ANALOG JOINT SOURCE-CHANNEL CODING IN RAYLEIGH FADING CHANNELS

Glauber Gomes de Oliveira Brante⋆, Richard Demo Souza⋆, Javier Garcia-Frias†

⋆ CPGEI, Federal University of Technology-Paraná (UTFPR), Curitiba, Brazil
† Dept. of ECE, University of Delaware, DE, USA
gbrante@ieee.org, richard@utfpr.edu.br, jgarcia@ee.udel.edu

ABSTRACT

We consider discrete-time all-analog-processing joint source-channel coding, using non-linear spiral-like curves. We assume a Rayleigh channel, where the receiver may employ or not multiple antennas. Maximum Likelihood (ML) and Minimum Mean Square Error (MMSE) detection are considered. Our results show that MMSE performs much better than ML in high CSNR in single-antenna wireless systems, while diversity combining is able to significantly reduce such performance gap, therefore making the low complexity ML decoding very attractive in the case of multiple receive antennas.

Index Terms— Joint source-channel coding, wireless channel, receive diversity, analog coding

1. INTRODUCTION

In traditional digital communication systems, the continuous source is first encoded up to the desired rate/distortion pair. Then, capacity approaching channel codes are applied. Such separate source and channel encoding is shown to be optimal [1]. The disadvantages of such systems is that, while we try to reach a performance close to the capacity, considerable complexity and delays are introduced due to the long block lengths required to approach such theoretical limits. Moreover, whenever we want to change the code rate or the distortion target, a full redesign of the system is required.

Analog communications based on the transmission of discrete-time continuous-amplitude sources can be considered an interesting alternative to digital systems. It is also well known that analog communications are optimal under certain circumstances, such as the direct transmission of uncoded Gaussian samples over AWGN channels where there is no bandwidth compression or expansion [2]. Recently, several works have focused on studying analog communications to find an efficient way to perfectly match sources with channels [3–5]. These analog joint source-channel coding schemes perform analog compression at the symbol level. Thus, introducing no delays to the communication.

We focus on systems proposed in [6, 7], which have recently acquired a renewed interest due to the works in [3–5] and many others. The idea is to represent $N$ source samples as a point in an $N$-dimensional space where a non-linear spiral-like surface lives. Then, the $N$-dimensional source sample is projected onto the spiral-like surface and the projection is transmitted through the channel. The literature [3–5] deals basically with the transmission over AWGN channels. In this paper, first we consider a two-user single-antenna scenario under a fast-fading Rayleigh channel, and Maximum Likelihood (ML) or Minimum Mean Squared Error (MMSE) decoding. Then, we investigate the effects of diversity combining at the receiver, by means of the use of multiple antennas. Our results show that MMSE performs much better than ML in the high channel signal-to-noise ratio (CSNR) region in single-antenna systems, which is much different than for the AWGN channel [8]. However, when diversity combining is applied, such gap between ML and MMSE is significantly reduced, which makes the low complexity ML interesting to practical implementation in such cases.

The rest of this paper is organized as follows. In Section 2 we describe the system model. Section 3 compares the signal-to-distortion performance of the ML and MMSE decoders in the wireless scenario. In Section 4 we analyze the effects of diversity combining. Finally, Section 5 concludes the paper.

2. SYSTEM MODEL

Consider the analog transmission of a discrete-time continuous amplitude memoryless Gaussian source over a wireless channel. At the transmitter, $N$ i.i.d. source symbols are encoded into $K$ channel symbols, and then transmitted through a real fast-fading Rayleigh channel with average energy $\sigma_h^2$. Additive white Gaussian noise (AWGN) is assumed, with variance $\sigma_w^2$. Without loss of generality, we assume that the source produces samples with zero mean and unity variance, and that the average power of the channel symbols is $\sigma_h^2 = 1$.

The source vector is denoted as $X = \{x_i\}_{i=1}^N$ (or $x$ when $N = 1$), while the received observations as $Y = \{y_i\}_{i=1}^K$ (or $y$ when $K = 1$). At the decoder, an estimate of $X$, namely $\hat{X}$, is obtained based on $Y$. The distortion of $\hat{X}$ with respect to $X$ is measured according to the mean square error (MSE):

$$MSE = \frac{1}{N} E\{||X - \hat{X}||^2\},$$  \hspace{1cm} (1)
where $E\{\cdot\}$ is the expected value and $||.||$ denotes the Euclidean distance. The system performance is measured in terms of the output signal-to-distortion ratio (SDR) versus the channel signal-to-noise ratio (CSNR). The SDR is defined as:

$$\text{SDR} = 10 \log_{10} \left( \frac{1}{\text{MSE}} \right),$$

while CSNR = $10 \log_{10}(\sigma_y^2/\sigma_n^2)$. Next we focus on the case of 2:1 compression systems, where $N = 2$ and $K = 1$.

