USER ADMISSION IN MIMO INTERFERENCE ALIGNMENT NETWORKS


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ABSTRACT
In this paper we consider an interference channel where a set of primary active users are cooperating through interference alignment over a constant multiple-input-multiple-output channel while a set of secondary users desire access to the channel. We present the conditions under which a secondary user can be admitted to the network. For the admitted users, we derive several beamforming designs maximizing approximately the secondary users’ sum-rate based on the number of the secondary users, the number of antennas at the secondary users and the total number of streams in the network of active users.

Index Terms — Interference Alignment, Multiple-input multiple-output (MIMO), User Admission, Zero Forcing

1. INTRODUCTION
Interference alignment (IA) can achieve the maximum degrees of freedom (DOF) of the K-user interference channel [1]. Given that current and future wireless communication networks will be endowed with multiple antennas [2], IA over multiple-input-multiple-output (MIMO) channels [3] is an attractive physical layer signaling approach. Most networks relevant for IA will be packet-switched with bursty data traffic, requiring frequent changes in the number of active users.

In this paper, we consider a network composed of a set of current active users utilizing IA and a further set of secondary candidate users who wish to access the network but are not allowed to impact the performance of the active users. More specifically, we require the secondary users to not degrade the sum-rate of the active users. Furthermore, we assume the active users do not change their precoding/beamforming matrices to help the transmission of the secondary users. Although this setup is also familiar in the context of cognitive radio, it more generally applies to most interference-limited wireless networks with dynamic user demands.

We present the conditions under which a set of secondary users can join the network of active users without affecting their performance. Then we derive secondary user beamforming designs which are sum-rate optimal in the case of one secondary user and sub-optimal for more.

Although there exist a few prior strategies for user admission and beamforming design for MIMO networks, they either do not consider an existing primary IA network (see [4] and references therein) or they are restricted to opportunistic access methods (e.g. see [5] and references therein).

2. SYSTEM MODEL
Consider a $K_a$-user MIMO interference channel where the $i^{th}$ transmitter/receiver pair is equipped with $M_i$ and $N_i$ antennas, respectively. In general, each transmitter $i$ uses a beamforming matrix $F_i$ to transmit $d_i$ streams to its corresponding receiver. For each time instant, the received signal at the $i^{th}$ receiver, assuming perfect timing and synchronization, can be written as

$$y_i = \sum_{k=1}^{K_a} H_{ik} F_k x_k + z_i \quad i = 1, \ldots, K_a,$$

where $H_{ik}$ is the matrix of channel coefficients of a block fading channel between transmitter $k$ and receiver $i$, the transmitted signal from the $i^{th}$ node is $x_i$ with power constraint $\mathbb{E}\{x_i^* x_i\} = P$ and $z_i$ is the additive white Gaussian noise with elements in $\mathcal{CN}(0, \sigma^2)$ where $\sigma^2$ is the noise power spectral density.

A system of IA between the $K_a$ users is feasible if there exists a set of matrices $\mathcal{W} = \{\mathcal{W}_1, \ldots, \mathcal{W}_{K_a}\}$ such that, given the system model of (1), the following constraints are met [1]

$$\left\{ \begin{array}{l}
\text{rank}(\mathcal{W}_i H_i F_i) = d_i \\
\mathcal{W}_i H_i F_k = 0 \ \forall k \neq i \ \forall i, k \in \{1, \ldots, K_a\}. \end{array} \right.$$  

