INTERFERENCE ALIGNMENT IN MIMO CELLULAR NETWORKS

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ABSTRACT

We explore the feasibility of linear interference alignment (IA) in MIMO cellular networks. Each base station (BTS) has $N_t$ transmit antennas, each mobile has $N_r$ receive antennas, and a BTS transmits a single beam to each active user. We present a necessary Zero-Forcing (ZF) condition for zero interference in terms of the number of users, the number of cells, $N_t$ and $N_r$. We then examine the performance of iterative (forward-backward) algorithms for jointly optimizing the transmit precoders with linear receivers. Modifications of the max-SINR and minimum leakage algorithms are presented, which are observed to converge to a ZF solution whenever the necessary conditions are satisfied. In contrast, convergence of the (original) max-SINR algorithm is problematic when the necessary conditions are satisfied with (near) equality. A more restrictive ZF condition is presented, which predicts when these convergence problems are unlikely to occur.

1. INTRODUCTION

The performance of cellular data networks is fundamentally limited by co-channel interference. By adding antennas to the Base Station (BTS) and the mobiles, Multi-Input Multi-Output (MIMO) links are created, which allow the BTS to transmit simultaneously to multiple mobiles without interference. Here we consider multiple cells and study how many independent data streams the BTS’s can transmit with zero interference across all mobiles.

Maximizing the number of data streams in a MIMO interference (peer-to-peer) network generally requires interference alignment. That is, the interference covariance matrix at each receiver must have less than full rank [1]. It is shown in [1] that a narrowband MIMO interference network with $N$ antennas per node can support $2N - 1$ users without interference, assuming i.i.d. channel coefficients. Here we wish to derive an analogous result for the narrowband MIMO cellular network. We further assume that the BTS transmits a single beam (data stream) to each mobile. Due to the broadcast nature of the downlink channels the previous results in [1] cannot be directly applied.

We present a necessary “Zero-Forcing (ZF)” condition for interference, obtained from examining the number of equations and variables in the corresponding multivariate polynomial system, as in [1]. This condition assumes the same number of transmit antennas in each cell $N_t$, the same number of receive antennas across users $N_r$, and gives an upper bound on the number of users $K$ which can transmit/receive with zero interference. (This applies to both uplink and downlink.) Although numerical examples suggest that the necessary conditions are also sufficient, this remains an open question.\(^1\)

Distributed iterative algorithms for achieving interference alignment have been presented in [2]. Here we examine the performance of the max-SINR algorithm, presented in [2], for the downlink cellular model. The algorithm consists of forward-backward iterations, and has the attractive property that it can be implemented adaptively (without channel state information) by transmitting training symbols in each direction in Time-Division-Duplex (TDD) mode [3].

Numerical results show that for the scenarios considered, ZF solutions exist whenever the necessary conditions are satisfied. However, if the necessary conditions are satisfied with (or near) equality, then the max-SINR algorithm often does not converge at high SNRs. We present modifications of both the max-SINR and minimum-leakage algorithms in [2] in which the intra-cell beams are constrained to be orthogonal, and for which convergence is substantially improved. Furthermore, by explicitly including ZF constraints for intra-cell users, we obtain a more restrictive ZF condition, and observe that when these conditions are satisfied, the (non-orthogonalized) max-SINR algorithm converges rapidly. Conversely, when these conditions are violated, convergence of the max-SINR algorithm becomes problematic.

Related work on IA for cellular systems is presented in [4, 5, 6]. ZF conditions for two cells are presented in [4], and are extended to an arbitrary number of cells for a certain class of “decomposable” channels (which does not include those with i.i.d. elements). An analogous ZF condition is presented in [6] allowing both time and frequency diversity (in addition

\(^1\)The difficulty is due to the dependence among downlink channel coefficients. This problem is analogous to providing ZF conditions for an interference network with multiple beams per user [1]. However, in contrast with that scenario, the structure of the cellular configuration can be exploited to establish necessary ZF conditions.
2. SYSTEM MODEL AND ZF CONDITIONS

We focus on the downlink although our results apply to both downlink and uplink. There are $G$ cells with $K$ users in cell $i$, each BTS has $N_t$ transmit antennas, and each mobile has $N_r$ receive antennas. Fig. 1 shows an example with $G = 3$ (cells labeled $a, b, c$) with $K_a = 2$, $K_b = 1$, and $K_c = 1$ users.

