DISTRIBUTED BEAMFORMING FOR MULTIUSER PEER-TO-PEER AND MULTI-GROUP MULTICASTING RELAY NETWORKS

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ABSTRACT
We generalize the concept of multiuser peer-to-peer (MUP2P) relay networks to that of a multi-group multicasting (MGM) relay network where each source may broadcast its message to a group of multiple users. State-of-the-art beamforming methods, which have been proposed for MUP2P relay networks, are shown to be straightforwardly extendable to such MGM networks. These methods aim to minimize the total transmitted relay power subject to receiver quality-of-service (QoS) constraints using convex approximations of the underlying non-convex problem. Due to the increase in number of receivers, these approximations may become inaccurate in the MGM case leading to severe performance degradation and problem infeasibility. To avoid this drawback, we propose an iterative method where the aforementioned convex approximations are successively improved. Our technique overcomes the difficulties emerging in the MGM and MUP2P relay networks for large numbers of users and outperforms the state-of-the-art methods developed for MUP2P relay networks.

Index Terms— Relay networks, distributed multiuser peer-to-peer beamforming, multicasting, feasibility, admission control

1. INTRODUCTION
User cooperation in wireless communication networks, where users act as relays and mutually assist each other in transmitting data through the network, is an emerging trend in wireless communications [1]-[2]. Several relaying protocols have been proposed among which the amplify-and-forward (AF) protocol is of particular interest due to its simple implementation [1]-[2].

AF beamforming has recently been applied in cooperative relay networks [3]-[6]. The single user peer-to-peer relay network, where a single source communicates with a single destination through several relays, has been studied in [3] and [4]. This network has been extended from the single source-destination pair to multiple pairs in [5] and [6]. A common beamforming approach for relay networks involves minimizing the total transmitted relay power subject to receiver quality-of-service (QoS) constraints. In the MUP2P relay network case, this results in a non-convex optimization problem for which convex approximations have been proposed based on semi-definite programming (SDP) [5] and second-order cone programming (SOCP) [6].

In this paper, we consider the concept of MGM relay networks (see Fig. 1) which is a generalization of that of the MUP2P relay networks to the case where each source may broadcast a message to multiple destination users. Note that this generalization is similar to the one in [7] where multiuser downlink beamforming has been generalized to transmit beamforming in MGM. Considering some fixed number of transmitters, the number of interfering sources in the resulting MGM relay network is the same as in the corresponding MUP2P relay network. However, the number of QoS constraints in MGM relay networks may be significantly larger than in MUP2P relay networks due to the increased number of receivers (destination users). In the special case when K = 1 and only a single group is present, the MGM relay network reduces to a broadcasting relay network.

We show that the generalization described above does not change the mathematical structure of the original power minimization problem. Thus, the suboptimal beamforming techniques proposed in [5] and [6] are straightforwardly applicable to MGM relay networks. However, due to the increased number of constraints, the approximations involved may be rather poor. This may lead to a severe performance degradation and to the situation when the approximated (reformulated) problem becomes infeasible. To overcome these difficulties, we propose an iterative procedure, where in each iteration a SOCP problem is solved and the problem approximation is successively improved. Simulation results show that our method outperforms the approaches of [5] and [6] in terms of both the transmitted power and problem feasibility. These improvements are quite significant in MGM networks but can also be noticeable in MUP2P networks.

