TRANSCEIVER OPTIMIZATION FOR MULTI-USER MULTI-ANTENNA TWO-WAY RELAY CHANNELS

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ABSTRACT

In this paper, we study a multi-user multi-antenna two-way relay system, where a multi-antenna base station (BS) exchanges uplink and downlink signals with multiple users via a multi-antenna amplify-and-forward relay station (RS). We jointly design the BS and RS transceivers, aiming to maximize the system bidirectional sum rate under inter-user interference free constraint. Since the optimization problem is non-convex, we employ alternating optimization algorithm to design the transmit and receive weight matrices at the BS and RS. Simulation results show that the proposed solution offers higher bidirectional sum rate than existing schemes.

Index Terms— two-way relay, multi-user, multi-antenna, transceiver

1. INTRODUCTION

Two-way relay network (TWRN) has attracted considerable interest because it overcomes the 1/2 spectral efficiency loss from the half-duplex in the conventional relay networks [1]. Most of the existing works focus on the systems with single user pair, where two users exchange information via a single relay station (RS). Various transmission strategies have been studied in different scenarios [2–4].

Recently, TWRN is extended to multi-user cases [5–9]. Basically we can classify these works into two categories based on the system topologies, symmetric and asymmetric systems. In the symmetric systems [5], multiple user pairs exchange information via the RS. Each user has private message for the other user in the same user pair. In the asymmetric systems [6–9], a base station (BS) exchanges information with multiple users via the RS. The BS transmits different messages to different users and at the same time all users transmit their respective messages to the BS. In this paper, we focus on the asymmetric multi-user TWRN.

Inter-user interference (IUI) exists in multi-user TWRN systems. To eliminate IUI, one natural approach is to allocate different users with different sub-channels, e.g., orthogonal sub-carriers [6]. Then the signal transmission on each sub-channel reduces to that in a single user pair TWRN. Another approach is to apply multi-antenna techniques to deal with the interference. In [7], zero-forcing (ZF) method is used to separate the uplink and downlink signals of different users. In order for ZF to cancel the interference, the BS needs to have at least $2N$ antennas for an asymmetric system with a $N$-antenna BS and $N$ single antenna users.

In the multi-antenna asymmetric systems, when the BS has full channel state information (CSI) of all links, the BS and the RS can work jointly to eliminate the interference. In [8, 10], a transmission scheme was proposed based on the signal alignment (SA). In the SA scheme, the downlink signal of each user arrives at the RS on the same direction of the same user’s uplink signal via proper BS precoding. Then for the asymmetric system with a $N$-antenna BS and $N$ single antenna users, the RS only requires $N$ antennas to separate the signals from multiple users. In [9], a balanced transmission scheme was proposed to further improve the system throughput. It first designs the transceivers to respectively maximize the uplink and downlink rate, then balances the uplink and downlink rate by adjusting the power allocation at the relay.

In this paper, we jointly design the BS and RS transceiver for the multi-antenna multi-user asymmetric TWRN, which involves the transceiver weighting matrix at the RS, and the transmit and receive weighting matrices at the BS. Amplify-and-forward (AF) protocol is considered. We first formulate the optimization problem, which aims at maximizing the bidirectional sum rate under the IUI free constraints. Unfortunately, it is a non-convex problem. We then use alternating optimization approach [11] to find the three matrices. Simulation results show that the proposed transceiver offers a higher sum rate than existing schemes.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a multi-user multi-antenna TWRN, which consists of a BS equipped with $N_B$ antennas, a RS equipped with $N_R$ antennas and $N_U$ single-antenna users, where $N_U \leq \min(N_B, N_R)$. The BS and $N_U$ users exchange downlink and uplink information via the RS. The bidirectional transmission takes place in two phases.

At the first phase, both the BS and users transmit to the RS. The received signal at the RS is given by

$$y_r = \sqrt{P_B} H_{br} W_{br} x_b + \sqrt{P_r} H_{ur} x_u + n_r,$$

where $H_{br}$ is the channel matrix from the BS to the RS, $H_{ur} = (h_{1r}, h_{2r}, \ldots, h_{N_{U}r})$, $h_{ir}$ is the channel vector from the $i$th user to the RS, $W_{br}$ is the precoder matrix at BS, which satisfies the transmit power constraint $\|W_{br}\|^2 \leq 1$. $\|\cdot\|$ denotes the norm of a matrix. $P_B$ and $P_U$ are the transmit power of the BS and a single user, $x_b, x_u$ are downlink and uplink signal vectors of the $N_U$ users, and $n_r$ is the noise vector at the RS.