The received signal at user 1 of BTS $a$ is:

$$y_{a1} = H^a_{a1}x_{a1} + H^a_{a2}x_{a2} + H^a_{a3}x_{b1} + H^a_{a1}x_{c1} \text{ (1)}$$

where $H^\beta_{ak}$ is the $N_r \times N_t$ channel from the BTS $\beta$ to user $k$ in cell $\alpha$ ($\alpha, \beta \in \{a, b, c\}$), $x_{ak} \in \mathbb{C}^{N_t \times 1}$ is the signal vector from BTS $\alpha$ to user $k$ in cell $\alpha$, and $y_{ak} \in \mathbb{C}^{N_r \times 1}$ is the received signal vector at user $k$ in cell $\alpha$. All matrices are assumed to have i.i.d. complex Gaussian elements with zero mean and unit variance. The first term on the right-hand side of (1) is the desired signal, the second term is the intra-cell interference from cells $b$ and $c$, respectively.

We wish to select the precoding vector $v_{ak} \in \mathbb{C}^{N_r \times 1}$ and receive vector $g_{ak} \in \mathbb{C}^{N_t \times 1}$ to satisfy the ZF conditions:

$$g_{ak}^H H^\beta_{ak} v_{ak} = 0, \forall \beta \neq \alpha \text{ or } l \neq k \text{ (2)}$$

$$g_{ak}^H H^\alpha_{al} v_{lk} \neq 0, \text{ (3)}$$

where $\beta \neq \alpha$ corresponds to inter-cell interference and $\beta = \alpha$ & $k \neq l$ corresponds to intra-cell interference.\(^2\) Defining the reciprocal channel for uplink transmission the same way as in [2], we obtain the analogous ZF conditions for the uplink. These are equivalent to the downlink conditions, where the roles of the receive filters and beams are swapped.

3. FEASIBILITY CONDITION

We start with a symmetric system in which there are $K$ users in each cell and subsequently extend the analysis to allow different numbers of users per cell. Since each BTS transmits $K$ linearly independent beams to the users in its cell, we must have $N_t \geq K$. An analogous constraint does not apply at the receiver, since it may be possible to align all of the interference in the appropriate receiver null space.

The ZF conditions in (2) contain $GK \times (GK - 1)$ equations with $GK(N_r + N_t)$ variables.\(^3\) However, in analogy with the ZF conditions for the interference network considered in [1], many of these variables are superfluous. Specifically, the precoding matrix $V = [v_{a1}, v_{a2}, \ldots, v_{aK}]$ at BTS $\alpha$ must have full column rank, which adds $K$ constraints. That is, following the argument in [1], $V$ can be linearly transformed to the matrix

$$\begin{bmatrix} \bar{v}_{a1} & \bar{v}_{a2} & \bar{v}_{a3} & \cdots & \bar{v}_{aK} \end{bmatrix}$$

where $I_{K \times K}$ is the $K \times K$ identity matrix and $ar{v}_{a\alpha}$ is $(N_t - K) \times 1, \alpha = 1, 2, \ldots, K$. Hence the precoding matrices across cells have $K \times (N_t - K)$ independent variables.

Because each receive filter can be scaled arbitrarily without affecting the ZF condition, this eliminates one variable from each filter $g_{ak}$ (e.g., we can set the top element to one). Hence adding the variables on the transmit and receive sides gives the total number of independent variables in the system

$$N_v = GK(N_t + N_r - K - 1). \text{ (4)}$$

We now wish to compare $N_v$ with the total number of independent equations $N_e$. The total number of equations in (2) is $GK(GK - 1)$. However, we observe that constraining the beams within a cell to be linearly independent, as was done to compute $N_v$, renders the intra-cell ZF conditions redundant. That is, if $v_{a1}, \ldots, v_{aK}$ are linearly independent, then there exists a linear transformation of the beams that makes the $K - 1$ interfering received vectors $H^\alpha_{ak} v_{al}$ orthogonal to $g_{ak}$, $\forall k \neq l$. Excluding the ZF conditions for intra-cell interference gives

$$N_e = G(G - 1)K^2. \text{ (5)}$$

If the channel coefficients were i.i.d. across transmitters and receivers, then the existence of a ZF solution would be guaranteed by Bezout’s theorem almost surely (a.s.), which requires $N_v \geq N_e$, i.e.,

$$N_t + N_r \geq GK + 1. \text{ (6)}$$

However, for the cellular downlink many direct and cross channels are the same, hence the theorem cannot be directly applied.

\(^2\) $A^H$ denotes the Hermitian transform of $A$.