2. SIGNAL MODEL
We consider a wireless relay network of single antenna nodes operating in the same frequency band which consists of K transmitters (sources), L relays and M receivers, with K ≤ M, as illustrated in Fig. 1. Each transmitter serves one out of K disjoint multicast groups, \{G_1, \ldots, G_K\}, where G_k is the index set of the receivers intended to receive the message of transmitter k. Each receiver participates in one multicast group only. Hence, G_k ∩ G_l = ∅, for l ≠ k and ∪_k G_k = \{1, \ldots, M\}. We assume no direct links between the transmitters and receivers. This model includes two special cases, the MUP2P relay network of [5] and [6] for K = M and the broadcasting relay network for K = 1.
Throughout the paper, we consider the two-step AF protocol [2]. In the first step, all transmitters broadcast their signals to the relays. The received signals at the relays are given by
\[ r = \sum_{k=1}^{K} f_k s_k + \eta \] (1)
where \( r \triangleq [r_1 \ldots r_L]^T \), \( r_i \) is the signal observed at the \( i \)-th relay, \( f_k \triangleq [f_{k,1} \ldots f_{k,L}]^T \), \( f_{k,i} \) is the complex channel gain between the \( k \)-th transmitter and the \( l \)-th relay, \( s_k \) is the information symbol emitted by the \( k \)-th transmitter, \( \eta \triangleq [\eta_1 \ldots \eta_L]^T \) is the noise relay vector, and \((\cdot)^T\) denotes the transpose. In the second step, the \( l \)-th relay multiplies its received signal \( r_l \) by a complex beamforming weight \( w_l^* \) and transmits the result \( t_l \) to the receivers. Hence, the vector of signals transmitted by the relays is given by
\[ t = W^H r \] (2)
where \( W \triangleq \text{diag} \{ w \} \), \( \text{diag} \{ a \} \) stands for a diagonal matrix whose diagonal entries are the elements of vector \( a \), \( w \triangleq [w_1 \ldots w_L]^T \), \( t = [t_1 \ldots t_L]^T \) and \((\cdot)^H\) denotes the Hermitian transpose. Let \( g_{m,l} \) denote the channel coefficient between the \( l \)-th relay and the \( m \)-th receiver and \( g_{m,l} \triangleq [g_{m,1} \ldots g_{m,L}]^T \). Assume that the \( m \)-th receiver belongs to multicast group \( G_m \) (i.e., \( m \in G_m \)). With (1) and (2), we can express the signal at the \( m \)-th receiver as
\[ y_m = g_m^T t + n_m = g_m^T (W^H \sum_{k=1}^{K} f_k s_k + W^H \eta) + n_m \]
\[ = w^H G_m f_{d,d} + w^H G_m \sum_{k \neq d} f_k s_k + w^H G_m \eta + n_m \] (3)
where \( G_m = \text{diag} \{ g_m \} \), \( n_m \) is the noise at the \( m \)-th receiver and \( y_m \), \( y_s \) and \( y_m \), \( y_n \) are the desired signal, the interference and the noise components at the \( m \)-th receiver, respectively.

Throughout the paper, we assume that the symbols of different transmitters are uncorrelated, the relay noise is spatially white and the information symbols, the relay noise, the receiver noise and the channel coefficients are mutually statistically independent [5], [6]. According to [3] and [6], we also assume that the instantaneous channel state information (CSI) in terms of the channel vectors \( f \) and \( g \) is available at the processing nodes. In a centralized relay system, there may be a single node which computes the relay weights and distributes them to the relays. Despite the centralized processing of the relay weights, the beamforming scheme can be considered as distributed since the relays do not process their received signals jointly. This is fundamentally different from the case of a single relay equipped with \( L \) antennas (see, e.g., [8]) since the multi-antenna relay has access to the received signal at each antenna.

3. POWER MINIMIZATION

The goal of our network beamforming approach is to minimize the total transmitted relay power subject to QoS constraints which guarantee that the signal-to-interference-plus-noise ratio (SINR) at the receivers does not fall below certain thresholds denoted by \( \gamma_1, \ldots, \gamma_M \). The latter problem can be written as
\[ \min_w P_T \quad \text{subject to } \text{SINR}_m \geq \gamma_m, \forall m \in G_d, \forall d \in K \] (4)
where \( K = \{1, \ldots, K\} \), \( \text{SINR}_m \) is the SINR at the \( m \)-th receiver defined as
\[ \text{SINR}_m = \frac{\mathbb{E} \{ |y_{m,s}|^2 \}}{\mathbb{E} \{ |y_{m,n}|^2 \} + \mathbb{E} \{ |y_{m,n}|^2 \}} \] (5)
and \( \mathbb{E}\{ \cdot \} \) stands for the statistical expectation. \( P_T \) in (4) is the total transmitted relay power given by
\[ P_T = \sum_{l=1}^{L} \mathbb{E} \{ |t_l|^2 \} = \sum_{l=1}^{L} |w_l|^2 |R_l|^2 = w^H D w \] (6)
where \( R_l \triangleq \mathbb{E} \{ r r^H \} \) is the covariance matrix of the signals received at the relays and \( D \) is the diagonal matrix with \( |D_{l,l}| = |R_l| \). Using (1), this covariance matrix becomes
\[ R_l = \sum_{k=1}^{K} P_k f_k f_k^H + \sigma_n^2 I \] (7)
where \( P_k = \mathbb{E} \{ |s_k|^2 \} \) is the transmitted power of the \( k \)-th transmitter, \( \sigma_n^2 \) is the variance of the relay noise and \( I \) denotes the \( L \times L \) identity matrix.

