JOINT TRANSMITTER AND RECEIVER DESIGN WITH ADAPTIVE BEAMFORMING IN MIMO SC-FDMA SYSTEMS

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ABSTRACT

This paper presents the joint transmitter and receiver design with adaptive beamforming for multiple-input multiple output (MIMO) single carrier-frequency division multiple access (SC-FDMA) systems. Overall signal to interference plus noise ratio (SINR) of estimated symbols is maximized by selecting appropriate transmit power, transmit and receive beamforming weights and frequency domain-linear equalization (FD-LE) weight. Closed-form solutions unveil a symmetric design of the transmitter and receiver. In addition, two suboptimal system designs with closed form solutions are investigated.

Index Terms— MIMO, SC-FDMA, adaptive beamforming, optimal power allocation.

1. INTRODUCTION

Single carrier-frequency division multiple access (SC-FDMA) has been selected as a promising uplink transmission scheme for the third generation partnership project (3GPP) long term evolution (LTE) standards, due to its lower peak to average power ratio (PAPR) characteristics compared with conventional orthogonal FDMA (OFDMA) techniques [1]. To improve system performance, adaptive antenna arrays can be employed in conjunction with SC-FDMA in LTE system, where base station and mobile station are equipped with multiple antennas [2].

In [3], Palomar et al. developed a unified framework in the joint design of transmit and receive beamforming (or precoding and equalization) for MIMO OFDM system based on convex optimization theory. In [4], the authors proposed a power allocation approach to maximize capacity in SC-FDMA systems when utilizing frequency domain-linear equalizer (FD-LE), including zero forcing (ZF) and minimum mean square error (MMSE) equalizer at the receiver. These studies focused on resource allocation in either conventional MIMO OFDM systems or single-input single-output (SISO) SC-FDMA systems. Based on our knowledge, the system design of a MIMO SC-FDMA system with adaptive antenna arrays has not yet been considered.

Motivated by [3], in this paper we investigate joint transmitter and receiver design with adaptive beamforming for MIMO SC-FDMA systems. In particular, we assume a fixed modulation scheme for all subcarriers and aim to adjust transmit power with adaptive beamforming at both the transmitter and receiver. Additionally, we employ FD-LE at the receiver due to its low complexity in practical implementations. Based on this system model, we formulate an optimization problem to maximize overall signal to interference plus noise ratio (SINR) of estimated symbols by jointly selecting the appropriate transmit power, transmit and receive beamforming weights and FD-LE weights for each subcarrier. We derive closed-form expressions, which unveil a symmetric design of the transmitter and receiver for MIMO SC-FDMA systems. Additionally, we investigate two suboptimal system designs for MIMO SC-FDMA systems.

Notations: column vectors are denoted by boldface lowercase letters, i.e., $x = [x_1, x_2, \cdots, x_N]^T$ and $x_i$ is the $i$th entry of $x$. $\mathbf{0} = [0, 0, \cdots, 0]^T$ and $\mathbf{1} = [1, 1, \cdots, 1]^T$. $[\mathbf{A}]_{ij}$ denotes the element in the $i$th row and $j$th column of $\mathbf{A}$. $\mathbf{I}_N$ is the $N \times N$ identity matrix. $(\cdot)^T$ and $(\cdot)^\dagger$ denote the transpose and conjugate transpose operation, respectively. $\otimes$ denotes the Kronecker product. $\text{tr}\{\mathbf{A}\}$ denotes the trace of $\mathbf{A}$. $||\mathbf{x}||_2$ denotes the $\ell_2$ norm of $\mathbf{x}$. $\mathbf{x} \succeq \mathbf{0}$ denotes the generalized inequality, i.e., $x_i \geq 0$. $|\mathcal{S}|$ denotes the cardinality of a set $\mathcal{S}$.

2. SYSTEM MODEL

We consider an uplink MIMO SC-FDMA system, where each user is equipped with $K_t$ transmit antennas and the base station is equipped with $K_r$ receive antennas.