\(^3\) With i.i.d. channels any non-trivial solution to (2) will satisfy (3) almost surely.
The following theorem states that the preceding condition is a necessary condition, and extends this to a cellular system with different numbers of users per cell. Although sufficiency remains an open question, numerical examples have indicated that if the necessary conditions are satisfied, then solutions to the ZF conditions can be found via the algorithms to be discussed.

**Theorem:** For a cellular network with $G$ cells, $N_t$ antennas at each BTS, and $N_r$ antennas at each mobile, necessary conditions for the existence of a ZF solution are

$$N_t \geq K_i \quad N_r \geq 1 \quad \forall i \in \{1, \ldots, G\} \quad (7)$$

$$N_r + N_t \geq \sum_{i=1}^G K_i + 1 \quad (8)$$

The conditions are again based on counting the number of independent variables and equations and setting $N_v \geq N_e$. To establish necessity, we first observe that if $N_v < N_e$, then we must have $N_v \leq N_e - \sum_{i=1}^G K_i$. We then choose a specific subset of the $N_e$ equations to discard and show that solutions to the remaining equations cannot be consistent with the discarded equations a.s. Details are omitted due to space constraints.

For a symmetric system with $G = 2$ the necessary ZF conditions say that we need at least $(K + 1) \times K$ channels to support $K$ users. We note that the IA algorithm presented in [4] assumes $(K + 1) \times (K + 1)$ channels, whereas the algorithm presented in the next section is observed to work with $(K + 1) \times K$ channels. With $N \times N$ channels the preceding condition requires $GK \leq 2N - 1$. This is exactly the same condition as obtained in [1] for MIMO interference networks with a single beam per user, although the counting argument for equations and variables appears to be different. Other related but different ZF conditions are given in [5] (assuming a particular class of “decomposable” channels), and in [6] (assuming time- and/or frequency-diversity with $K \geq N_t$).

We note that including the ZF condition for intra-cell interference, which were excluded in (5), gives the more restrictive ZF condition

$$N_t + N_r \geq (G + 1)K. \quad (9)$$

Although this is not a necessary ZF condition, it will be useful for predicting the performance of iterative algorithms for joint optimization of beams and receive filters.

### 4. Iterative Algorithms

We now present iterative algorithms for jointly optimizing the beams and receive filters. These are modifications of the minimum leakage and max-SINR algorithms presented in [2] for interference networks. They have the advantage of being distributed (requiring local knowledge of direct- and cross-channels), and exploit knowledge of the direct- as well as cross-channels. All schemes consist of iterating a forward optimization, in which the receive filters are optimized with fixed beams, with a backward optimization in which data flows in the reverse direction and the beams are optimized with fixed receiver filters.

#### 4.1. Orthogonalized Minimum Leakage (OR-ML)

For the forward direction, we minimize the inter-cell interference at each user $k$ given by

$$I_{ok} = Tr[g_{ok}^H Q_{ok} g_{ok}] \quad (10)$$

where $Q_{ok} = \sum_{x \neq \alpha} \sum_{j=1}^K H_{xk}^R v_{xj} v_{xj}^H H_{xk}^E$. Minimizing $I_{ok}$ over $g_{ok}$ subject to $\|g_{ok}\|_2 = 1$ gives $g_{ok} = \nu_k [Q_{ok}]$ where $\nu_k [A]$ is the eigenvector corresponding to the $d^{th}$ smallest eigenvalue of $A$. In the backward direction, we minimize the inter-cell interference at BTS $\alpha$,

$$\hat{I}_\alpha = Tr[V_{\alpha}^H \hat{Q}_{\alpha} V_{\alpha}] \quad (11)$$

where $\hat{Q}_{\alpha} = \sum_{x \neq \alpha} \sum_{j=1}^K H_{xz}^R g_{zj} g_{zj}^H H_{xz}^E$ (includes only inter-cell interference) and $V_{\alpha}$ is the $N_t \times K$ precoding matrix with full column rank. The solution is $V_{\alpha} = [\nu_1 [Q_{\alpha}], \ldots, \nu_K [Q_{\alpha}]]$. This guarantees linear independence of the columns. Motivated by the IA scheme presented in [4], to eliminate both the inter- and intra-cell interference we apply the precoding matrix

$$V_{\alpha} = V_{\alpha}[g_{\alpha 1}^H H_{\alpha 1}^\top V_{\alpha}; \ldots; g_{\alpha K}^H H_{\alpha K}^\top V_{\alpha}]^\top \quad (12)$$

where “$\top$” denotes pseudo-inverse and semicolon separates rows. The proof of convergence in [2] can be used to show that the sum rate converges when iterating the forward and backward optimizations.

