Consider a robust downlink beamforming optimization problem for secondary multicast transmission in a multiple-input multiple-output (MIMO) spectrum sharing cognitive radio (CR) network. The minimization problem of transmit power is formulated subject to both the quality-of-service (QoS) constraints on the secondary receivers and the interference temperature constraints on the primary users, under the assumption of imperfect channel state information (CSI). The problem is non-convex quadratically constrained quadratic program (QCQP), and it is hard to achieve the global optimality. As a compromise, we present a randomized approximation algorithm for the problem via convex optimization techniques. In particular, we point out that the robust beamforming problem is efficiently solvable when the number of primary and secondary links in the CR network is not larger than three. Simulation results are presented to demonstrate the performance gains of the proposed algorithm over an existing robust design.

Index Terms—Robust multicast beamforming, MIMO cognitive radio networks, spectrum sharing, semidefinite programming relaxation, imperfect channel state information.

1. INTRODUCTION

In cognitive radio (CR) networks, the idea of spectrum sharing using multiple transmit antennas has drawn much research interest recently. Spectrum sharing allows secondary and primary users to access the same channel simultaneously, by using the spatial degree of freedom at the secondary transmitter to avoid excessive interference to the primary users. For a comprehensive coverage of the recent advances, readers are referred to [1]; [2, 3] for some recent specific works on CR transmit beamforming.

In this work we are interested in a CR multicast transmit beamforming problem setting. Consider a CR network that has a secondary transmitter using $N$ antennas to transmit common information to $M$ secondary receivers, and that there are $K$ primary users operating in the same spectrum. Let $\mathbf{H}_m \in \mathbb{C}^{N \times N_m}$ be the multiple-input multiple-output (MIMO) channel from the secondary transmitter to the $m$th secondary receiver, where the number of receive antennas is denoted by $N_m$. Similarly, let $\mathbf{G}_k \in \mathbb{C}^{N \times N'_k}$ be the MIMO channel from the secondary transmitter to the $k$th primary user, where the number of receive antennas is denoted by $N'_k$. The signal received by secondary receiver $m$ is given by $x_m(t) = \mathbf{H}_m^H \mathbf{y}(t) + n_m(t)$, where $\mathbf{y}(t) \in \mathbb{C}^N$ is the secondary transmit signal vector, and $n_m(t) \in \mathbb{C}^{N_m}$ is Gaussian noise vector, assumed to have zero mean and covariance $\sigma^2_n \mathbf{I}$. The secondary transmitter employs the multicast transmit beamforming scheme, in which we have $\mathbf{y}(t) = s(t) \mathbf{w}$, where $\mathbf{w} \in \mathbb{C}^N$ is the beamformer weight and $s(t) \in \mathbb{C}$ is the information signal. We assume zero mean and unit variance with $s(t)$, without loss of generality. Moreover, assuming maximum ratio combining (MRC) receive beamforming for all the secondary receivers, the received SNR of secondary user $m$ is $\text{SNR}_m = \frac{\| \mathbf{H}_m \mathbf{w} \|_2^2}{\sigma^2_n}$, where $\| \cdot \|$ denotes the Euclidean norm for a vector or the Frobenius norm for a matrix. The multicast beamformer design may be formulated as [3,4]:

\[
\begin{align*}
\text{minimize} & \quad \mathbf{w}^H \mathbf{w} \\
\text{subject to} & \quad \| \mathbf{H}_m^H \mathbf{w} \|^2_2 \geq \sigma^2_n \tau_m, \quad m = 1, \ldots, M, \\
& \quad \| \mathbf{G}_k^H \mathbf{w} \|^2_2 \leq \eta_k, \quad k = 1, \ldots, K,
\end{align*}
\]

where, $\tau_m$ specifies the minimal quality of service (QoS) of the secondary user $m$, in terms of SNR, and $\eta_k$ specifies the maximal allowable interference level from the secondary transmitter to primary user $k$. Problem (1) has been considered in [3], where an effective approximation method called semidefinite programming (SDP) relaxation (see, e.g., [5] and the references therein) has been applied. Herein we consider the imperfect CSI case. In practice, one may not have perfect knowledge of the CSI especially for the primary users. The CSI errors may be caused by inaccurate channel estimation, quantization in channel feedback, and outdated CSI effects. Let $\Delta_m \in \mathbb{C}^{N \times N_m}$ and $\Delta'_k \in \mathbb{C}^{N \times N'_k}$ denote the CSI errors associated with $\mathbf{H}_m$ and $\mathbf{G}_k$, respectively. By assuming that the errors $\Delta_m$ and $\Delta'_k$ are deterministic norm-bounded, a worst-case robust version of (1) is given by [3] (also [6] for a similar robust formulation in non-CR MIMO transmission):