At the second phase, the RS precodes and broadcasts its received signals to the BS and users. The received signal at the BS is

$$y_b = W_{br}^T (H_{br}^H \sqrt{P_B} W_r y_r + n_b)$$

$$= \sqrt{P_B} P_U W_{br}^T H_{br} W_r y_r + \sqrt{P_U} P_B W_{br}^T H_{br} x_b + n_b$$

$$+ \sqrt{P_U} P_B W_{br}^T H_{ur} W_r x_u + \sqrt{P_U} P_B W_{br}^T n_r,$$

(2)

where $(\cdot)^T$ denotes the transpose of a matrix, $W_r$ is the weighting matrix at the RS, which satisfies the transmit power constraint...
∥W_iy_r∥ ≤ 1. P_R is the transmit power of the RS. W_{br} is the receive weighting matrix at the BS, and n_0 is the noise vector at the BS. Note that the first term is the BS transmitted signal in the first phase and thus can be removed by self-interference cancelation [1]. The second term is the desired uplink signal. The last two terms are the noise. To eliminate the interference among the N_U uplink signals, the following constraint should be satisfied,

\[ w_{br}^T H_r^T W_i h_{jr} = 0, \quad i \neq j, \]  

where \( w_{br} \) is the \( i \)th column of \( W_{br} \).

The received signal at the \( i \)th user is

\[ y_{ui} = h_{ri}^T \sqrt{P_R} W_i y_r + n_{ui} = \sqrt{P_R} P_0 h_{ri}^T W_i H_r^T W_{br} x_b + \sqrt{P_R} P_0 h_{ri}^T W_i H_{ur} x_u + \sqrt{P_R} P_0 h_{ri}^T W_i n_r + n_{ui}, \quad (1 ≤ i ≤ N_U), \]

where \( n_{ui} \) is the noise at the \( i \)th user. Note that the first two terms correspond to the downlink and uplink signals of all users, and the last two terms are the noise. The uplink and downlink signals of other users are the interference to the \( i \)th user. To eliminate the IUI, the transceivers should satisfy,

\[ h_{ri}^T W_i H_r^T W_{bdj} = 0, \quad i \neq j, \]  

\[ h_{ri}^T W_i h_{jr} = 0, \quad i \neq j, \]

where \( W_{bdj} \) is the \( j \)th column of \( W_{bd} \).

We assume that the transmitted signals of the BS and users have unit average power, and all the noise signals subject to a complex Gaussian distribution with zero mean and variance \( N_0 \). We also assume that the BS and RS have all the CSI as in [8].

When the constraints (3), (5) and (6) are met, and considering that the self-interference can be canceled in TWRN [1], the BS and each user will receive their uplink and downlink signals without interference. Then the received signal-to-noise-ratio (SNR) of the \( i \)th uplink and downlink signal can be obtained as,

\[ SNR_{ui} = \frac{P_R P_0 |w_{br}^T H_r^T W_i h_{ri}|^2}{N_0 P_R |w_{br}^T H_r^T W_i W_{br}|^2 + N_0 |w_{br}^T|^2}, \]

\[ SNR_{di} = \frac{P_R P_0 |h_{ri}^T W_i H_r^T W_{bdj}|^2}{N_0 P_R |h_{ri}^T W_i H_r^T W_{bdj}|^2 + N_0 |h_{ri}^T|^2}. \]

and the system bidirectional sum rate is as follows,

\[ R_S = R_U + R_D = \sum_{i=1}^{N_U} (R_{ui} + R_{di}) = \sum_{i=1}^{N_U} \left( \frac{1}{2} \log_2 (1 + SNR_{ui}) + \frac{1}{2} \log_2 (1 + SNR_{di}) \right), \]

where \( R_U \) and \( R_D \) denote the system uplink and downlink sum rate, and \( R_{ui} \) and \( R_{di} \) are respectively the uplink and downlink data rate of the \( i \)th user. The factor 1/2 is due to the two-phase half-duplex transmission.