### 2.1. 2:1 Compression

We encode the $X = (x_1, x_2)$ source samples using a type of space-filling continuous curves called spiral-like curves. The mapping function which projects the pair of coordinates $x_1$ and $x_2$ into the spiral is defined as [4]:

$$
\begin{align*}
\hat{x}_{\theta,1} &= \text{sign}(\theta) \frac{\Delta}{\pi} \sin \theta \\
\hat{x}_{\theta,2} &= \frac{\Delta}{\pi} \cos \theta \\
\end{align*}
$$

for $\theta \in \mathbb{R}$, where $\Delta$ is the distance between the two neighboring arms of the spiral, and $\theta$ is the angle from the origin to the point $X_\theta = (\hat{x}_{\theta,1}, \hat{x}_{\theta,2})$, which is the mapped version of $X$ on the curve. Each pair of source samples, $x_1$ and $x_2$, represents a specific point in $\mathbb{R}^2$ that is matched to the closest point $X_\theta$ on the spiral. The angle from the origin to that point on the spiral, $\theta$, will be the channel symbol for $x_1$ and $x_2$, so that:

$$\hat{\theta} = M_\Delta(X) = \min\{\hat{\theta}_1, \hat{\theta}_2\},$$

where $M_\Delta$ is the function which maps the source pair $X = (x_1, x_2)$ into the spiral-like function, with $\hat{\theta}_1$ and $\hat{\theta}_2$ representing $X$ mapped in each of the spiral arms, given by:

$$\hat{\theta}_1 = \arg \min_{\theta \geq 0} \{(x_1 - \frac{\Delta}{\pi} \theta \sin \theta)^2 + (x_2 - \frac{\Delta}{\pi} \theta \cos \theta)^2\},$$

$$\hat{\theta}_2 = \arg \min_{\theta \leq 0} \{(x_1 + \frac{\Delta}{\pi} \theta \sin \theta)^2 + (x_2 + \frac{\Delta}{\pi} \theta \cos \theta)^2\}.$$

To ensure the transmit power constraint, we use the invertible function $T_\alpha(\zeta) = \text{sign}(\zeta)|\zeta|^{\alpha}$ [8], and we apply a normalization factor $\sqrt{\gamma}$. Thus, the channel symbol is denoted by $T_\alpha(\hat{\theta})/\sqrt{\gamma} = \varphi$. This symbol is transmitted through the wireless channel and then de-normalized at the receiver, so that $y = (\varphi h + w) \sqrt{\gamma}$, where $h$ is the fading coefficient and $w$ is the Gaussian noise. This operation is equivalent to:

$$y = T_\alpha(M_\Delta(X))h + w \sqrt{\gamma}. \quad (7)$$

Considering the above transmission scheme, the optimal performance theoretically attainable (OPTA) can be calculated by equating the rate distortion function to the channel capacity [9], which depends on the distribution of $h$:

$$N \log \left( \frac{1}{\text{MSE}} \right) = K \int h \log \left( 1 + \frac{h^2}{\sigma_n^2} \right) p(h)dh. \quad (8)$$

A practical receiver can be designed by considering the recovery of $X$ from $Y$ by using either Maximum Likelihood (ML) or Minimum Mean Square Error (MMSE) decoding.

### 2.2. ML Decoder

The ML estimate is obtained as the source pair $\hat{X}_{ML} = \{\hat{x}_1, \hat{x}_2\}$ belonging to the curve, and satisfying:

$$\hat{X}_{ML} = \{X|X \in \text{curve} \text{ and } T_{\alpha}(M_\Delta) = y\},$$

which is equivalent to applying the inverse function $T_{\alpha}^{-1}(.)$ to the received sequence and then performing the inverse mapping to find the estimated $\hat{X}_{ML}$. The inverse is:

$$\hat{\theta}' = T_{\alpha}^{-1}\left(\frac{y}{\gamma}\right) = \text{sign}\left(\frac{y}{h}\right) \left|\frac{y}{h}\right|^\frac{1}{\alpha}, \quad (10)$$

and $\hat{X}_{ML}$ is found from $\hat{\theta}'$ according to (3). Note that perfect channel state information (CSI) at the receiver is assumed.