\[
\begin{align*}
\text{minimize} & \quad \mathbf{w}^H \mathbf{w} \\
\text{subject to} & \quad \text{minimize} \quad \| (\mathbf{H}_m + \Delta_m) \mathbf{w} \|^2_2 \geq \sigma^2_n \tau_m, \\
& \quad \text{maximize} \quad \| (\mathbf{G}_k + \Delta'_k) \mathbf{w} \|^2_2 \leq \eta_k \\
& \quad m = 1, \ldots, M, \\
& \quad k = 1, \ldots, K,
\end{align*}
\]

where $\epsilon_m$ and $\epsilon'_k$ specifies the bound, or the worst-case magnitude, of the CSI errors $\Delta_m$ and $\Delta'_k$, respectively. The robust beamforming problem (2) guarantees that for all admissible channel errors, all
the secondary users must be served with QoSs no less than the specification \( \gamma_m \), and the interferences to all the primary users must be kept below \( \{ \nu_k \} \).

This work concentrates on the robust multicast problem (2). Before presenting our method, we should mention related works. In [7], the robust multicast beamforming problem without the CR interference limiting constraints; i.e., problem (2) without the second set of constraints, has been considered. The method there gives an interesting connection between non-robust and robust beamforming problems. In [3], problem (2) is handled by applying a conservative bound on both the QoS constraints and the interference suppressing constraints; the subsequent formulation is a quadratically constrained quadratic program (QCQP) and can be effectively approximated by SDP relaxation.

In this paper, we propose a new randomized approximation algorithm for the robust CR multicast downlink beamforming problem (2). Particularly, we take into account an equivalent QCQP reformulation of problem (2). By dropping the rank-one constraint on the beamforming vector, we obtain a parameterized SDP which is a relaxation of (2), and this parameterized SDP can be solved by searching one-dimensional parameter over an interval, and a feasibility checking routine using SDP. Based on the solution of the parameterized SDP, a Gaussian randomization procedure is presented to give approximate solutions of (2) in a near way. The numerical simulation results show the outperformance of the proposed beamformer over the robust design in [3].

2. EFFICIENT ALGORITHMS FOR THE ROBUST BEAMFORMING PROBLEM

In this section, we will propose an approximate solution scheme for the robust beamforming problem (2), and discover an interesting particular subclass of (2), which can be solved efficiently. Let us start with an equivalent non-convex QCQP reformulation of (2).

2.1. An equivalent QCQP reformulation of robust optimal beamforming problem (2)

Consider the first robust QoS constraint of problem (2) in a slightly more general form, and set

\[
 f_1(w) = \min \| (H_1 + \Delta_1)H w \|^2, \\
 \Delta_1 \in E_1^{-1/2} \Delta_1 \leq \epsilon_1
\]

where \( E_1 > 0 \) governs the ellipsoid shape of the error set (or termed as the perturbation set in some robust optimization literature, e.g., [8]) and \( w \neq 0 \). We claim that \( f_1(w) \) has a closed-form expression as stated in the following lemma.

**Lemma 2.1** It holds that

\[
 f_1(w) = \left( \max \left\{ \frac{\| H_1 w \|^2}{\| E_1^{-1/2} w \|}, 0 \right\} \right)^2. \tag{3}
\]

**Proof:** When \( \| H_1^H w \| \leq \epsilon_1 \| E_1^{-1/2} w \| \), we select \( \Delta_1 = -\frac{E_1^{-1/2} H_1^H H_1}{\| E_1^{-1/2} w \|^2} \). It is easily verified that \( \| E_1^{1/2} \Delta_1 \| = \| H_1^H w \| / \| E_1^{-1/2} w \| \leq \epsilon_1 \) and \( \| (H_1 + \Delta_1)H w \| = 0 \). Suppose \( \| H_1^H w \| > \epsilon_1 \| E_1^{-1/2} w \| \). It then follows that

\[
 \| (H_1 + \Delta_1)H w \| \geq \| H_1^H w \| - \epsilon_1 \| E_1^{-1/2} w \| \geq \| H_1^H w \| - \epsilon_1 \| E_1^{-1/2} w \| \geq \| H_1^H w \| - \epsilon_1 \| E_1^{-1/2} w \| > 0
\]

where the first inequality is due to the triangle inequality and the second inequality is due to the Cauchy-Schwartz inequality. Clearly, the above inequality chain becomes equality chain when \( \Delta_1 = -\epsilon_1 E_1^{-1/2} H_1 / \| H_1^H w \| / \| E_1^{-1/2} w \| \). \( \square \)