![Fig. 1. MIMO SC-FDMA transmitter for one user.](image)

The transmitter for MIMO SC-FDMA system is shown in Fig. 1. The time domain transmitted signal before cyclic prefix (CP) insertion is given as

$$x = (\mathbf{I}_{K_t} \otimes \mathbf{W}_N^\dagger)(\mathbf{I}_{K_r} \otimes \mathbf{D})\mathbf{FPW}_M s. \quad (1)$$

Here $s$ is the $M \times 1$ data symbol vector. Without loss of generality, we assume that $\mathbb{E}\{ss^\dagger\} = \sigma_s^2 \mathbf{I}_M$. $\mathbf{W}_L$ is
the L × L discrete Fourier transform (DFT) matrix, i.e.,
\[ W_L \] \[ ik = \frac{1}{\sqrt{L}} e^{-j \frac{2\pi}{L} (i-1)(k-1)}. \]
P = diag\{ p_1, p_2, \ldots, p_M \} is the M × M power allocation matrix, where \( p_i \) is the power assigned to symbol \( i \). In this paper, we assume a total transmit power constraint, i.e., \( \text{tr}\{P^2\} = M \). D is the N × M subcarrier mapping matrix. Without loss of generality, we assume localized subcarrier mapping and the user of interest is assigned subcarrier from 1 to M, i.e.,
\[ D = \text{diag}\{ \omega_1, \omega_2, \ldots, \omega_M \}. \]
F is the (K,M) × M transmit beamforming matrix such that \( F = [F_1, F_2, \ldots, F_K] \) with \( F_j = \text{diag}\{ \alpha_{1,j}, \alpha_{2,j}, \ldots, \alpha_{M,j} \} \), where \( \alpha_{i,j} \) is the transmit beamforming weight for the \( i \)th subcarrier and \( j \)th transmit antenna. We define the transmit beamforming vector for \( i \)th subcarrier as \( \alpha_i = [\alpha_{1,i}, \alpha_{2,i}, \ldots, \alpha_{K,i}] \). To ensure the same transmit power with multiple antennas as with one antenna, we assume \( ||\alpha|| = 1 \).

\[ y = \tilde{H}x + v, \quad (2) \]

where \( \tilde{H} \) is a (K,N) × (K,N) block circulant channel matrix; \( n \) is the Gaussian noise vector with zero mean and \( \mathbb{E}\{vv^\dagger\} = \sigma_n^2 I_K \). After performing N-point FFT, subcarrier demapping, weighted combining for each subcarrier, FD-LE and M-point IDFT, the estimated symbol vector for the user of interest can be given as

\[ \hat{s} = W_M^\dagger \Omega^\dagger (I_K \otimes D^\dagger) (I_K \otimes W_N) y, \quad (3) \]

where \( \Omega = \text{diag}\{ \omega_1, \omega_2, \ldots, \omega_M \} \); \( G \) is the (K,M) × M weight receiving beamforming matrix and \( G = [G_1, G_2, \ldots, G_K] \). Specifically, \( G_j = \text{diag}\{ \beta_{1,j}, \beta_{2,j}, \ldots, \beta_{M,j} \} \), where \( \beta_{i,j} \) is the receive beamforming weight for the \( i \)th subcarrier and \( j \)th receive antenna. Let us define the receive beamforming vector for \( i \)th subcarrier as \( \beta_i = [\beta_{1,i}, \beta_{2,i}, \ldots, \beta_{K,i}] \). Similar to the transmit beamforming vector \( \alpha_i \), we assume \( ||\beta|| = 1 \).

Plugging (1) and (2) into (3), the estimated symbol vector can be represented as

\[ \hat{s} = W_M^\dagger \Omega^\dagger \text{UPW}_M s + n = \Theta s + n, \]

where \( U = G^\dagger (I_K \otimes D^\dagger) (I_K \otimes W_N) \tilde{H} (I_K \otimes W_N^\dagger) (I_K \otimes D) F \) and \( n = W_M^\dagger \Omega^\dagger (I_K \otimes D^\dagger) (I_K \otimes W_N) v. \)

Next, we study methods to maximize the overall SINR of the estimated symbols by jointly accounting for the transmit power, transmit and receive beamforming and FD-LE weights. In this paper, we assume that channel state information is available at the base station, and is fed back to the mobile user via a perfect feedback channel.