We jointly design the BS and RS transmit and receive weighting matrices to maximize the bidirectional sum rate \( R_S \) under the IUI free constraints as well as the transmit power constraints. The optimization problem is formulated as

\[ \begin{align*}
\max_{W_r, W_{br}} & \quad R_S \\
\text{s.t.} & \quad (3), (5) \text{ and } (6), \\
\|W_{bd}\|^2 & \leq 1, \\
\|W_r y_r\|^2 & \leq 1.
\end{align*} \]

### 3. Transceiver Optimization for TWRN

The optimization problem in (10) is non-convex, which is difficult to solve. We employ the alternating optimization approach [11] to design the three matrices, \( W_r \), \( W_{br} \) and \( W_{bd} \). We first assign an initial value for each of them, which should satisfy all the constraints in (10). Then we alternatively optimize one of the three matrices by fixing the other two. The iterative optimization of the three matrices continues until the bidirectional sum rate does not increase any more. Note that the alternating optimization does not guarantee a global optimal solution, but it at least achieves a local optimal solution [11].

#### 3.1. Optimization of RS Weighting Matrix \( W_r \)

We first design \( W_r \) to maximize the bidirectional sum rate with fixed \( W_{bd} \) and \( W_{br} \). The optimization problem is given as follows.

\[ \begin{align*}
\max_{W_r} & \quad R_S \\
\text{s.t.} & \quad (3), (5), (6) \text{ and } (10d).
\end{align*} \]

Since \( R_S \) is not a convex function of \( W_r \), this problem is not convex. Borrowing the idea from [4, 7], the maximum sum rate can be found by searching over the system achievable rate region boundary. We introduce a vector \( \beta = (\beta_1, \ldots, \beta_{2N_U}) \), where \( \sum_{i=1}^{2N_U} \beta_i = 1 \) and \( \beta_i ≥ 0 \). By solving the original problem (11) with additional rate ratio constraints as follows,

\[ \begin{align*}
R_{ui} & \geq \beta_i R_S, \quad R_{di} \geq \beta_i + N_U R_S, \quad 1 ≤ i ≤ N_U, \quad (12)
\end{align*} \]

a boundary point of the achievable rate region will be achieved. Each possible \( \beta \) corresponds to a boundary point. We need to search the optimal \( \beta \) corresponding to the maximum sum rate. Although it is hard to rigorously prove that the achievable rate region boundary is a convex hull, simulation results show that efficient algorithms, e.g., bisection method, offer the same result as that of brute-force searching. Therefore, we use bisection method here to search the optimal \( \beta \) as in [7].

Analogous to the approach in [4], the original problem (11) with extra constraint (12) can be resolved by solving the following power minimization problem.

\[ \begin{align*}
\min_{W_r} & \quad \|W_r y_r\|^2 \\
\text{s.t.} & \quad (3), (5), (6) \text{ and } (12).
\end{align*} \]

For a given \( R_S \), if the minimum power found from (13) is less than 1, the given \( R_S \) is a feasible solution of the original problem. We can solve the original problem by searching the largest \( R_S \) that can keep the minimum power of (13) lower than 1, where bisection method is applied to search \( R_S \).

Define \( x_{wr} = \text{vec}(W_r) \), where \( \text{vec}(\cdot) \) means the vectorization of a matrix. The problem (13) can be reformulated into the following form (see [12] for detailed derivation).

\[ \begin{align*}
\min_{x_{wr}} & \quad K_{wr} x_{wr} \\
\text{s.t.} & \quad x_{wr}^H K_{wr} x_{wr} \geq c_{ui}, \quad x_{wr}^H K_{di} x_{wr} \geq c_{di}, \quad 1 ≤ i ≤ N_U, \\
K_{di} x_{wr} & = 0.
\end{align*} \]

where we define

\[ g(X_1, X_2) \triangleq (X_1 \otimes X_2)^H (X_1 \otimes X_2), \]

\[ K_{wr} \triangleq P_R g(W_{bd}^H H_{br}, I_{N_R}), \]

\[ K_{ui} \triangleq P_R P_0 g(h_{ri}^H W_{br}, H_{ur}), \]

\[ K_{di} \triangleq P_R P_0 g(w_{br}^T H_{bd}, H_{ur}) - r_{ui} N_0 P_R g(I_{N_R}, H_{ur}), \]
c_{ui} \triangleq r_{ui} N_0 \|w_{bri}^T\|^2, \quad r_{ui} \triangleq 2^{2\beta_i} R_S - 1,
\quad c_{di} \triangleq r_{di} N_0, \quad r_{di} \triangleq 2^{2\beta_i+NU} R_S - 1,
K_\Omega \triangleq (h_{jr} \otimes H_{brwbri}, H_{brwbti} \otimes h_{jr}, h_{jr} \otimes h_{ri})^T.