Note that the closed-form expression for the optimal value \( f_1(w) \) in (2.1) remains unchanged whenever \( \| E_1^{1/2} \Delta_1 \| \) means the spectral norm (the maximal singular value). Nevertheless, we always consider the Frobenius norm for a matrix in this paper. Specially, if \( N_1 = 1 \) (\( H_1 \) becomes \( h_1 \) in this case), we have that

\[
 \min \frac{\| (H_1 + \delta_1)H w \|}{\| \delta_1 \|/\| E_1^{1/2} \delta_1 \| \leq \epsilon_1}
\]

has the optimal value \( \max \{ |h_1^H w| - \epsilon_1 \| E_1^{-1/2} w \|, 0 \} \).

From Lemma 2.1, we see that the first QoS constraint of problem (2) amounts to

\[
 \| H_1^H w \| \geq \sigma_1 \sqrt{t} + \epsilon_1 \| w \|. \tag{4}
\]

Like the proof in Lemma 2.1, the maximization problem in the first interference-power constraint of (2) has the optimal value \( (\| G_k^H w \| + \epsilon_1 \| w \|)^2 \). Thus it follows that the robust beamforming problem (2) can be recast into

\[
 \begin{align*}
 & \text{minimize} \quad w^H \delta w \\
 & \text{subject to} \quad \| H_m^H w \| \geq \sigma_m \sqrt{t}, \quad m = 1, \ldots, M, \\
 & \quad \| G_k^H w \| \leq \sqrt{t} - \epsilon_k', \quad k = 1, \ldots, K, \\
 & \quad \| w \| = t.
\end{align*}
\]

Note that problem (17) of [3] has approximate constraints which are more conservative than those of problem (4) herein. It is known (cf. [9]) that problem (4) is NP-hard (in fact, problem (4) has been proved NP-hard, when \( \epsilon_m = 0, \forall m \), and \( G_k = 0, \epsilon_k = 0, \forall k \). Instead, one may resort to efficient suboptimal solutions.

2.2. A randomized approximation algorithm for the robust beamforming problem with ball perturbation

Problem (4) is tantamount to the following QCQP problem

\[
 \begin{align*}
 & \text{minimize} \quad w, t \\
 & \text{subject to} \quad \| H_m^H w \| \geq \sigma_m \sqrt{t}, \quad m = 1, \ldots, M, \\
 & \quad \| G_k^H w \| \leq \sqrt{t} - \epsilon_k', \quad k = 1, \ldots, K, \\
 & \quad \| w \| = t.
\end{align*}
\]

Note that any feasible point \( (w, t) \) of (5) must satisfy

\[
 \begin{align*}
 & \sqrt{\lambda_{\max}(H_m H_m^H)} \geq \frac{\| H_m^H w \|}{\| w \|} \geq \sigma_m \sqrt{t} + \epsilon_m, \forall m \tag{6}
\end{align*}
\]

and

\[
 \begin{align*}
 & \frac{\lambda_{\min}(G_k G_k^H)}{\| G_k^H w \|} \leq \frac{\| G_k^H w \|}{\| w \|} \leq \sqrt{t} - \epsilon_k', \forall k. \tag{7}
\end{align*}
\]

Suppose that \( \sqrt{\lambda_{\max}(H_m H_m^H)} - \epsilon_m > 0 \) for all \( m \) (otherwise problem (5) would be infeasible). It follows that a necessary condition for \( t \) to be feasible for (5) is

\[
 t_0 \leq t \leq t_1, \tag{8}
\]

where the lower bound \( t_0 \) and the upper bound \( t_1 \) are respectively given by

\[
 t_0 = \max_{1 \leq m \leq M} \left\{ \frac{\sigma_m \sqrt{t}}{\lambda_{\max}(H_m H_m^H) - \epsilon_m} \right\}. \tag{9}
\]
\[ t_1 = \min_{1 \leq k \leq K} \left\{ \frac{\sqrt{\pi}}{\lambda_{\min}(G_k G_k^H) + \epsilon'_k} \right\}. \quad (10) \]

