COOPERATIVE SENSING WITH SEQUENTIAL ORDERED TRANSMISSIONS TO SECONDARY FUSION CENTER

Laila Hesham† Ahmed Sultan† Mohammed Nafie† Fadel Digham⋆

† Wireless Intelligent Networks Center (WINC), Nile University, Egypt
⋆ National Telecom Regulatory Authority (NTRAG), Egypt

ABSTRACT

Successful spectrum sharing in a cognitive radio network depends on the correct and quick detection of primary activity. Cooperative spectrum sensing is therefore suggested to enhance the reliability of such detection. However, it renders another significant problem of increased detection delay and traffic burden. Moreover, efficient schemes for multi-sensor data fusion should be designed. In this paper, we employ sequential detection scheme together with ordered transmissions from cognitive detectors. We derive two sequential schemes, one that is capable of achieving the minimum probability of error, and another that trades-off performance with delay. For the latter we derive expressions for the likelihood functions of ordered observations and compute the thresholds via backward induction. Simulation results demonstrate the relative performance of the approaches proposed in the paper.

Index Terms—Cognitive Radio, cooperative spectrum sensing, sequential detection, detection delay

1. INTRODUCTION

Cognitive radio technology is one of the key candidate technologies for improving the utilization of scarce radio spectrum [1]. Cognitive networks consist of licensed primary users and secondary users known as cognitive detectors. These cognitive detectors are required to efficiently detect spectral holes that dynamically appear when licensed users are not using their assigned spectrum. Therefore, spectrum sensing becomes a fundamental and challenging task to reliably detect a vacant channel. Shadowing and deep fading on the link between a primary transmitter and a secondary sensor may cause individual sensors to falsely decide that the probed channel is vacant. Secondary transmission on this channel would then cause interference to existing communications which makes spectrum sensing relying on one sensor highly unreliable [2]. Exploiting spatial diversity among a system of multiple sensors has been therefore proposed, because cooperation of these spatially distributed cognitive detectors together dramatically improves the sensing reliability and detection delay. However, to get the desired results from such cooperation, efficient schemes should be designed to combine the sensing information from individual sensors [3]. Central processing of diverse sensing results is done at a fusion center which is authorized to make the final decision regarding the occupancy of the sensed channel. However, reporting these local observations to the fusion center over orthogonal channels increases the traffic burden which is a significant drawback.

In contrast to block detection schemes, sequential detection schemes would reach a quick final decision while maintaining some detection performance measures (see, e.g., [4, 5]) where the number of observations samples is determined according to the reliability of the observations, and it is therefore a random variable. Applying sequential analysis to distributed detection problems has become of considerable interest (see, e.g., [6, 7]), specifically in cognitive radio context as in [8]. A sequential detection scheme was proposed in [8], where each sensor computes the log-likelihood ratio (LLR) for its observations and reports it to the fusion center over a perfect reporting channel. The LLR’s are accumulated sequentially at the fusion center till their sum is found sufficient to cross either of two predefined thresholds. In [9], the authors have proposed the idea of ordering the transmissions from local detectors. By means of timer backoff mechanism, the local sensor with the most informative observation has the priority to report it to the fusion center. Consequently, transmissions are restricted to highly informative and confident observations only. This scheme gives the added advantage of dramatical reduction of the average number of transmissions required till the final decision is reached. It is to be noted that the sensor selection criterion is based on its instantaneous observation quality and not the average SNR as in [10] for example.

In this paper, we adopt the ordering scheme proposed in [9]. We extend the work in [9] to the cases where a decision has to be made when only K readings out of the M sensors have been received at the fusion center where K ≤ M. Moreover, since we consider the detection delay to be one of our main concerns, we propose another technique where a sequential test is performed at the fusion center, and decision thresholds required are obtained via backward induction often utilized in dynamic programming. The proposed scheme improves the performance without requiring the participation of all secondary users in the reporting process. In Section 2 we describe the system model of cooperative spectrum sensing in a cognitive radio network. Section 3 describes the previous work utilizing the ordered transmissions scheme. We also describe our two schemes. In Section 4 we demonstrate the difference in performance via simulation results.