Examples of $\mathcal{W}_i$ in (2) are the linear equalizer presented in [7] and the projection matrix presented in [3]. We assume the set of $\{M_i, N_i, d_i, K_a\}, \forall i$, is feasible [6]. Through IA, at the $i^{th}$ active user receiver, $i = 1, \ldots, K_a$, the interfering signal from the other active users span a $N_i - d_i$ subspace of the received signal space. Assume a non-unique basis for the interference subspace at the $i^{th}$ active user receiver is given as the columns of $C_i$. We assume, each active user is transmitting the maximum number of allowed streams and then, each
active user receiver utilizes a zero-forcing equalizer given by
\[ W_i = [I_{d_i}, 0_{d_i}, N_i - d_i] ([H_{ii} F_i, C_i])^{-1} \] [7] to recover its data. Note that, from (2)
\[ W_i \sum_{k=1}^{K_a} H_{ik} F_k x_k = 0 \quad i, k \in \{1, \ldots, K_a\}. \] (3)

In this case, achievable sum-rate of the active users’ network in the absence of any interference from the secondary users is given by [7]
\[ R_{\text{sum}}^a = \sum_{i=1}^{K_a} \sum_{n=1}^{d_i} \log \left(1 + \frac{\gamma_0/(d_i M_i)}{[P_i H_{ik} F_k^* P_i H_{ik} F_k]^{-1} n, n} \right), \] (4)
where \( P_i = (I_{N_i} - C_i C_i^*) \) is the projection matrix into the null-space of the interference sub-space at the \( i^{th} \) user receiver and \( \gamma_0 = \frac{P_i}{\sigma^2} \).

3. SECONDARY USER ADMISSION

Assume \( K_s \) secondary users request access to the network resources. Define \( K = K_a + K_s \), and without loss of generality, assume the last \( K_s \) users in the ordered set of user indices \( K = \{1, \ldots, K_a, K_a + 1, \ldots, K\} \) are the secondary users. If the secondary users are not allowed to degrade the sum-rate of the first \( K_a \) users, comparing (6) and (4) implies that \( \gamma_0 \) should be larger than its row rank, i.e. \( d_i < M_k \), which implies that rank \( [P_i H_{ik} F_k] \) = \( d_i \). Hence, a non-zero \( F_k \) satisfying (9) can be found if and only if \( M_k > \sum_{i=1}^{K_s} d_i \). Moreover, if each secondary user wants to transmit \( d_s \) streams to its corresponding receiver, \( F_k \) (the null-space of \( H_k \)) should have rank of \( d_k \) and the result follows.

In general, in addition to not impacting the performance of the active users, the secondary users want to achieve good sum-rate performance, given by
\[ R_{\text{sum}}^s = \sum_{k=K_a+1}^{K} \log \det \left(I + \frac{\gamma_0}{M_k} H_{kk} F_k F_k^* H_{kk}^* B_k \right), \] (11)
where \( B_k = (I_{N_k} + \sum_{i=1,i \neq k}^{K} \gamma_i M_i F_i F_i^* H_{ik}^* H_{ik})^{-1} \) captures the aggregate interference from all the transmitting nodes. Note that (9) restricts the precoder of the \( k^{th} > K_a \) transmitter should satisfy
\[ \begin{bmatrix} P_i H_{ik} \\ P_{K_a} H_{k,s} \end{bmatrix} F_k = 0 \] (9)

Based on (8), the precoder of the \( k^{th} > K_a \) transmitter should satisfy
\[ \begin{bmatrix} P_i H_{ik} \\ P_{K_a} H_{k,s} \end{bmatrix} F_k = 0 \] (9)

Using (9), Lemma 1 provides the minimum required number of antennas at each secondary user.

**Lemma 1.** The \( k^{th} \) secondary user, \( k \in \{K_a + 1, \ldots, K\} \), can transmit \( d_k \) streams without potentially degrading (4) if
\[ M_k \geq \sum_{i=1}^{K_a} d_i + d_k \quad k \in \{K_a + 1, \ldots, K\}. \] (10)

**Proof.** In constant channel MIMO IA networks \( d_i < N_i \), so rank \( [P_i H_{ik} F_k] \) = min \( \{M_k, N_i, d_i\} \) = min \( \{M_k, d_i\} \). In addition, in order to have a non-trivial answer for \( F_k \) in (9), the column rank of \( P_i H_{ik} F_k \) should be larger than its row rank, i.e. \( d_i < M_k \), which implies that rank \( [P_i H_{ik} F_k] \) = \( d_i \). Hence, a non-zero \( F_k \) satisfying (9) can be found if and only if \( M_k > \sum_{i=1}^{K_s} d_i \). Moreover, if each secondary user wants to transmit \( d_s \) streams to its corresponding receiver, \( F_k \) (the null-space of \( H_k \)) should have rank of \( d_k \) and the result follows.