I_N is an identity matrix of size N, and \otimes is the Kronecker product.

This is a NP-hard problem [13]. Here we apply the widely used semidefinite relaxation (SDR) and the randomization method to handle it. It shows that in most cases, the optimized results are quite close to the global optimal solution [13].

### 3.2. Optimization of BS transmit weighting matrix \( W_{bt} \)

If \( W_{br} \) and \( W_r \) are fixed, \( W_{bt} \) only affects the downlink data rate. Therefore, we design \( W_{bt} \) to maximize the downlink sum rate \( R_D \).

Among the five constraints in (10), \( W_{bt} \) is related to (5) and (10c). In addition, since the RS received signal \( y_r \) is a function of \( W_{bt} \) according to (1), \( W_{bt} \) is also related to the RS power constraint (10d), which can be rewritten as follows after substituting (1),

\[
P_{\text{D}} \|W_r H_{br} W_{bt}\|^2 + P_{\text{D}} \|W_r H_{brw}\|^2 + N_0 \|W_r\|^2 \leq 1. \quad (15)
\]

Therefore, the optimization problem is formulated as

\[
\max_{W_{bt}} R_D \quad \text{s.t.} \quad (5), (10c) \text{ and } (15). \quad (16)
\]

This is also a non-convex problem. We apply the same method that we used to solve (11). Define \( \beta = [\beta_1, \ldots, \beta_N] \), where \( \beta_i \geq 0 \) and \( \sum_{i=1}^{N_U} \beta_i = 1 \). Then we can find the solution of the problem (16) by solving the following problem and searching the optimal \( \beta \),

\[
\max_{W_{bt}} R_D \quad \text{s.t.} \quad R_{di} \geq \beta_i R_D, \quad 1 \leq i \leq N_U, \quad (17a)
\]

\[
\max_{W_{bt}} R_D \quad \text{s.t.} \quad (5), (10c) \text{ and } (15). \quad (17b)
\]

Again, efficient searching algorithms, e.g. bisection, can be applied to search the optimal \( \beta \).

Note that any phase rotation on each column of \( W_{bt} \), i.e., \( w_{bt} \), will not affect the constraints and objective function in (17). By adding a proper phase rotation on each \( w_{bt} \), (17b) can be reformulated into linear constraint [14]. Then all constraints in (17) form a second-order-cone (SOC) feasible region [14]. If \( R_D = R_D^* \) is a feasible solution of (17), the SOC feasible region must be empty. Therefore, we solve (17) by searching the largest \( R_D \) that guarantees a non-empty feasible region. Bisection method is used to search \( R_D \). We use the CVX tool [15] to check whether the SOC feasible region is empty or not. If it is not empty, the CVX tool will return a value of \( W_{bt} \) in the feasible region. Finally, we will obtain both the maximum \( R_D \) and the optimal \( W_{bt} \).

### 3.3. Optimization of BS receive weighting matrix \( W_{br} \)

If \( W_{bt} \) and \( W_r \) are fixed, \( W_{br} \) only affects the uplink data rate. We design \( W_{br} \) to maximize the uplink sum rate. The optimization problem is formulated as,

\[
\max_{W_{br}} R_U \quad \text{s.t.} \quad w_{bri}^H H_{brw} W_r h_{jr} = 0, \quad i \neq j. \quad (18)
\]

According to (7) and (9), the data rate of each uplink stream, \( R_{usi} \), is only a function of \( w_{bri} \). Therefore, the above problem can be decoupled into \( N_U \) subproblems, where each optimizes a column of the \( W_{br}, w_{bri} \), to maximize \( R_{usi} \). Since \( R_{usi} \) is a monotonic increasing function of \( SN R_{usi} \), each subproblem can be given by

\[
\max_{w_{bri}} \quad SN R_{usi} \quad \text{s.t.} \quad w_{bri}^H H_{brw} W_r h_{jr} = 0, \quad i \neq j. \quad (19a)
\]