#### 4.2. Orthogonalized Max-SINR (OR-MSINR)

Rather than finding a ZF solution, the max-SINR (MSINR) algorithm iteratively updates the receive filters and beams to maximize the received SINR in the forward and backward directions. Simulations of the MSINR algorithm for the cellular model have shown that when the ZF necessary condition (6) is satisfied with equality (or close to equality), the MSINR algorithm does not converge for a significant fraction of randomly selected channel realizations at high SNRs. What typically happens in those examples is that the optimized beams in the reverse direction are not linearly independent, which creates intra-cell interference. Hence we modify the MSINR algorithm in the backward direction to maximize the SINR at

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4This requires feedback of the direct channels. Note also that the approach in [4] applies only to two cells.
each BTS with only inter-cell interference subject to the intra-cell ZF constraint. Specifically, the backward SINR objective at BTS $\alpha$ is

$$\max_{v_{\alpha i}} \frac{v_{\alpha i}^H B_{\alpha i}^{-1} H_{\alpha i}^{\alpha H} g_{\alpha i} H_{\alpha i}^\alpha v_{\alpha i}}{\sum_{m \neq \alpha} v_{m i}^H H_{\alpha i}^{\alpha H} g_{\alpha i} H_{\alpha i}^\alpha v_{\alpha i}}$$

(13)

with the constraint $v_{\alpha i}^H H_{\alpha i}^{\alpha H} g_{\alpha i} = 0$, $i \neq i$.

The solution is given by $v_{\alpha i} = aB_{\alpha i}^{-1} H_{\alpha i}^\alpha g_{\alpha i}$, where $B_{\alpha i}$ is the appropriately defined covariance matrix at the transmitter and $a$ is a normalization constant. This solution requires that intra-cell interference be distinguished from inter-cell interference at the receivers and BTS’s. Hence the algorithm cannot be implemented adaptively by synchronously transmitting training symbols in TDD mode, as proposed in [3] for the MSINR algorithm. Instead, cells would have to alternate the transmission of training symbols.

5. NUMERICAL RESULTS

Fig. 2 shows sum rate vs SNR for the OR-ML, OR-MSINR, and MSINR algorithms. The parameters are $G = 3$, $N_t = N_r = 5$ ($5 \times 5$ channels), and $K = 3$ in each cell, so that the necessary condition (6) is satisfied with equality. The results are averaged over 20 channel realizations. At high SNRs the MSINR algorithm often does not converge (approximately 50% of the time at SNR = 50 dB). For those scenarios the sum rate is computed after a fixed number of iterations, resulting in the degradation shown in the figure. In contrast, the OR-MSINR was observed to converge for all channel realizations. As expected, the performance of the OR-ML and MSINR algorithms converge at high SNRs.

We omit convergence plots (sum rate vs number of forward-backward iterations) due to space constraints, and instead briefly summarize the results. For the $5 \times 5$ system in Fig. 2 at SNR = 50 dB the OR-MSINR algorithm takes approximately 5000 forward-backward iterations to converge to within 20% of the maximum sum rate, and about 5000 more iterations to converge to within 2%. The MSINR algorithm requires about the same number of iterations when it converges, but it often does not converge for different channel realizations. (Also, the number of iterations typically increases when $N_r < N_t$.)

In contrast, if $N_t = N_r = 6$, which satisfies the stronger ZF condition (9), then both algorithms consistently converge in much less than 100 iterations. Additional numerical examples have indicated that when the necessary condition (6) is satisfied, but (9) is not satisfied, the OR-MSINR algorithm converges very slowly and the MSINR often does not converge at high SNRs. A heuristic explanation is that the additional dimensions provided by the stronger condition guarantee the intra-cell beams to be linearly independent when optimized in the reverse direction.

6. CONCLUSIONS

We have presented necessary ZF conditions for a cellular system with i.i.d. channels. Although proving sufficiency appears to be difficult, numerical examples indicate that the modified OR-MSINR algorithm is able to find a ZF solution whenever the necessary conditions are satisfied. A more restrictive condition for ZF has also been presented, which indicates when the MSINR can be expected to converge rapidly. Since the MSINR algorithm can be implemented adaptively [3], this stronger condition can be viewed as a more significant practical constraint on system dimensions.

7. REFERENCES


