RESOURCES ALLOCATION FOR OFDMA COGNITIVE RADIOS UNDER CHANNEL UNCERTAINTY
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ABSTRACT
A weighted sum-rate maximization problem is considered for an access point (AP) allocating resources to wireless cognitive radios (CRs) communicating using orthogonal frequency-division multiple access (OFDMA). To protect the incumbent primary user (PU) system operating over the same frequency band, the interference inflicted to the PU receiver must be regulated. Since the channel gain estimates from the AP to the PU receiver may well be inaccurate in practice, a probabilistic interference power constraint is adopted. Although the latter translates to a second-order cone constraint, the overall optimization problem for joint transmit-power and subcarrier allocation is non-convex, and coupled over all subcarriers in general. To circumvent this hurdle, a tight polyhedral approximation of the second-order cone is employed. A near-optimal solution can then be efficiently obtained using the dual method.

Index Terms— cognitive radios, resource allocation, OFDMA, channel uncertainty, optimization.

1. INTRODUCTION

Through by sensing and dynamic resource allocation (RA), cognitive radios (CRs) aim to opportunistically reuse spectral resources licensed to primary user (PU) systems. Under a spectrum underlay model, CRs can transmit concurrently with the PUs, provided that the interference experienced by the PU system is strictly regulated. In order to utilize a large swath of bandwidth efficiently, CR systems are often designed based on multi-channel access such as OFDMA. The key RA issue in OFDMA is to optimally determine power allocation across the subcarriers, and assign disjoint sets of subcarriers to individual users. Extensive research is already available on this topic [1], [2].

In the context of CR, it has been recognized that RA must account for channel uncertainty [3], [4]. Particularly challenging is accurate estimation of CR-to-PU channels, since the incumbent PU system often does not explicitly support channel estimation for the CR system. In addition, the CR system might not have prior knowledge of PU signal characteristics, and thus needs to resort to less efficient channel estimation techniques. Nevertheless, PU transmissions have to be strictly protected from the interference due to the CRs.

Very few works have addressed RA problems for OFDMA-based CRs under channel uncertainty. Such a problem was considered in [5] with perfect channel gain knowledge, and also in [6] under imperfect sensing (PU presence detection). Spatial beamformer design was pursued for CRs under channel uncertainty in [3], [4]. This work is supported by NSF grants CCF-0830480, 1016605, and ECCS-0824007, ECCS-1002180. N. Soltani gratefully acknowledges funding support from Iran Telecommunication Research Center (ITRC).

The present paper formulates a weighted sum-rate maximization problem to be solved by the access point (AP) under a probabilistic interference constraint, which accounts for the uncertainty in the CR-to-PU channel. The interference constraint can be re-written as a second-order cone constraint, which is convex. However, the overall RA problem is non-convex in general, and does not have separable structure. To mitigate these difficulties, a tight linear approximation for the interference constraint is introduced at a modest (polynomial) increase in the problem dimension. The approximated problem may readily separable via the dual decomposition method, which leads to a near-optimal but computationally efficient solution [1].

The rest of the paper is organized as follows. Sec. 2 states the problem, and Sec. 3 briefs a polyhedral approximation technique for second-order cones. The RA algorithm is developed in Sec. 4. Numerical results are presented in Sec. 5, followed by conclusions in Sec. 6.

2. PROBLEM STATEMENT

Consider an AP allocating resources to K CR receivers (users) employing OFDMA using N subcarriers. The channel gain h_k(n) between the AP and the k-th receiver for each k ∈ K = {1, 2, ..., K}, and each subcarrier n ∈ {1, 2, ..., N} is assumed to be perfectly known. Suppose also that during the sensing phase, the CR AP has detected the presence of an active PU. Let g(n) denote the channel gain from the AP to the PU receiver on subcarrier n. Due to the lack of cooperation from the PU system, it is difficult to estimate {g(n)} precisely. To capture the uncertainty, it is assumed that g ≜ [g(1) g(2) ... g(N)]' is a random vector with mean g and covariance C_g.