2. SYSTEM MODEL

We consider a slotted primary system where the primary activity does not change during the time slot duration, τs, and switches independently from one slot to the next. There are M cognitive sensors which take a number of measurements at the beginning of each time slot and compute a function of these measurements. A maximum of K sensors among the M sensors forward the results sequentially to a fusion center at which the final decision regarding primary activity is taken. We consider binary hypothesis testing at the fusion center with the following two hypotheses:

H0: Sensed channel is free; H1: Sensed channel is busy

The prior probabilities of each, denoted by π0 and (1 − π0), respectively, are assumed to be known. Observations from different sen-
sors are conditionally independent given either hypothesis but can be non-identical. Let \( X_i(n) \) be the received signal at the \( i^{th} \) sensor at instant \( n \), where \( i = 1, 2, \ldots, M \). At each sensor \( i \), \( \{ X_i(n) \}_{n=1}^{N} \), are independent given each hypothesis and are identically distributed, such that a total of \( N \) samples are taken over a time duration \( \tau_N \). Under the two hypotheses, \( X_i(n) \) is given by

\[
H_0 : X_i(n) = W_i(n), \quad n = 1, 2, \ldots, N \\
H_1 : X_i(n) = S_i(n) + W_i(n), \quad n = 1, 2, \ldots, N
\]

(1)

where \( W_i \) is additive white Gaussian noise (AWGN) with the same noise power, \( \sigma^2 \), at all sensors. Without loss of generality, the received primary signal \( S_i(n) \) is assumed to be real zero-mean Gaussian random variable. The conditional probability distributions of \( X_i(n) \) given \( H_0 \) and \( H_1 \) are described by \( f_{X_i(n)}(x_n | H_0) \) and \( f_{X_i(n)}(x_n | H_1) \), respectively, such that

\[
f_{X_i(n)}(x_n | H_0) \sim N(0, \sigma^2) \quad f_{X_i(n)}(x_n | H_1) \sim N(0, \sigma^2_i + \sigma^2)
\]

(2)

where \( \sigma^2_i \) is defined as the average received primary signal power at the \( i^{th} \) local sensor, and is assumed to be known at the local sensors and fusion center. Moreover, they are assumed to be fixed over the time slot duration. The local observations at sensors are defined as

\[
Y_i = \sum_{n=1}^{N} \log \left( \frac{f_{X_i(n)}(x_n | H_1)}{f_{X_i(n)}(x_n | H_0)} \right)
\]

(3)

such that \( Y_i \) is the LLR computed by the \( i^{th} \) local detector, then reported to the fusion center sequentially as explained below. Defining the local signal-to-noise ratio (SNR) as \( \gamma_i = \frac{\sigma^2_i}{\sigma^2} \), we can compute the LLR at the \( i^{th} \) sensor as

\[
Y_i = \frac{1}{2\sigma^2} \cdot \frac{\gamma_i}{\gamma_i + 1} \sum_{n=1}^{N} | X_i(n) |^2 - \log(1 + \gamma_i) \frac{N}{2}
\]

(4)

It is clear that the likelihood functions of \( Y_i \) given \( H_0 \) or \( H_1 \) are shifted and scaled chi-square distributions with \( N \) degrees of freedom. Note that the information exchange between the cognitive sensors and the fusion center is assumed to be perfect and occurs on a low-rate common control channel that is not on the band being sensed. We assume that the procedure of seizing the control channel and sending the LLR observation obtained by one sensor requires an amount of time which we denote as \( \tau \).