Define \( A_i = (H_{brw}^H W_r h_{jr})_{j \neq i} \) as a matrix consisting of all \( H_{brw}^H W_r h_{jr} \) for \( j \neq i \), and denote \( U_i^+ \) as a matrix consisting of all the singular vectors of \( A_i \) corresponding to its zero singular values. Then any feasible \( w_{bri} \) satisfying (19b) can be written as \( w_{bri} = U_i^+ x \), where \( x \) is an arbitrary vector. Substitute \( w_{bri} \) and \( h_{jr} \) is also related to the RS power constraint (10d), which can be rewritten as an unconstrained Rayleigh ratio maximization problem,

\[
\max_{x} \quad x^T U_i^+ W_r^H H_{brw} W_r U_i^+ x \quad \text{s.t.} \quad x^T U_i^+ W_r H_{brw} W_r U_i^+ x = 1 \quad (20)
\]

where \((\cdot)^*\) denotes conjugate of a complex variant, and

\[
K_{\Omega} \triangleq P_B P_R H_{brw} H_{brw}^H H_{brw}^H + N_0 N_0^T
\]

The optimal \( x \) is the eigenvector of \( U_i^T K_{\Omega} U_i^+ (U_i U_i^+)^{-1} \) corresponding to its largest eigenvalue [16]. Following \( w_{bri} = U_i^+ x \), we finally obtain the optimal \( W_{br} \).

By now we have solved all the three problems (11), (16) and (18), each of which optimizes one of \( W_r, W_{bt} \) and \( W_{br} \) with the other two being fixed.

### 4. SIMULATIONS

In this section, we investigate the performance of our proposed transceivers by simulations. We assume that all the channels are independently identically distributed Rayleigh fading channels, and that the noise variance \( N_0 \) is identical at the BS, RS and each user. The transmit power of each user is normalized as 1. The BS and RS transmit power are respectively denoted as \( P_B \) and \( P_R \). We define \( 1/N_0 \) as the transmit SNR.

First we study the convergence characteristics of the alternating optimization solution. The antenna number at the BS and RS, and the user number are set as 2, 4 and 2, respectively. The transmit power of the BS and RS are both set as 2. The transmit SNR is 30dB. We respectively use the ZF, SA [8], balanced transceiver scheme [9] and random weighting matrices as the initial value for the iterative optimization. Fig. 1 shows the system bidirectional sum rate versus the iteration number.

In each iteration, each of the three transceiver weighting matrices is alternatively optimized once with the other two being fixed. Since we require that after each iteration, the system sum rate must be higher than that in previous iteration, the system sum rate will surely converge to a local maximum. Since the original problem is not a convex problem, the optimization result depends on the initial values. Among the four compared initial values, the balanced transceiver scheme [9] converges to the highest bidirectional sum rate. In the following simulation, we use that scheme as the initial value for the alternating optimization.

Then we compare the rate performance of the optimized transceivers with the existing ones. We set the BS number and the user number as \( N_B = N_U = 2 \). The normalized transmit power
of the BS and RS are both fixed as 2. The transmit SNR is 30dB. We change the RS antenna number and simulate the sum rate of all the schemes, respectively. The simulation result is shown in Fig. 2, where “Al−Opt” denotes the alternating optimization.

Fig. 1. Convergence characteristic of the proposed alternating optimization solution with different initial values

All the four compared schemes are designed under the same interference free constraints, however, the sum rates of them vary a lot. Fig. 2 shows that neither the existing ZF nor SA scheme is always a good approach to achieve zero-interference. In TWRN, the joint BS and RS processing offers the system more choices rather than the ZF and SA structure to cancel all the interference. The performance of proposed alternating optimization shows that it is possible to design a joint processing structure, which can fulfill the zero-interference constraint and offer higher sum rate than both the existing structures. The performance of the balanced scheme [9] and the alternating optimization are quite close. It indicates that the balanced scheme [9] is at least a near-local-optimal transceiver structure.

Fig. 2. Sum rate versus RS antenna number

5. CONCLUSIONS

In this paper, we optimized the transceivers for multi-user multi-antenna TWRN, aiming at maximizing the bidirectional sum rate under the constraint of zero interference. To solve the non-convex optimization problem, we apply alternating optimization to jointly optimizes three transceiver weighting matrices at the BS and RS. Though the approach cannot be ensured to converge to the global maximal value, simulation results showed that the proposed solution offers higher bidirectional sum rate than the existing schemes when proper initial values are employed. The result indicates the possible existence of a novel interference zero-forcing transceiver structure which offers higher sum rate than existing structures.

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7. REFERENCES