The relevant RA problem is to maximize the weighted sum-throughput of the CR system while adhering to a transmit-power constraint, and a PU interference constraint. Let p(n) denote the transmit-power loaded on subcarrier n, and P_max the maximum transmit-power allowed on subcarrier n. Vectors p and P_max collect {p(n)} and {P_max}, respectively. Also, let k(n) ∈ K = {1, 2, ..., K} represent the index of the user served on subcarrier n, and define k≜[k(1),...,k(n)]'. The positive weight of user k ∈ K is denoted by w_k. Then, the following chance-constrained optimization problem is of interest:

(P1) \[ \max_{0 \leq p_k \leq P_{\text{max}}, k \in K} \sum_{n=1}^{N} w_{k(n)} \log(1 + h_{k(n)}(n)p_{k(n)}) \] subject to \[ \sum_{n=1}^{N} p_{k(n)} \leq P_{\text{max}} \] \[ \Pr \left\{ \sum_{n=1}^{N} g_{k(n)}p_{k(n)} > I_{\text{max}} \right\} \leq \epsilon \]
where (3) enforces the interference afficted to the PU not to exceed $I_{\text{max}}$ with probability at least $1 - \epsilon$. By invoking the central limit theorem, the latter chance constraint can be equivalently written as
\[
P \in C \triangleq \left\{ p \left| f_{\text{max}} - g^T p \geq Q^{-1}(\epsilon) \sqrt{p^T C_p p} \right. \right\}
\] (4)
where $Q(\cdot)$ is the standard Gaussian tail function. Note that (4) is a second-order cone constraint. Thus, the overall optimization problem consisting of (1), (2) and (4) is convex provided $K = 1$, i.e., for a single-user system. In the general case of multiple users with $K > 1$, (P1) is non-convex due to the combinatorial assignment of users on each subcarrier.

Consider approximating the constraint (4) so as to obtain an optimization problem that is separable into $K$ per-subcarrier subproblems via dual decomposition.

To this end, a general result that approximates second-order cone constraints by polyhedral constraints is employed, which is briefed next.

3. POLYHEDRAL APPROXIMATION OF SECOND-ORDER CONES

Motivated by the availability of efficient large-scale linear program (LP) solvers, a second-order cone problem was tackled via an approximate LP in [7]. To this end, it was shown that the second-order Lorentz cone
\[
\mathcal{L}^N \triangleq \left\{ (y_0, y) \in \mathbb{R} \times \mathbb{R}^N : ||y|| \leq y_0 \right\}
\] (5)
admits a polyhedral approximation of accuracy $\delta$, comprising variables and constraints whose number is polynomial in $N$ and $\log(1/\delta)$; see also [8] for a refinement of this result. Here, we briefly recap the idea behind this approximation.

Direct construction of a polyhedral approximation circumstances $\mathcal{L}^N$ in the $(N + 1)$-dimensional space is bound to have its number of facets growing exponentially with the dimension $N$, which implies that its needs an exponentially growing number of linear inequalities to define. The key idea is to reduce the number of inequalities by lifting the polyhedron to a higher-dimensional space by introducing additional variables, and considering its projection onto the $(N + 1)$-dimensional subset. Since a projection of a higher-dimensional polyhedral set can significantly multiply the number of facets, this approach yields a relaxation that is “efficient” in the sense that it is very tight, yet it is defined using a relatively small number of constraints and extra variables.

The first step is to decompose the $(N + 1)$-dimensional Lorentz cone $\mathcal{L}^N$ to a number of $3$-dimensional Lorentz cones using the “tower of variables” concept. Suppose for simplicity that $N = 2^d$ for some integer $d$. Then, by introducing a vector of $N/2$ new variables $\hat{\rho}^{(1)} = \left[ \rho_1^{(1)}, \rho_2^{(1)}, \ldots, \rho_{N/2}^{(1)} \right]^T$, where the superscript $(i)$ in $\rho^{(i)}$ denotes the $i$-th layer of the “tower,” $\mathcal{L}^N$ can be equivalently written as
\[
\mathcal{L}^N = \left\{ (y_0, y) \in \mathbb{R} \times \mathbb{R}^N : \exists \rho^{(1)} \in \mathbb{R}^{N/2}, \sum_{n=1}^{N/2} \rho_n^{(1)T} \leq y_0, \right. \\
y_0^2 + y_{2i-1}^2 + y_{2i}^2 \leq \rho_i^{(1)2}, i = 1, 2, \ldots, \frac{N}{2} \right\}
\] (6)
Decomposing further the $(N + 1)$-dimensional Lorentz cone in (7) by applying the idea repeatedly, one obtains eventually $(N - 1)$ three-dimensional second-order cone constraints via $(N - 2)$ new variables $\rho^{(i)} = \left[ \rho_1^{(i)}, \rho_2^{(i)}, \rho_{N/2}^{(i)} \right]^T$, $i = 1, 2, \ldots, d - 1$.