Since the transmission of each computed LLR value takes some time \( \tau \), eliciting another measurement from the sensors, though improving the reliability of detection, wastes a duration of \( \tau \) from the potential secondary transmission opportunity in case the primary is off. This causes a decrease in secondary throughput. In other words, we have a reliability-throughput trade-off which we can control by allowing only the subset of the cognitive detectors with the most reliable observations to transmit their LLR’s to the fusion center. This is implemented by having two thresholds at the fusion center. The LLR with maximum magnitude is transmitted first to the fusion center. A decision can be made, but if the metric is between the two thresholds, the second highest LLR in magnitude is transmitted to the fusion center and is combined with the first, and then a decision is attempted again. This continues until a decision in favor of \( H_0 \) or \( H_1 \) is made, or \( K \) LLR’s are accumulated at the fusion center. If \( k \) sensors, \( 1 \leq k \leq K \), are probed before a decision is reached, the time to make a decision is \( \tau_N + k\tau \). Since the slot duration is \( \tau_N \), this leaves \( \tau_N + k\tau \) for secondary transmission. Given the durations \( \tau_N \), \( \tau \) and \( \tau_s \), it is obvious that \( K \) must satisfy the inequality \( \tau_s \geq 0 \).

3. PROPOSED RANKED SEQUENTIAL SCHEME

In this section we describe the ordered transmissions scheme. A backoff timer is set at each of the \( M \) sensors according to the magnitude of the locally computed LLR. Specifically, the timer is inversely proportional to the absolute value of the sensor’s LLR \( [9] \). Consequently, transmissions are restricted to the more confident observations such that only highly informative measurements are processed at the fusion center. These combined observations are then compared to two thresholds and one of three decisions is made accordingly; declare \( H_0 \), continue taking the next ranked observations, or declare \( H_1 \). In \([9]\), this idea was adopted such that the processing done at the fusion center is the accumulation of the LLR observations. Let \( Y_{[m]} \) denote the LLR of rank \( m = 1, 2, \ldots, K \), where \( m = 1 \) is the highest rank. For \( 1 \leq k \leq K \), the decision strategy is expressed as

\[
\text{If } \sum_{m=1}^{k} Y_{[m]} \in \{ t_k^L, t_k^H \} \text{ then } \begin{cases} 
\text{Declare } H_0 \\
\text{Continue} \\
\text{Declare } H_1
\end{cases}
\]

(5)

Note that the thresholds are generally time-dependent, i.e., their values depend on the stage index \( k \). When the statistics evidently favor one hypothesis over the other, the decision is made and LLR transmissions from the local sensors stop. Otherwise, more observations are taken possibly till the end of the time slot of primary activity. These data-dependent thresholds \( t_k^L \) and \( t_k^H \) are given by

\[
t_k^L = \log \frac{1 - \pi_0}{\pi_0} - (M - k) \left| Y_{[k]} \right| \\
t_k^H = \log \frac{1 - \pi_0}{\pi_0} + (M - k) \left| Y_{[k]} \right|
\]

(6)

where \( M - k \) is the number of sensors that have not transmitted their LLR’s yet at the \( k^{th} \) stage, and \( \left| Y_{[k]} \right| \) is the absolute value of the LLR received at stage \( k \). However, if this scheme is forced to take a decision in favor of either \( H_0 \) or \( H_1 \) at stage \( K \), then it should work only for the case \( K = M \), as can be seen by the fact that only \( t_M^L = t_M^H \). The transmission is ordered in terms of the absolute LLR value. That is, \( \left| Y_{[m]} \right| < \left| Y_{[m']} \right|, \forall m > k \). It can be shown that this choice of thresholds ensure that this scheme has the same average probability of error as maximum a posteriori (MAP) procedure where all the LLR’s are summed and compared to \( \log \frac{\pi_0}{1 - \pi_0} \). The average probability of error when the guessed hypothesis is not the true one is given by

\[
P_e = (1 - \pi_0) \Pr (H_0|H_1) + \pi_0 \Pr (H_1|H_0)
\]