Next we solve (13) for few special cases. Without loss of generality, to simplify the equations, we assume the first \( M_a \times N_a K_a \) users and the last \( M_s \times N_s K_s \) users are each transmitting \( d_a \) and \( d_s \) streams, respectively.
3.1. Single New User

Assume $K_s = 1$. In this case, the optimum secondary user beamformer solving (13) is given by Lemma 2.

Lemma 2. For $K_s = 1$, columns of $G_k$ solving (13) are the $d_s$ most significant eigenvectors of $V_k^* H_{kk}^* B_k H_{kk} V_k$.

Proof. The proof is given in [9].

3.2. Two New Users

For $K_s = 2$ and $d_a = 1$, Lemma 3 gives an approximate solution for (13) in the high signal-to-interference-plus-noise (SINR) regime.

Lemma 3. In the high SINR regime, when $K_s = 2$ and $d_a = 1$, (13) is approximately solved by setting the columns of $G_k$ for $k \in \{K_a + 1, K\}$ to $d_s$ most significant eigenvectors of

$$\left( M_q + \frac{\gamma_o}{M_a} \tilde{V}_k^* H_{qk}^* H_q \tilde{V}_k \right)^{-1} \tilde{V}_k^* H_{kk}^* H_{kk} \tilde{V}_k,$$

for $k, q \in \{K_a + 1, K\}$ and $k \neq q$ where

$$M_q = I_{K_a} - K_a d_a + \frac{\gamma_o}{M_a} \sum_{i=1}^{K_a} F_i^T H_{qi}^* H_q F_i.$$

Proof. The proof is given in [9].

3.3. More Than Two New Users

As solving (13) for more than two secondary users is equivalent to solving the general capacity of the MIMO interference channel, to date, a closed-form solution directly solving (13) for $K_s > 2$ does not exist. We provide, however, a beamforming design maximizing the pre-log factor of (13) asymptotically high $\gamma_o$ for a special case of network configuration.

Assume $M_s = N_s$. The secondary users have to use $K_a d_a$ of their transmit and receive antennas to cancel the interference they cause to and the interference they receive from the active users respectively. So each secondary user has $M_s = N_s = M_a - K_a d_a$ antennas available for its own communications. Now for arbitrary $d_a$ if $K_s \leq 2M_s/d_a - 1$ [6], or $M_s = \lceil (1 + K_a)/2 \rceil$ (when $d_a = 1$) [3], we can perform another level of IA between the secondary users. For doing so, we construct effective channels between the secondary users by replacing $H_{kk}$, for $k \in \{K_a + 1, \ldots, K\}$, with $W_k H_{kk} V_k$, where columns of $V_i$ and $W_k$ span the null-spaces of $[(C_i H_{kj})]^T, \ldots, (C_{K_a} H_{K_a})^T]^T$ and $[(H_{kj} F_i)]^T, \ldots, (H_{K_a K_a} F_{K_a})^T]^T$, respectively. Then, we treat the effective channels as the actual channels and perform any of the various IA beamforming designs to construct a system of IA between the secondary users. In this fashion, we will have two levels of IA, one among the active users and one among the secondary users. We name this method successive IA. Interestingly, in [9] we show that in special cases, successive IA achieves the same DOF as all the $K$ users had performed IA together.