### 2.3. MMSE Decoder

In the MMSE decoding, estimated points are no longer constrained to lie on the spiral curve. The decoding is the expected value of a probability density function in which the a priori information of the source is involved, so that:

$$\hat{X}_{\text{MMSE}} = E\{X|y\} = \int Xp(X|y)dX = \frac{1}{p(y)} \int Xp(y|X)p(X)dX. \quad (11)$$

Supposing perfect CSI at the receiver:

$$p(y|X) = \frac{1}{2\pi (\gamma \sigma_n^2)} e^{-\frac{\|y - X_{\text{OPTA}}(\theta)\|^2}{\gamma \sigma_n^2}}. \quad (12)$$

Notice that, the conditional probability $p(y|X)$ is discontinuous and highly non-linear, since it involves the mapping $M_\Delta(.)$, and (11) can only be calculated numerically. An approximate way to calculate the integral is to perform a simple discretization. First, $X$ is discretized using a uniform step, and a mapped value for each discretized point is calculated according to (4), which leads us to a discretized version of $p(y|X)$. Then, calculating $p(X)$ for each discretized point, the solution of the above integral is simplified to multiplicative and additive operations. In addition, since this last discretization does not depend on the received symbol, once it is done off-line, it can be stored in the decoder, reducing the complexity of the MMSE decoding.

### 2.4. Parameter Optimization

The system performance is fully characterized by the curve parameters $\Delta$ and $\alpha$. Given a decoding technique, different encoding parameters yield different system performances. For ML decoding and high CSNR in the AWGN channel, the optimal parameter $\Delta'$ when $\alpha = 2$ is [3, 4]:

$$\Delta' = 2\pi \sqrt{\frac{6 \cdot 0.16^2}{\text{CSNR}}}. \quad (13)$$
In case of instantaneous CSI at the transmitter:
\[ \Delta_h = 2\pi \sqrt{\frac{6 \cdot 0.162 \cdot \sigma^2_w}{h^2}}. \]  

(14)

It is not realistic to assume that the transmitter has perfect CSI. A more practical consideration is the knowledge of the CSNR. Since \( \sigma^2_w = 1 \), the optimal \( \Delta \) is given by (13).

If \( \alpha \neq 2 \) or if the CSNR is too low, (13) and (14) are no longer optimal. In such cases, the analytical optimization of the system parameters is impractical. Instead, the optimization is performed numerically, by extensively calculating the SDR over a wide range of \((\Delta, \alpha)\) pairs for each CSNR.

In ML decoding, decoded points lie on the non-linear curve. In MMSE decoding, however, decoded points can be at any place in the source plane, which allows to reduce the approximation distortion. Specifically, for a received observation, the MMSE solution is always closer to the origin when compared to the ML decoded point [8]. Again, it is not possible to perform the optimization analytically for MMSE, and we must perform it numerically.

### 3. NUMERICAL RESULTS

We evaluate the performance of a 2:1 compression system in a Rayleigh fading scenario. Both ML and MMSE decoders are analyzed considering three sets of parameters: i) \( \Delta' \), as in (13), and \( \alpha = 2 \); ii) \( \Delta_h \), as in (14), and \( \alpha = 2 \); iii) \( \Delta \) and \( \alpha \) numerically optimized for each CSNR for each decoding strategy. The SDR of the ML and MMSE decoders, as well as the theoretical limit (OPTA), are shown in Figure 1. The OPTA was obtained via the Monte Carlo method.

#### Fig. 1. Performance of the 2:1 system in Rayleigh fading channels with ML and MMSE decoders.

From the figure we can see that MMSE performs much better than ML, mostly in the low CSNR region. This increased performance can be observed specially when CSI is available only at the receiver (\( \alpha = 2 \) and \( \Delta' \)) and the transmitter has knowledge only of the CSNR. In this case, in the high CSNR region, MMSE is up to 2.2dB better than ML. When we assume that CSI is available also at the transmitter (\( \alpha = 2 \) and \( \Delta_h \)), the performance of ML is increased in high CSNR, approaching that of MMSE. When the system parameters are numerically optimized for each decoder, we can observe that the ML performance is increased specially in the low CSNR region, with low impact on high CSNR. Moreover, the numerical optimization of the MMSE parameters leads to performance improvements of around 1dB in the whole CSNR range. At high CSNR, the optimized MMSE SDR is 5.5dB away from the OPTA.