The advantage of this system is that the average number of transmissions needed to reach a decision is about half the total number of the sensors communicating with the fusion center. Here, we considered extending the scheme in \([9]\) to work for \( K \leq M \) case such that we only process the \( K \) LLR’s with highest magnitudes outside of the \( M \) LLR’s. Assume that the probability density function of LLR value \( Y_i \) under hypothesis \( H \) (either \( H_0 \) or \( H_1 \)) is given by \( f_{Y_i}(y | H) \), Define \( \beta_{i,b,H} = \Pr \left[ \left| Y_i \right| > |b| \mid H \right] \). The value of \( \beta_{i,b,H} \) can be readily computed knowing the probability density functions of \( Y_i \). In this work, we consider only the case of identical sensors, therefore, \( f_{Y_i}(y | H) = f_Y(y | H) \) and \( \beta_{i,b,H} = \beta_H \).

The optimal probability of error MAP block detector when the best \( K \) out of \( M \) LLR values (highest in magnitude) are received
can be written as
\[
\log \frac{f_{Y_1, \ldots, Y_K | H_1}(y_1, y_2, \ldots, y_K \mid H_1)}{f_{Y_1, \ldots, Y_K | H_0}(y_1, y_2, \ldots, y_K \mid H_0)} \geq \frac{\pi_0}{1 - \pi_0} - \log \pi_0
\] (7)
where, using the independence of LLR’s at various sensors, (7) can be given by
\[
\log \left[ \frac{f_y(y_{1} | H_1) \cdots f_y(y_{K} | H_1) (1 - \beta_{y_{k} | H_1})^{M - K}}{f_y(y_{1} | H_0) \cdots f_y(y_{K} | H_0) (1 - \beta_{y_{k} | H_0})^{M - K}} \right] \geq \frac{\pi_0}{1 - \pi_0} \log \pi_0
\] (8)
such that the given sequence of observations is \(Y_1 = y_1, Y_2 = y_2, \ldots, Y_K = y_K\) [11].

In order to perform sequential detection, we compute the accumulated sum at the fusion center at stage \(k\), which is given by
\[
\sum_{m=1}^{k} \log \left[ \frac{f_y(y_m | H_1)}{f_y(y_m | H_0)} \right].
\]
If this accumulated sum is \(\ell_k^{(L)}(H_1)\), \(H_1\) is declared, and if it is \(\ell_k^{(L)}(H_0)\), then \(H_0\) is declared. Otherwise, if it does not cross either thresholds, the fusion center continues to accumulate the LLR’s from the next ranked sensors. These thresholds, \(\ell_k^{(L)}\) and \(\ell_k^{(H)}\), would be given by
\[
\ell_k^{(L)} = \log \frac{1 - \beta_1 y_k H_1}{1 - \beta_1 y_k H_0} - (K - k) Z_{k+1}^{\max} - (M - K) \max_{0 \leq y \leq |y_k|} \rho(y)
\]
and
\[
\ell_k^{(H)} = \log \frac{1 - \beta_1 y_k H_1}{1 - \beta_1 y_k H_0} - (K - k) Z_{k+1}^{\min} - (M - K) \min_{0 \leq y \leq |y_k|} \rho(y)
\] (9)
for \(1 \leq k \leq K - 1\), such that \(\rho(y) = \log \frac{1 - \beta_1 y_k H_1}{1 - \beta_1 y_k H_0}\) is a correction term that appears at each stage \(k\) to account for the probability that all the other \((M - K)\) sensors would have LLR values less than \(|y_k|\). Since it is not a monotonic function, we need to span over the range from 0 < \(y < |y_k|\) and choose the maximum and the minimum value of \(\rho(y)\). However, at \(k = K\), \(\ell_k^{(H)} = \ell_k^{(L)} = \log \frac{1 - \beta_1 y_k H_1}{1 - \beta_1 y_k H_0} - (M - K)\rho(y_K)\). Moreover, \(Z_{k+1}^{\max} = \frac{\log f_y(y_k \mid H_1)}{f_y(y_k \mid H_0)}\) and \(Z_{k+1}^{\min} = \frac{\log f_y(-y_k \mid H_1)}{f_y(-y_k \mid H_0)}\) due to the monotonicity of \(f_y(y \mid H_1)\) and \(f_y(y \mid H_0)\). This term represents the maximum possible contribution of the non-transmitting sensors to the received LLR at stage \(k\). These thresholds guarantee achieving the same error probability as the MAP block detector.