4. NUMERICAL RESULTS

In this section we assume both the channel and the AWGN coefficients are distributed as $CN(0, 1)$. Assume $K_s = 1$, $M_a = N_s = 5$, $K_a = 2$, $M_a = N_a = 3$ and $d_a = d_a = 1$. The secondary user beamformer can be designed using several methods: I-selfish beamforming: the secondary user ignores the active users and maximizes its own rate as if no interference was present, i.e. $F_k$ equal to the right singular vector of $H_{kk}$ corresponding to its most significant singular value; II-self-optimizing null-space beamforming: as presented in Section 3.1; III-random unitary null-space beamforming: the secondary user selects a random beamforming vector in the receive interference sub-space of the active users. The total sum-rate of this network for various secondary user beamforming designs versus $P$ is shown in Fig. 1. The depicted upper-bound is the aggregate sum-rates of the active users and the secondary users when they see no interference from each other. As expected, selfish beamforming has the worst performance where, due to the uncoordinated interference, the DOF of the whole network is less than the original network of the active users. Moreover, both the self optimizing and random unitary null-space beamforming achieve the total DOF of 4 with a total sum-rate close to the upper-bound. Note that at low $\gamma_o$, where interference and noise are indistinguishable, one rather use the spatial dimensions to increase the received signal power than aligning the interference and thus the selfish design has the best performance.

![Fig. 1. Total achievable sum-rate versus transmit power for varying beamforming designs for $K_a = 3$, $M_a = N_a = 2$, $K_s = 1$ and $M_s = N_s = 5$.](image.png)

Now, consider two $M_s \times N_s$ MIMO secondary users where $M_s = N_s$. Similar to the single secondary user case, the two secondary users can adopt selfish beamforming, random unitary null-space beamforming or the high-SINR joint-optimum beamforming of Lemma 3. The total achievable sum-rate of this network versus $M_s$ for various beamforming designs at two values of $\gamma_o$ is shown in Fig. 2. Note that when $M_s = K_a + 1$, after enforcing a zero-interference-to-active-users constraint on the beamforming vectors of the
secondary users, no further optimization of the beamforming vectors is possible ($G_k$ has to be ±1) which explains why random unitary null-space beamforming and high-SINR joint-optimum beamforming have the same performance for $M_s = N_s = 4$. In addition, although random unitary null-space beamforming requires considerably less overhead compared to the method of Lemma 3, it is achieving an acceptable performance. Moreover, the diminishing returns of increasing secondary user antennas is more severe at low γ_o.

For the case of three $5 \times 5$ MIMO secondary users where $K_a = 3$, $M_a = N_a = 2$, $d_a = d_s = 1$ and two different transmit power values.

As can be seen, only successive IA achieves more DOF (6 DOF here) than the IA network of active users (3 DOF), and all the other methods, because of uncoordinated interference at the secondary user receivers, cannot achieve more DOF than the original network of the active users. In fact, it can be shown that when $M_s < (K_a d_a + (K - K_a) d_s)$ and $d_a = d_s = 1$, it is only through successive IA that one can achieve more DOF than the original network of IA.

5. CONCLUSION

With the increasing interest in employing interference alignment in wireless communications networks emerges the need to develop user admission and control strategies. User admission in a MIMO interference alignment network was studied in this paper and several transmission techniques for the secondary users were developed. It was shown that coordination between the secondary nodes plays an important role in maximizing the sum-rate of the network.

6. REFERENCES

[1] V. Cadambe and S. Jafar, “Interference alignment and degrees of freedom of the K-user interference channel,”


Fig. 2. Total achievable sum-rate versus number of secondary users’ transmit antennas for $K_a = 3$, $M_a = N_a = 2$, $K_s = 2$, $d_a = d_s = 1$ and two different transmit power values.

Fig. 3. Total achievable sum-rate versus transmit power for varying beamforming designs for $K_a = 3$, $M_a = N_a = 2$, $K_s = 2$ and $M_s = N_s = 5$. 


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