf{1}_j)
\]
(30)
and a block-wise MSE averaged across all tones
\[
\text{MSE}_{j} = \frac{1}{M} \sum_{m=0}^{M-1} \text{MSE}_{m}^{(j)} = \frac{1}{\beta^l_j} - (1 - \frac{\rho(l-1)^2}{\sigma^s_s (l-1)^2}) \sigma^2_s.
\]
(31)
As the iteration increases, the estimate improves, and $(1 - \frac{\rho(l-1)^2}{\sigma^s_s (l-1)^2}) \sigma^2_s$ approaches 0 accordingly. The block-averaged MSE (31) in this case approximates $1/\beta^l_j$, which in turn indicates that the SINR at equalizer output approaches the matched filter bound SNR (MFBSNR)
\[
\text{SINR}^l_j \approx \beta^l_j = \sum_{i=1}^{N_R} |h_{i,j}|^2 / \sigma^2_n,
\]
(32)
where
\[
\sum_{i=1}^{N_R} |h_{i,j}|^2 = \frac{1}{M} \sum_{m=0}^{M-1} (\mathcal{H}_m \mathbf{1}_j)\mathbb{R}_{m}^{(j)\dagger} (\mathcal{H}_m \mathbf{1}_j).
\]
(33)

4. PERFORMANCE EVALUATION

We consider an uncoded block transmission system over frequency selective Rayleigh fading channels. We also assume that the number of users and receiver antennas are $N_T = 4$ and $N_R = 4$, respectively. The data is QPSK-modulated, the block length $N_f = 256$ symbols, and a frame of 4 blocks is processed in each simulation, which is equivalent to a system of 1024-symbol block length. Channel state information and timing detection is assumed to be perfectly known at receiver side.

Fig.1 shows the average SER performance of the 4 users using the proposed MUD as a function of the average received $E_b/N_0$, with the arrival timing difference among users as a parameter. A symbol-spaced 16-tap frequency-selective block Rayleigh-fading channel is considered to have an exponential power delay profile with a normalized
E_b/N_0 (dB)
SER
QPSK−SC−FDE over MIMO Rayleigh Fading Channel

Fig. 1. Average SER performance for asynchronous MU systems in a 16-tap ISI channel with rms = 2.

For comparison, the SER performance of the conventional MUD based on the CP is shown when the arrival timing differences are within the CP period \(((\tau_1, \tau_2, \tau_3, \tau_4) = (0, 0, 0, 0))\) and exceed the CP period. We assume that CP length is equal to the channel excessive length \(\nu\), and consider two asynchronous settings: (1) \((0, 2\nu, 2\nu, 2\nu)\), and (2) \((0, \tau_2, \tau_3, \tau_4)\) where \((\tau_2, \tau_3, \tau_4)\) are randomly changed between \((\nu, 2\nu)\) in each simulation, respectively. As shown in the figure, the conventional MUD is unable to recover the transmitted signals due to the severe interference components from adjacent symbol blocks, when the timing difference exceeds the CP period, whereas the proposed MUD significantly improves the SER performance, and approaches the MFB in moderate to high \(E_b/N_0\) region.

Fig. 2 shows the average SER performance for a symbol-spaced 64-tap frequency-selective block Rayleigh-fading channel, which have an exponential power delay profile with a normalized rms delay spread of 20. While the conventional MUD shows SER floor again, when the timing difference exceeds the CP period, the proposed MUD, without any CP, achieves near-ideal (synchronous) SER performance.

5. CONCLUSIONS

In this paper, we proposed a MUD technique for the asynchronous uplink MU-MIMO systems. The proposed method requires no timing control and CP insertion at transmitter side, and treats the precanceled-distortion as colored noise to derive the iterative FDE filters suitable for eliminating the interference components from adjacent symbol blocks. The asymptomatic MSE performance is shown to approach MFB, which is also confirmed by simulation in various system settings.

6. REFERENCES