\section{3. OPTIMAL SYSTEM DESIGN IN MIMO SC-FDMA SYSTEMS}

Based on the system model as described in Section 2, the SINR for \( i \)th estimated symbol in MIMO SC-FDMA system can be calculated as

\[ \Gamma_i = \frac{\mathcal{P}_{d,i}}{\mathcal{P}_{r,i} + \mathcal{P}_{n,i}}, \quad (4) \]

where \( \mathcal{P}_{d,i} \) is the desired signal power of \( i \)th estimated symbol and can be calculated as \( \mathcal{P}_{d,i} = \mathbb{E}\{|s_i|^2||\Theta||^2 = \sigma_x^2||\Theta||^2\}; \mathcal{P}_{r,i} \) is the ISI power due to off diagonal element in \( \Theta \), i.e., \( \mathcal{P}_{r,i} = \mathbb{E}\{|s_i\Theta\Theta^\dagger s_i\}| - \mathcal{P}_{d,i} = \sigma_x^2||\Theta\Theta^\dagger||^2 - ||\Theta||^2; \) and \( \mathcal{P}_{n,i} \) is the noise power of \( i \)th estimated symbol, i.e., \( \mathcal{P}_{n,i} = \sigma_n^2 ||R_n||_i^2 \).

It is interesting to note that when \( B = W_M^\dagger \text{AW}_M \), then B is a circular matrix and the diagonal elements in B are same. Based on this, we see that \( \Theta, \Theta\Theta^\dagger \) and \( R_n \) are all circular matrices, which implies that \( \mathcal{P}_{d,i} = \mathcal{P}_d, \mathcal{P}_{r,i} = \mathcal{P}_{SI} \) and \( \mathcal{P}_{n,i} = \mathcal{P}_n, i = 1,\ldots, M \). Therefore, SINR is same for all estimated symbols, i.e., \( \Gamma_i = \Gamma, i = 1,\ldots, M \). To obtain SINR in (4), we need to compute \( |\Theta||_i^2, |\Theta\Theta^\dagger||_i^2 \) and \( |R_n||_i^2 \), i.e., [4]

\[ |\Theta||_i^2 = \frac{1}{M} \text{tr}\{\Theta\} = \frac{1}{M} \text{tr}\{W_M^\dagger \Omega^\dagger \text{UPW}_M\} = \frac{1}{M} \text{tr}\{\Omega^\dagger \text{UP}\} = \frac{1}{M} \sum_{k=1}^M \omega_k u_k p_k. \quad (5) \]

The above equation is valid because \( \text{tr}\{AB\} = \text{tr}\{BA\} \). Similarly, we have

\[ |\Theta\Theta^\dagger||_i^2 = \frac{1}{M} \text{tr}\{\Theta\Theta^\dagger\} = \frac{1}{M} \sum_{k=1}^M |\omega_k u_k p_k|^2, \]

\[ |R_n||_i^2 = \frac{1}{M} \text{tr}\{\Omega^\dagger \Omega\} = \frac{1}{M} \sum_{k=1}^M |\omega_k|^2. \quad (6) \]
Plugging (5) and (6) into (4), the SINR can be obtained as
\[
\Gamma = \frac{1}{M} \sum_{k=1}^{M} \omega_k^* u_k p_k \bigg| \bigg( \frac{1}{M} \sum_{k=1}^{M} \omega_k^* u_k p_k \bigg)^2 - \sum_{k=1}^{M} \omega_k^2 \bigg] \bigg( \frac{1}{M} \sum_{k=1}^{M} \omega_k^* u_k p_k \bigg)^2 + \frac{\sigma_n^2}{\sigma_s^2} \sum_{k=1}^{M} \omega_k^2 \bigg]^{1/2}. \tag{7}
\]