These results indicate an important difference from the AWGN results obtained in [8]. When fading is not present, the performance of ML and MMSE decoders is very similar in the high CSNR region. Therefore, in a practical implementation system and an AWGN channel, ML should be preferred, since it presents similar SDR performance with a reduced complexity. However, in the fading scenario presented in this paper, the performance of ML is degraded when compared to MMSE if CSI is available only at the receiver. The rationale behind this observation is that, as ML always estimates the transmitted pair on the spiral arms, when the channel fading is severe, the noise is amplified and the distortion of \( X \) with respect to \( X \) increases. On the other hand, the MMSE decoder is more sophisticated, in which the channel statistics are used in the decoding process. Therefore, the utilization of the MMSE receiver would be preferable due to its increased performance in the single antenna wireless scenario.

### 4. DIVERSITY COMBINING ON 2:1 SYSTEMS

Now consider that the source maps a sample \( X = (x_1, x_2) \) in a spiral-like curve and transmits this information to the destination using two independent channels, \( h_1 \) and \( h_2 \), which is equivalent to a system with a single antenna at the transmitter and two antennas at the receiver. At the destination we have:

\[ y_i = T_\alpha(M_\Delta(X))h_i + w_i\sqrt{\gamma}, \quad i \in \{1, 2\}, \]

(15)

where \( w_i \) is the noise associated to the \( i \)-th antenna. The received sequences \( y_1 \) and \( y_2 \) can be combined by means of at least: Selection Combining (SC), Equal Gain Combining (EGC) and Maximal Ratio Combining (MRC) [10]. In SC, we simply choose the sequence which was transmitted by the best link. Therefore, the combined sequence is \( y_{SC} = y_1 \) if \( h_1 > h_2 \), otherwise \( y_{SC} = y_2 \). The equivalent channel is \( h_{SC} = \max\{h_1, h_2\} \), while the noise variance remains unchanged, given by \( \sigma^2_{SC} = \sigma^2_w \).

EGC considers both received sequences by adding them with unitary weight. Thus, the received sequence and the equivalent channel are given by \( y_{EGC} = y_1 + y_2 \) and \( h_{EGC} = h_1 + h_2 \), respectively. By adding them at the receiver, the noise of each transmission is also added, so that \( \sigma^2_{EGC} = 2 \cdot \sigma^2_w \).
Finally, MRC combines the two sequences weighted by each fading coefficient [10], in order to take advantage from the sequence transmitted by the best channel. Thus \( y_{\text{MRC}} = y_1 h_1 + y_2 h_2 \) and \( h_{\text{MRC}} = h_1^2 + h_2^2 \). The noise variance will also reflect the combination, being expressed as \( \sigma_{\text{MRC}}^2 = h_{\text{MRC}} \cdot \sigma_n^2 \). MRC is the optimum receiver in this case [10].

4.1. SDR Performance

In the case of diversity combining, the OPTA must be modified to include both channel realizations, thus:

\[
N \log \left( \frac{1}{\text{MSE}} \right) = K \int_{h_1} \int_{h_2} \log \left( 1 + \frac{h_1^2 + h_2^2}{\sigma_n^2} \right) p(h_1)p(h_2) dh_2 dh_1.
\]

Figure 2 shows the SDR performance with diversity combining for both ML and MMSE decoding as a function of the CSNR. We consider the most common case, when CSI is available only at the receiver. Thus, both methods are parametrized with \( \Delta' \) and \( \alpha = 2 \).

![Fig. 2. Performance of the 2:1 system of the single channel transmission compared to that of SC, EGC and MRC.](image)

It is interesting to notice from the figure that, in the high CSNR region, diversity combining is able to reduce the gap between ML and MMSE performance, when compared to the case of a single antenna. This indicates that ML, being a simpler decoding technique, does not present relevant losses in the SDR performance, reducing the complexity of the receiver. At high CSNR, the distance to OPTA is reduced to 4dB with the diversity combining. Such gap can be reduced even more by parameter optimization.

5. CONCLUSION

We consider a discrete-time all-analog-processing joint source-channel coding scheme, based on the use of spiral-like curves. We assume a fast-fading Rayleigh channel, and ML or MMSE decoding to recover the original data. In the case of a single antenna system we show that MMSE considerably outperforms ML decoding, what is significantly different than the conclusion obtained for the AWGN channel in [8]. Additionally, our results show that in the case of diversity combining at the receiver, the performance gap between MMSE and ML is significantly reduced, which makes ML quite interesting to practical implementation systems, due to its reduced complexity. Moreover, it is also interesting to note that with diversity combining the gap to the OPTA is reduced compared to the single antenna case.

6. REFERENCES