The remaining task is to approximate $\mathcal{L}^2$ using a polynomial number of variables and constraints. Consider a polyhedral $\delta$-relaxation $\Pi_\delta^N$ of $\mathcal{L}^N$ with $\delta > 0$ in the sense that
\[
\mathcal{L}^N \subset \Pi_\delta^N \subset \mathcal{L}^N := \left\{ (y_0, y) \in \mathbb{R} \times \mathbb{R}^N : ||y|| \leq (1 + \delta)y_0 \right\}
\] (7)
Then, for an integer $q$ with
\[
\delta = \frac{1}{\cos \left( \frac{\pi}{2q} \right)} - 1
\] (9)
it can be shown that the set of points $(y_0, y_1, y_2) = (\alpha_{q+1}, \alpha_0, \beta_0)$ satisfying the following set of linear constraints is a $\delta$-relaxation of $\mathcal{L}^2$ [7], [8]:
\[
\begin{align*}
\alpha_{i+1} &= \alpha_i \cos \left( \frac{\pi}{2q} \right) + \beta_i \sin \left( \frac{\pi}{2q} \right), & i = 0, 1, \ldots, q \\
\beta_{i+1} &\geq \beta_i \cos \left( \frac{\pi}{2q} \right) - \alpha_i \sin \left( \frac{\pi}{2q} \right), & i = 0, \ldots, q - 1
\end{align*}
\] (10)
where $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_q]^T$ and $\beta = [\beta_1, \beta_2, \ldots, \beta_q]^T$ are extra variables introduced to ”lift” the approximation to a higher-dimensional space. Thus, $2q$ extra variables have been introduced to form $(q + 1)$ equality constraints and $2q$ inequality constraints. One can further reduce the number of variables and constraints by eliminating $\alpha$ and $\beta_q$ using the equalities (10). The resulting set of linear constraints contains $(q - 1)$ extra variables and only $2q$ linear inequality constraints.

Overall, using $\delta_q$-relaxations for the cones in the $\ell$-th layer, a polyhedral $\delta_q$-relaxation of $\mathcal{L}^N$ can be obtained as
\[
\Pi_{\delta_q}^N := \left\{ (y_0, y) = (\rho^{(d)}, \rho^{(0)}) \in \mathbb{R} \times \mathbb{R}^N : \exists \rho^{(1)}, \ldots, \rho^{(d-1)}, \right. \\
\left. \rho_1^{(\ell)}, \rho_2^{(\ell)}, \rho_{N/2}^{(\ell)}, i = 1, \ldots, N, \ell = 1, \ldots, d \right\}
\] (12)
where $\delta = \prod_{\ell=1}^d (1 + \delta_\ell) - 1$ holds. Thus, the overall approximation introduces $\nu(N) := (N - 2) + \sum_{\ell=1}^d (q_\ell - 1)\frac{N}{2q_\ell}$ extra variables, and $c(N) := \sum_{\ell=1}^d \frac{N}{2q_\ell}$ inequality constraints. Given the overall accuracy requirement $\delta$, $\delta_\ell$ were optimized in [8], and can be obtained by plugging in $q_\ell = \left[ \frac{\pi}{2q} \right] - \left[ \log_q \left( \frac{16}{2q} \pi^2 \log(1 + \delta) \right) \right]$ to (9).