### 3.2. Optimal System Design in MIMO SC-FDMA System

In the optimal system design, our objective is to choose appropriate power allocation vector \( p \), transmit beamforming vector \( \alpha_i \), receive beamforming vector \( \beta_i \), for subcarrier \( i \) and linear equalization weight vector \( \omega \) that jointly maximize the overall SINR of the estimated symbol, where \( p = [p_1, p_2, \cdots, p_M]^T \) and \( \omega = [\omega_1, \omega_2, \cdots, \omega_M]^T \). By incorporating the constraints imposed on these parameters, the optimization problem can be formulated as:

\[
\max_{\mathbf{p}, \alpha_i, \beta_i, \omega} \Gamma \quad \text{s.t. constraints in (8).} \tag{15}
\]

It is worth mentioning that this optimization problem is more general than [4], where only optimal transmit powers are considered for SISO SC-FDMA systems. Let us define a set \( S = \{i|p_i > 0\} \). Then, the optimal solution of \((p, \alpha_i, \beta_i, \omega)\) can be found as follows:

**Theorem 3.1.** Optimal solution of \((p, \alpha_i, \beta_i, \omega)\) in (8) is: 1) when \( i \notin S, p_{opt,i} = 0; \) 2) when \( i \in S,
\[
\begin{align*}
p_{opt,i} &= (\mu \rho_{max,i} - \rho_{max,i}^{1/2}), & (9) \\
\alpha_{opt,i} &= \mathcal{L}_{max}(\mathbf{H}_i^* \mathbf{H}_i), & (10) \\
\beta_{opt,i} &= \frac{\mathbf{H}_i \mathcal{L}_{max}(\mathbf{H}_i^* \mathbf{H}_i)}{\| \mathbf{H}_i \mathcal{L}_{max}(\mathbf{H}_i^* \mathbf{H}_i) \|}, \quad \text{(10)} \\
\omega_{opt,i} &= \zeta_0 \big( \mu \rho_{max,i} - \rho_{max,i}^{1/2} \big)^{1/2}, \quad \text{(11)}
\end{align*}
\]

where \( \rho_{max,i} = \lambda_{max}(\mathbf{H}_i^* \mathbf{H}_i) \sigma^2 / \sigma_n^2 \) is the equivalent maximum signal to noise ratio (SNR) for ith subcarrier and \( \lambda_{max}(\mathbf{A}) \) is the maximum eigenvalue of \( \mathbf{A} \); \( \mathcal{L}_{max}(\mathbf{A}) \) is the normalized eigenvector of \( \mathbf{A} \) corresponding to \( \lambda_{max}(\mathbf{A}) \); \( \zeta_0 \) is an arbitrary constant and \( \mu = (1 + \varphi_1)\varphi_2^{-1}. \) Here, \( \phi_1 = (1/M) \sum_{i \in S} \rho_{max,i}^{-1} \) and \( \varphi_2 = (1/M) \sum_{i \notin S} \rho_{max,i}^{-1/2} \).

Due to space limitations, we omit the proof. For detailed proof, please refer to [5]. Next, we need to determine the set \( S \) to obtain the closed-form solutions for \( p \) and \( \omega \). To do this, let us define \( \epsilon_i = \rho_{max,i}^{-1/2} \). Without loss of generality, we assume \( \epsilon_1 \leq \epsilon_2 \leq \cdots \leq \epsilon_M \). Then, the set \( S \) can be obtained in the following lemma:

**Lemma 3.2.** Let us define
\[
\mathcal{F}(i) = \frac{\epsilon_i \sum_{j=1}^{i} \rho_{max,j}^{-1/2}}{M + \sum_{j=1}^{M} \rho_{max,j}}, \tag{12}
\]

then the set \( S \) is given as
\[
S = \{ i | \mathcal{F}(i) < 1, \mathcal{F}(i+1) \geq 1 \} \cup \{ 1, \cdots, M \}, \quad \text{otherwise.}
\]