In addition to this extension, we consider another method of combining ranked observations at the fusion center. We use dynamic programming to calculate the thresholds that allows throughput formulation to be addressed, which is the focus of current work. This would allow to trade-off probability of error with sensing delay, and hence, one could design an operation function that maximizes the secondary throughput. Specifically, backward induction technique is employed to provide the optimal action to be taken in order to minimize the overall decision cost [12]. In the analysis below we assume that the fusion center knows the statistics of all detectors but does not use sensors’ identifiers or indices to avoid complicated analysis and computational requirements.

Since we adopt the ordered transmission idea, if an LLR value is received, and since we assume perfect reporting, all subsequent LRR’s would have values with a lesser magnitude. Recall that \(y_k\) is the LLR from the \(k^{th}\) sensor, \(1 \leq k \leq M\), whereas \(Y^{[k]}\) is the LLR with the \(k^{th}\) highest magnitude that is transmitted to the fusion center at the \(k^{th}\) stage with \(k = 1, 2, \ldots, K\).

Let \(\pi_k\) be the probability of the channel being idle at stage \(k\) given the sequence of observations \(Y^{[1]} = y_1, Y^{[2]} = y_2, \ldots, Y^{[k]} = y_k\). Probability \(\pi_k\) can be obtained recursively as follows:
\[
\pi_k = Pr(H_0 | y_1, y_2, \ldots, y_k) = \frac{f_y(v | y^{[k-1]}, H_0) f_y(v | y^{[k-1]}, H_0) \pi_{k-1}}{\sum_{v=1}^{M} f_y(v | y^{[k-1]}, H_0) f_y(v | y^{[k-1]}, H_0) (v + (1 - 2v) \pi_{k-1})}
\] (10)
where in the last step, we use the property induced by ordered transmissions that \(Y^{[k]}\) is independent of \(Y^{[1]}, Y^{[2]}, \ldots, Y^{[k-2]}\) given \(Y^{[k-1]}\) and either hypothesis. What is needed for backward induction are the conditional probabilities of \(Y^{[m]}\) given \(Y^{[m-1]}\) under both \(H_0\) and \(H_1\). However, the dependence on the last received observation would make it a computationally extensive task. To reduce the complexity, we propose here to use the distributions of \(Y^{[k]}\) without conditioning to make the problem considerably more manageable. Distribution \(f_y(v | y^{[k-1]}, H_0)\) can be readily obtained as
\[
f_y(v | y^{[k-1]} | H_0) = M \frac{f_y(v | y^{[k-1]}, H_0) (M - 1) (\beta_{y, H_0} - \beta_{y, H_1})}{(m - 1)}
\] (11)
Assume that the cost of deciding \(i\) at stage \(k\) when \(j\) is the true hypothesis is \(\lambda_{ij}\), and the cost to continue taking observations is \(c \geq 0\). Define \(J_{k}^{(K)}\) as the minimum cost-to-go at stage \(k\) of the finite horizon \(K\).
\[
J_{k}^{(K)}(\pi_k) = \min \{ \lambda_0^k \pi_k + \lambda_0^{K} (1 - \pi_k), \lambda_1^k \pi_k + \lambda_1^{K} (1 - \pi_k), c + \mathbb{E}y_{k+1} \left[ J_{k+1}^{(K)}(\pi_{k+1}) \right] \}
\] (12)
where \(\pi_k\) and \(\pi_{k+1}\) are related through the expression
\[
\pi_{k+1} = \frac{\pi_k f_y(v | y^{[k]}, H_0) + (1 - \pi_k) f_y(v | y^{[k]}, H_1)}{\sum_{v=1}^{M} f_y(v | y^{[k]}, H_0) + (1 - \pi_k) f_y(v | y^{[k]}, H_1)}
\] (13)
The third term in (12) is interpreted as the expected cost when the fusion center decides that it should continue taking more observations from local sensors. Thus, we have
\[
\mathbb{E}y_{k+1} \left[ J_{k+1}^{(K)}(\pi_{k+1}) \right] = \int J_{k+1}^{(K)}(\pi_{k+1}) f_y(v | y^{[k]}, H_0) \; dy
\] (14)
and
\[
f_y(v | y^{[k]}, H_1) = \pi_k f_y(v | y^{[k]}, H_0) + (1 - \pi_k) f_y(v | y^{[k]}, H_1)
\] (15)
The parameter \(c\) represents the trade-off between the time taken till a decision is made and the average probability of error. As \(c\) increases, the fusion center becomes more likely to favor one of the two hypotheses in a shorter time using only a few LLR’s from the cognitive detectors. However, this cost has no effect at the last stage when \(k = K\), where the fusion center has two choices only; to declare \(H_0\) or \(H_5\) allowing backward induction from the last stage where \(J_K^{(K)}(\pi_K) = \min \{ \lambda_0^K \pi_K + \lambda_0^{K} (1 - \pi_K), \lambda_1^K \pi_K + \lambda_1^{K} (1 - \pi_K) \}\).