Then problem (5) indeed amounts to

\[ \begin{align*}
\text{minimize} & \quad w, t \\
\text{subject to} & \quad w^H H_m H_m^H w \geq (\sigma_m \sqrt{\tau_m} + \epsilon_m t)^2, \quad m = 1, \ldots, M, \\
& \qquad w^H G_k G_k^H w \leq (\sqrt{\eta_k} - \epsilon'_k t)^2, \quad k = 1, \ldots, K, \\
& \qquad w^H w = t^2, \\
& \qquad 0 \leq t \leq t_1, \\
& \qquad 0 \leq \lambda (w_i) \leq \lambda (w_i)_{\text{opt}} \leq \eta_k, \quad i = 1, \ldots, I, \\
& \quad \forall k \leq K.
\end{align*} \quad (11) \]

the equivalent matrix form of which is expressed as

\[ \begin{align*}
\text{minimize} & \quad W, t \\
\text{subject to} & \quad H_m H_m^H \cdot W \geq (\sigma_m \sqrt{\tau_m} + \epsilon_m t)^2, \quad m = 1, \ldots, M, \\
& \qquad G_k G_k^H \cdot W \leq (\sqrt{\eta_k} - \epsilon'_k t)^2, \quad k = 1, \ldots, K, \\
& \quad I \cdot W = t^2, \\
& \quad W \succeq 0, \quad \lambda (W) = 1, \quad 0 \leq t \leq t_1.
\end{align*} \quad (13) \]

where \( A \cdot B = \text{tr}(A B) \) for Hermitian matrices \( A \) and \( B \). Let us consider to drop the rank-one constraint on \( W \) in (12), obtaining the following optimization problem

\[ \begin{align*}
\text{minimize} & \quad t \\
\text{subject to} & \quad (12b), (12c), (12d) \text{ satisfied}, \\
& \quad W \succeq 0, \quad 0 \leq t \leq t_1.
\end{align*} \quad (14) \]

Fixing \( t \), problem (13) is an SDP feasibility problem. Now, let \( g(t) \) be the optimal value of such a feasibility problem. In other words, we have \( g(t) = t \) if (13) is feasible for a given \( t \) (any feasible \( W \) is thus optimal), and \( g(t) = +\infty \) if it is infeasible at point \( t \). Therefore, (13) amounts to the one-dimensional optimization problem:

\[ \begin{align*}
\text{minimize} & \quad g(t) \\
\text{subject to} & \quad 0 \leq t \leq t_1.
\end{align*} \quad (15) \]

In other words, (13) can be solved by solving (14): fixing \( t \), solving the SDP feasibility problem (obtaining \( g(t) \)), and reducing \( t \) iteratively. In the optimization literature there are some derivative-free methods for solving the one-dimensional optimization problem (14). One of these methods is called compass or coordinate search (cf. [10, Algorithm 3.1 and Section 8.1]). In practice, we adopt either the uniform sampling or the Matlab function fminbnd, in order to output a satisfactory solution.

Once such a solution \((W^*, t^*)\) of (13) is obtained, we retrieve a rank-one approximate solution of (13) by making use of \( W^* \). Particularly, a randomization procedure is proposed as follows: Take random vectors \( w_i, i = 1, \ldots, I \), from the complex normal distribution \( \mathcal{N}(0, W^*) \), and compute

\[
\lambda(w_i) = \min_{1 \leq m \leq M, 1 \leq k \leq K} \left\{ \frac{w_i^H H_m w_i - \sigma_m \sqrt{\tau_m}}{\epsilon_m}, \frac{w_i^H G_k G_k^H w_i}{\epsilon'_k} \right\}.
\]

Clearly, if \( \|w_i\| \leq \lambda(w_i) \), then \((w_i, \|w_i\|)\) is feasible for (11) or (5). Algorithm 1 summarizes the procedure to generate a randomized approximate solution of problem (4).

Algorithm 1 Gaussian randomization procedure for robust beamforming problem (4)

1. solve (14), finding an optimal solution \((W^*, t^*)\);
2. if Rank \((W^*) = 1\), then output \( w^* \) with \( w^* w^H = W^* \) and terminate;
3. draw random vectors \( w_i \in \mathcal{N}(0, W^*), i = 1, \ldots, I, \) and compute \( \lambda(w_i) \) by (15);
4. pick up \( w_{i_0} \) such that \( i_0 = \arg \min \{ \|w_i\| : \|w_i\| \leq \lambda(w_i), i = 1, \ldots, I \}\).

2.3. A solvable scenario of the robust beamforming problem

In this subsection, we shall elaborate that robust beamforming problem (2) can be solved efficiently with parameters such that \( M + K = 3 \) and \( N \geq 3 \) (i.e., the number of primary and secondary receivers equal to three and the number of the transmit antennas is not less than three), or with parameter condition \( M = K = 2 \).