We omit the proof and refer the reader to [5]. Comparing (11) with (9), we see that the optimal FD-LE weight for each subcarrier is a scaled version of the optimal transmit power. This unveils a symmetric design of the transmitter and receiver for MIMO SC-FDMA systems. Moreover, the optimal transmit power and FD-LE weights for each subcarrier follow a water-filling strategy, i.e., with larger \( \epsilon_i \), the chance for the ith subcarrier not to transmit the data symbol is higher. Note that \( \epsilon_i = \rho_{max,i}^{-1/2} \). Hence, when the equivalent maximum SNR is low, the ith subcarrier tends not to transmit the data symbol.

After some manipulations, we note that the MMSE equalizer coefficient is a scaled version of optimal linear equalization weight for each subcarrier. This indicates that when combining with the optimal transmit power in (9) and transmit and receive beamforming in (10), the MMSE equalizer is the optimal FD-LE in the sense that it maximizes the overall SINR in MIMO SC-FDMA systems. This is also valid in MIMO OFDM systems [3].

Given \((p_{opt}, \alpha_{opt,i}, \beta_{opt,i}, \omega_{opt})\), we see that the optimal SINR can be computed as \( \Gamma_{opt} = \frac{(1+\varphi_1) \varphi_2^{-1}}{(1-\theta)(1+\varphi_1)+\varphi_2^{-1}} \), where \( \theta = |S|/M \) is the ratio of the number of subcarriers being assigned with power to the total number of subcarriers. Based on this, the achievable capacity per unit bandwidth (b/s/Hz) can be obtained as
\[
C_{opt} = \log_2 \left( \frac{1+\varphi_1}{(1-\theta)(1+\varphi_1)+\varphi_2^{-1}} \right). \tag{13}
\]

### 4. SUBOPTIMAL SYSTEM DESIGN IN MIMO SC-FDMA SYSTEMS

In this section, we investigate two suboptimal system designs for MIMO SC-FDMA systems: 1) employing ZF equalizer; 2) assuming uniform power allocation and uniform transmit beamforming.

#### 4.1. Scenario I: System Design with ZF Equalizer

We can show that the ZF equalizer weight can be easily obtained as \( \Omega_{ZF} = [\mathbf{P}_u^\dagger \mathbf{U}_p]^{-1} \mathbf{P}_u^\dagger \), which leads to \( \omega_i = u_i / (p_i |u_i|^2) \). Plugging this into (7), we see that the overall SINR with ZF equalizer can be given as
\[
\Gamma^{(1)} = \frac{\sigma_n^2}{\sigma_s^2} \left( \frac{\epsilon_i \sum_{j=1}^{i} \rho_{max,j}^{-1/2}}{M + \sum_{j=1}^{M} \rho_{max,j}} \right)^{-1}. \tag{14}
\]

Hence, the optimization problem can be formulated as:
\[
\max_{\mathbf{p}, \alpha_i, \beta_i} \Gamma^{(1)} \quad \text{in (14)} \tag{15}
\]

The optimal solution of \((p, \alpha_i, \beta_i)\) can be found as follows:
Theorem 4.1. Optimal solution of \((p, \alpha_i, \beta_i)\) in (15) is

\[
\begin{align*}
p^{(1)}_{\text{opt},i} &= \zeta_1 \rho_{\text{max},i}^{-1/4}, \\
\alpha^{(1)}_{\text{opt},i} &= \mathcal{L}_{\text{max}}(H_i^\dagger H_i), \\
\beta^{(1)}_{\text{opt},i} &= H_i \mathcal{L}_{\text{max}}(H_i^\dagger H_i) / \|H_i \mathcal{L}_{\text{max}}(H_i^\dagger H_i)\|
\end{align*}
\]

where \(\zeta_1\) is a constant that ensures \(\|p^{(1)}_{\text{opt}}\|^2 = M\).