4. RESOURCE ALLOCATION ALGORITHM

In order to obtain a feasible solution to the original robust RA problem, the set of linear constraints approximating (4) must be tighter than the original constraint (4). Thus, for a given small positive constant $\delta$, consider a tightened constraint set given by
\[
\mathcal{C}' \triangleq \left\{ p \left| f_{\text{max}} - g^T p \geq Q^{-1}(\epsilon) \sqrt{p^T C_p p} \right. \right\}
\] (13)
Then, a $\delta$-relaxed lifted polyhedral approximation of the form
\[
C'_{\delta} \triangleq \left\{ p \mid \exists q \in \mathbb{R}^{n_q}, A p + B q \preceq b \right\}
\]
exists, where $A \in \mathbb{R}^{n_x \times N}, B \in \mathbb{R}^{n_y \times n_q}$, and $b \in \mathbb{R}^{n_c}$ are obtained from the procedure outlined in Sec. 3, and $q \in \mathbb{R}^{n_q}$ is the vector of additional variables, with $n_c = c(N)$ and $n_q = v(N)$. From (8), it can be seen that
\[
C' \subset C'_{\delta} \subset C
\]
holds. Therefore, the following optimization problem is a conservative surrogate for (P1):
\[
\begin{align}
\text{(P2)} & \quad \max_{p, q, k \in \mathbb{K}} \sum_{n=1}^{N} w_k(n) \log(1 + h_k^{(n)} p^{(n)}) \\
& \quad \text{subject to} \quad \sum_{n=1}^{N} p^{(n)} \leq P_{\max} \\
& \quad \quad 0 \leq p^{(n)} \leq P_{\max}, \quad n = 1, 2, \ldots, N \\
& \quad \quad A p + B q \preceq b.
\end{align}
\]
Problem (P2) is again non-convex. However, it can be shown that the duality gap vanishes asymptotically as $N \to \infty$ [1]. Therefore, (P2) can be solved efficiently using the dual method. Introducing dual variables $\lambda \geq 0$ and $\mu \triangleq [\mu_1, \mu_2, \ldots, \mu_n]^T \geq 0$, one can write the (partial) Lagrangian as
\[
L(p, q) = \sum_{n=1}^{N} w_k(n) \log(1 + h_k^{(n)} p^{(n)}) - \lambda \left( \sum_{n=1}^{N} p^{(n)} - P_{\max} \right) - \mu^T (A p + B q - b) \\
= \sum_{n=1}^{N} \left\{ w_k(n) \log(1 + h_k^{(n)} p^{(n)}) - \lambda + \mu^T A(:, n) p^{(n)} \right\} + \lambda P_{\max} - \mu^T B q + \mu^T b
\]
where $A(:, n)$ denotes the $n$-th column of matrix $A$. The dual function is given by
\[
D(\lambda, \mu) = \sup_{0 \leq p \leq P_{\max}, k \in \mathbb{K}} L(p, q)
\]
\[
= \left\{ \begin{array}{ll}
\sup_{0 \leq p \leq P_{\max}, k \in \mathbb{K}} \sum_{n=1}^{N} w_k(n) \log(1 + h_k^{(n)} p^{(n)}) \\
- (\lambda + \mu^T A(:, n) p^{(n)}) + \lambda P_{\max} + \mu^T b, \quad \text{if } B^T \mu = 0
\end{array} \right.
\]
(23)
and thus the dual optimization problem by
\[
\inf_{\lambda, \mu} D(\lambda, \mu)
\]
subject to $\lambda \geq 0$, $\mu \geq 0$, $B^T \mu = 0$.
(25)
It is interesting to note that the auxiliary variables $q$ introduced for the lifted polyhedral relaxation do not need to be determined in order to obtain the dual function. Moreover, it is immediate that the optimization in (23) can be decomposed into individual tones. Define
\[
L_n(p^{(n)}, k) \triangleq w_k \log(1 + h_k^{(n)} p^{(n)}) - (\lambda + \mu^T A(:, n)) p^{(n)}.
\]
1. Initialize $\Sigma$ and $\nu \triangleq [\lambda, \theta^T]^T$. Set tolerance $\tau$.
2. Repeat
3. If $\lambda < 0$, set $d = -i_1$ (first canonical basis).
4. Or, if $Z(i, :) < 0$ for some $i \in \{1, 2, \ldots, n_c\}$, set $d = -Z(i, :) T$.
5. Otherwise:
6. Find $k^*$ and $p^*$ from (29)–(30).
7. Set $d$ equal to (31).
8. If $\sqrt{d^T \Sigma d} < \tau$, stop.
9. Perform the ellipsoid update:
10. $d \leftarrow d/\sqrt{d^T \Sigma d}$
11. $\nu \leftarrow \nu - \Sigma d/(\nu_d + 2)$
12. $\Sigma \leftarrow \Sigma - \frac{2}{\nu_d + 2} \Sigma d d^T \Sigma$

Table 1. Algorithm for solving (P2).