4. SIMULATION RESULTS

In this section, we investigate via numerical simulations the performance of our proposed schemes. Assume \(\sigma^2 = 1\) and \(\tau = 0.1\). The time taken at the beginning of a time slot with duration \(\tau_s = 0.5\) to collect \(N\) observations is \(\tau_N = \tau\) which leaves \(K = 4\) stages in
the slot. We consider equal local signal to noise ratio over the channels between the local sensors and the primary user, $\sigma^2_{si} = 2$, $i = 1, 2, ..., M$ and $\pi_0 = 0.5$. Each sensor takes $N = 3$ samples in the duration $\tau_N$. The cost, $c$, to continue without deciding on one of the two hypotheses is set to 0.01. We have performed computer simulations to plot the average error probability and average sensing time versus the number of local sensors in the network. The decision costs are assumed to be as follows: $\lambda^k_{ij} = 1$, $i \neq j$ and $\lambda^k_{ij} = 0$, $i = j$, where $\lambda^k_{ii}$ = 0 means that no cost is incurred if the guessed hypothesis is true.

In Figure 1, it is clear that the modified scheme of [9], which we hereafter refer to as B&S, achieves a lower probability of error since it achieves the same probability of error as the MAP block detector, while the dynamic programming scheme, denoted as DP scheme, achieves a higher error probability. However, as elaborated in Figure 2, the number of sensors involved in the sensing process in the dynamic programming scheme is lower than that in the B&S Modified scheme. Therefore, the dynamic programming scheme represents a trade-off of the sensing time with the error probability, via the $c$ parameter. This can be a potentially significant enhancement if the secondary throughput is to be considered, which is the focus of our current work.

5. CONCLUSION

In this paper, we have considered ordering the transmissions from cognitive users according to the information they carry about the primary user. We devised two sequential schemes, one achieves the optimum probability of error of a MAP block detector for the best $K$ out of $M$ LLR’s, and the other would trade-off the error probability for less average sensing time. Simulation results have demonstrated that the dynamic programming scheme achieves a significant decrease in the delay of a decision at the expense of an increased probability of error.

6. REFERENCES