We omit the proof and refer the reader to [5]. Given \((p^{(1)}_{\text{opt}}, \alpha^{(1)}_{\text{opt},i}, \beta^{(1)}_{\text{opt},i})\), we see that the optimal SINR is

\[
\Gamma_{\text{opt}}^{(1)} = \omega_{\text{opt}}^{(1)} - 1/2 \sum_{i=1}^{M} 1 / \rho_{\text{max},i}^{-1/2},
\]

and the achievable capacity per unit bandwidth (b/s/Hz) is

\[
C_{\text{opt}}^{(1)} = \log_2 \left[ 1 + \left( \frac{1}{M} \sum_{i=1}^{M} \rho_{\text{max},i}^{-1/2} \right)^{-2} \right].
\] (18)

4.2. Scenario II: System Design with Uniform Power Allocation and Uniform Transmit Beamforming

When there is no CSI available at the transmitter, uniform power allocation and uniform transmit beamforming are usually utilized at the transmitter. In this scenario, \(p = 1\), \(\alpha = \zeta_0^{-1/2} 1\) and the optimization is:

\[
\begin{align*}
\max_{\beta_{\text{t},i}, \omega} & \quad \Gamma^{(2)} \text{ in (7)} \\
\text{s.t.} & \quad \mathbf{u}_i = \zeta_0^{-1/2} \beta_{\text{t},i} \mathbf{H}_i \mathbf{1}, \\
& \quad \|\beta_i\| = 1, \; i = 1, \ldots, M.
\end{align*}
\] (19)

The optimal solution of \((\beta_{\text{t},i}, \omega)\) can be found as follows:

Theorem 4.2. Optimal solution of \((\beta_{\text{t},i}, \omega)\) in (19) is

\[
\begin{align*}
\beta^{(2)}_{\text{opt},i} &= \mathbf{H}_i \mathbf{1} / \|\mathbf{H}_i \mathbf{1}\|, \\
\omega^{(2)}_{\text{opt}} &= \zeta_2 \rho_{i}^{-1/2} (K_l + \tilde{\rho}_i)^{-1},
\end{align*}
\]

where \(\tilde{\rho}_i = \|\mathbf{H}_i \mathbf{1}\|^2 2 \sigma_i^2 / \sigma_n^2\) and \(\zeta_2\) is an arbitrary constant.

We omit the proof and refer the reader to [5]. Given \((\beta^{(2)}_{\text{opt},i}, \omega^{(2)}_{\text{opt}})\), we see that the optimal SINR is

\[
\Gamma_{\text{opt}}^{(2)} = \sum_{i=1}^{M} \tilde{\rho}_i (K_l + \tilde{\rho}_i)^{-1},
\]

and the achievable capacity per unit bandwidth (b/s/Hz) is

\[
C_{\text{opt}}^{(2)} = -\log_2 \left( \frac{1}{M} \sum_{i=1}^{M} \frac{K_l}{K_l + \tilde{\rho}_i} \right).
\] (22)

5. SIMULATION RESULTS

We simulate a MIMO SC-FDMA system assuming\(^2\): QPSK modulation scheme, \(N = 256\), \(M = 64\) and CP length \(l_{\text{CP}} = 16\). Additionally, we assume a 6-tap fading channel and each tap is modeled as a complex independent and identical distributed (i.i.d.) Rayleigh fading coefficient. We further assume the fading coefficients are normalized to ensure that the channel gain for transmitted symbol is equal to unity.

\(^2\) We also studied the PAPR characteristics for system designs in MIMO SC-FDMA systems. Due to space limitations, we omit it in this paper.

In Fig. 3, we plot the average capacity performance of the optimal and suboptimal system designs in MIMO SC-FDMA systems. As expected, we see that the average capacity performance can be significantly improved by increasing the number of antennas. Additionally, we note that when \(K_t > 1\), without optimal power allocation and adaptive beamforming at the transmitter, the system design in scenario II has the worst capacity performance compared to other system designs. Furthermore, we see that the average capacity performance of the optimal design is close to that of the suboptimal design in scenario I. This is primarily due to the fact that when the number of antennas increases, the fluctuation of the channel gains in the frequency domain becomes moderate.

6. REFERENCES