Then, at each subcarrier $n \in \{1, 2, \ldots, N\}$
\[
\max_{0 \leq p^{(n)} \leq P_{\max}, k \in \mathbb{K}} L_n(p^{(n)}, k(n))
\]
needs to be solved. If $k(n) = k \in \mathbb{K}$, the optimal power loading $p^*(n)[k]$ can be shown to be
\[
p^*(n)[k] = \left\{ \begin{array}{ll}
[w_k] - 1/h_k \mu_{\max} - \frac{1}{h_k} \mu_{\max}^T A(:, n) p_{\max}^n, \text{ if } \lambda + \mu^T A(:, n) > 0 \\
\mu_{\max}, \text{ otherwise}
\end{array} \right.
\]
where $[w_k] \triangleq \min\{\max\{0, a\}, b\}$. Thus, the optimal user allocation $k^*$ and power loading $p^*$ are given, respectively, by
\[
k^*(n) \in \arg\max_{k \in \mathbb{K}} L_n(p^*(n)[k], k), \quad n = 1, 2, \ldots, N
\]
\[
p^*(n) = p^*(n)[k^*(n)], \quad n = 1, 2, \ldots, N.
\]
(30)
The optimal solution of (24)–(25) can be obtained via iterative optimization methods for non-differentiable objectives, such as the subgradient method or the ellipsoid method. To ensure $B^T \mu = 0$ as required in (25), $\mu$ is parametrized by $\theta \in \mathbb{R}^{n_u}$ as $\mu = Z \theta$, where the columns of $Z$ constitute the basis vectors of the null space of $B^T$. It can be shown that
\[
- \sum_{n=1}^{N} p^*(n) - P_{\max} \left[ \begin{array}{c}
\theta^T \\
\frac{2}{\nu_d + 2} \Sigma d d^T \Sigma
\end{array} \right]
\]
is a subgradient of $D(\lambda, Z \theta)$ w.r.t. $[\lambda \theta^T]^T$. The overall procedure for solving (P2) using the ellipsoid method is presented in Table 1, where $Z(i, :)$ is the $i$-th row of $Z$ for $i = 1, 2, \ldots, n_c$.

5. NUMERICAL TESTS

The proposed RA algorithm is verified using numerical tests. In order to validate the polyhedral approximation of the second-order cone constraint, the single-user case is first examined. Recall that when $K = 1$, the original problem (P1) is convex, and can be solved efficiently to obtain the optimal power allocation over the subcarriers. $N = 16$ subcarriers were assumed, and a Rayleigh fading, 4-path channel was simulated. The covariance of the channel estimation error followed the model in [9]. A single realization of the
A weighted sum-rate maximization problem was formulated for a CR system employing OFDMA. Due to the uncertainty present in the CR-to-PU channel, a probabilistic interference constraint was imposed to protect the PU system, and shown to be equivalent to a second-order cone constraint. When only one CR user is served by the AP, the optimization problem is convex. On the other hand, when multiple users are present, the combinatorial subcarrier assignment task renders the problem non-convex and hard to solve. In fact, since the second-order cone constraint lacks separable structure, the optimization variables are coupled over all subcarriers. A polyhedral approximation was introduced to break the coupling. The dual method was then applied to decompose the overall problem to per-subcarrier sub-problems, which can be easily solved. Since complexity of the polyhedral approximation scales modestly (polynomially) in the number of subcarriers, the overall algorithm can efficiently find the near-optimal power loading and subcarrier assignment. Numerical tests verified the efficacy of the novel algorithm.

6. CONCLUSIONS

A weighted sum-rate maximization problem was formulated for a CR system employing OFDMA. Due to the uncertainty present in the CR-to-PU channel, a probabilistic interference constraint was imposed to protect the PU system, and shown to be equivalent to a second-order cone constraint. When only one CR user is served by the AP, the optimization problem is convex. On the other hand, when multiple users are present, the combinatorial subcarrier assignment task renders the problem non-convex and hard to solve. In fact, since the second-order cone constraint lacks separable structure, the optimization variables are coupled over all subcarriers. A polyhedral approximation was introduced to break the coupling. The dual method was then applied to decompose the overall problem to per-subcarrier sub-problems, which can be easily solved. Since complexity of the polyhedral approximation scales modestly (polynomially) in the number of subcarriers, the overall algorithm can efficiently find the near-optimal power loading and subcarrier assignment. Numerical tests verified the efficacy of the novel algorithm.

7. REFERENCES