Let \( t' \) be a numerical minimizer for \( g(t) \) over the interval \([t_0, t_1]\) as in problem (14), and \( W^* \) be a corresponding feasible solution, namely, \((W^*, t')\) complies with the constraints of (13). To proceed, let us assume \( M = 2 \) and \( K = 1 \) without loss of generality. Then, it follows that

\[
H_m H_m^H \cdot W^* \geq (\eta_k - \epsilon'_k t^*)^2, \quad I \cdot W^* = (t^*)^2.
\]

It is verified immediately that the conditions of the specific rank-one matrix decomposition theorem 2.3 of [11] are satisfied, and thus one is able to polynomially construct a matrix \( w^* w^H \) according to the rank-one decomposition theorem, such that

\[
w^* H_m H_m^H w^* = H_m H_m^H \cdot W^*, \quad m = 1, 2,
\]

\[
w^* H_m H_m^H w^* = H_k H_k^H \cdot W^*, \quad \|w^*\|^2 = I \cdot W^*.
\]

This implies that \((w^* w^H, t^*)\) is feasible for (13); thus \((w^*, t^*)\) is feasible for (5). Therefore, we conclude that \( w^* \) is optimal for (5) since the problem shares the same optimal value \( t^* \) with its relaxation problem (13). For the scenario with parameters fulfilling \( M + K = 2 \), it can be shown in a similar way by the decomposition theorem 2.1 of [12] that (11) can be solved efficiently.

3. Numerical examples

We consider a CR network with 3 primary single antenna receivers \((K = 3 \) and \( N_k = 1, \forall k)\) and four-antenna secondary transmitter \((N = 4)\) serving either 3 secondary users \((M = 3)\) or 5 secondary users \((M = 5)\), all with single antenna \((N_m = 1, \forall m)\). The elements of the channels (from the secondary transmitter to either the primary users or the secondary users) are assumed to be i.i.d. complex Gaussian distributed with mean 0 and variance 1. Suppose that the secondary receivers have the noise variance \( \sigma_k^2 = \cdots = \sigma_M^2 = 1 \) and the threshold value of SNR \( \tau_1 = \cdots = \tau_M = 10 \) dB, and that the upper bound of tolerable interference power for the primary receivers is \( \eta_1 = \cdots = \eta_K = 0 \) dB. The same channel perturbation level is assumed for all primary and secondary channels, i.e., \( \epsilon_m = \epsilon_k = \epsilon, \forall m, k \). A total of 2000 channel realizations (each with 10000 Gaussian randomizations) are tested.

Fig. 1 examines how the average transmit power is affected by the radius of the channel perturbation set. We compare our proposed robust beamforming design (i.e. problem (5)) with an existing robust design provided by problem (17) of [3]. It should be noted that
problem (17) of [3] is a homogeneous QCQP, and thus a randomized heuristic procedure was proposed upon a solution of its SDP relaxation problem. Moreover, as power minimization problem with QoS constraints could be intrinsically infeasible, the average transmit power in Fig. 1 is obtained by averaging only those channel realizations for which both problem (17) of [3] and problem (5) herein are feasible in the sense that a feasible randomized solution can be found. It can be seen from Fig. 1 that higher transmit power is required to assure larger radius of the channel error set (i.e., provide more robust beamformer), as well as to serve more secondary receivers. Since our problem (5) is somehow less conservative than (17) of [3] (i.e., the feasible region of the former is bigger than that of the latter), the average transmit power by our proposed robust beamformers would be lower in general, which is confirmed in Fig. 1. To get a better understanding of the conservativeness, Fig. 2 plots the feasibility rate of the two designs with the same setup as Fig. 1. Here the feasibility rate is denoted as the ratio of the number of channel realizations, for which we can generate a feasible beamformer via randomization, over the total 2000 channel trials. It can be seen from Fig. 2 that the proposed robust design has a much higher feasibility rate than that of [3] over the whole perturbation radiuses tested, for different number of secondary users.

![Fig. 1: Average transmit power versus the radius of channel perturbation set, with different numbers of secondary receivers.](image)

4. CONCLUSIONS

We have considered the robust secondary multicast beamformer design problem for spectrum sharing in a MIMO CR network. Due to NP-hardness of the problem, for a general setting, we have proposed a randomized approximation algorithm, which includes solving a one-dimensional optimization problem, checking the feasibility of SDPs, and a Gaussian randomized procedure. In the special case of “not too many” primary and secondary receivers, we have also proved that the robust optimal beamforming problem can be solved efficiently. The performance of the proposed beamforming design has been demonstrated by simulations.

5. REFERENCES