Making use of (3), we next derive expressions for the desired signal power, the interference power and the total noise power in (5). The desired signal power at the \( m \)-th receiver can be written as
\[ \mathbb{E} \{ |y_{s,m}|^2 \} = \sum_{k \neq d} P_k h_{k,m} h_{k,m}^H w^H Q_m w \] (8)
where \( h_{k,m} \triangleq G_f d \) and \( Q_m \triangleq \sum_{k \neq d} P_k h_{k,m} h_{k,m}^H \). Finally, the total noise power at the \( m \)-th receiver can be expressed as
\[ \mathbb{E} \{ |y_{n,m}|^2 \} = \mathbb{E} \{ |\eta^H G_m w w^H G_m \eta| \} + \mathbb{E} \{ |n_m|^2 \} \] (9)
\[ = \text{tr} \{ \mathbb{E} \{ \eta^H G_m w w^H G_m \eta \} \} + \sigma_n^2 \]
\[ = \sigma_n^2 w^H G_m G_m^H w + \sigma_n^2 = w^H D_m w + \sigma_n^2 \]
where \( \text{tr} \{ \cdot \} \) denotes the trace of a matrix and \( D_m \) is the diagonal matrix with \( |D_{m,l,l}| = \sigma_n^2 |g_m|^2 \).

Using (5), (6) and (8)-(10), the optimization problem in (4) can be written as
\[ \min_w w^H D w \] (11)
s.t. \[ P_d w^H h_{d,d} h_{d,d}^H w \] \[ w^H (Q_m + D_m) w + \sigma_n^2 \geq \gamma_m, \forall m \in G_d, \forall d \in K. \]

Problem (11) is generally non-convex. We note that it has a similar mathematical structure as the MUP2P beamforming problem in [5] and [6], and either the SDP approach of [5] or the SOCP approach of [6] can be straightforwardly used to approximately solve it. In the sequel, we will review some aspects of these two methods which are relevant for our approach.

According to [5], the semi-definite relaxation technique can be used to approximate problem (11) by an SDP problem. The relaxation consists in a convex approximation of the non-convex feasible set of problem (11) which contains the original set as a subset. In the case when the solution of the SDP problem is not contained in this subset, a feasible solution can be generated from the solution of the SDP using randomization techniques (see, e.g., [7] and references therein). However, in this way, only solutions which are suboptimal for problem (11) are generated and in some cases they cannot be made feasible for problem (11) at all.

As an alternative to this relaxation approach, we can exploit the rank-one property of the matrix \( P_f h_{d,d} h_{d,d}^H \) and approximate (11) by a SOCP problem [6]. Towards this aim, let us introduce
and rewrite the constraints of problem (11) as
\[
[w] \geq \sqrt{\gamma_m/P_d}||u_m\tilde{w}||, \quad \forall m \in G_d, \quad \forall d \in K \quad (12)
\]
where \(|| \cdot ||\) denotes the Euclidian norm of a vector. With \(w^H\tilde{h}_{d,m} \geq \Re\{\tilde{w}^H\tilde{h}_{d,m}\}\), where \(\Re\{ \cdot \}\) denotes the real part of a complex variable, the non-convex constraints in (12) can be strengthened as
\[
\Re\{\tilde{w}^H\tilde{h}_{d,m}\} \geq \sqrt{\gamma_m/P_d}||u_m\tilde{w}||, \quad \forall m \in G_d, \quad \forall d \in K \quad (13)
\]
The constraints in (13) are second-order cone constraints that are generally convex. Note that
\[
||\cdot||_V \leq \gamma_{d,m}\exp(-j\alpha_{d,m})
\]
(14) compared to that of iteration \(i\) as long as \(\alpha_{d,m}^i \neq 0\) for all active constraints in iteration \(i\). To prove this, first note that it can be shown that at optimum of (14) at least one QoS constraint is active, i.e. satisfied with equality. Then the statement can be proved by contradiction. Let us assume that in some iteration \((i+1)\) the solution of (14) is the same as that in the previous iteration \(i\) (i.e. \(w_{d,m}^{(i)} = w_{d,m}^{(i+1)}\)). Let us further assume that \(\alpha_{d,m}^i \neq 0\) for all active constraints in iteration \(i\). Then, we observe from the inequality in (16) that for these constraints, the left-hand parts of (13) in iteration \((i+1)\) are greater than those in iteration \(i\). Thus, all the QoS constraints in (14) are inactive, contradicting optimality of \(w_{d,m}^{(i+1)} \neq w_{d,m}^{(i)}\) for (14) in iteration \((i+1)\). Further, since \(w_{d,m}^{(i)}\) is feasible for (14) in iteration \((i+1)\), the obtained transmitted power must decrease monotonically with the iterations as long as \(\alpha_{d,m}^i \neq 0\) for all active constraints in the current iteration \(i\). Note, however, that convergence to the optimum of problem (11) is not guaranteed in general.

### Table 1: Proposed iterative procedure

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
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<tbody>
<tr>
<td>Initialization</td>
<td>(\tilde{h}<em>{d,m}^{(1)} = \tilde{h}</em>{d,m}, \forall m \in G_d, \forall d \in K)</td>
</tr>
<tr>
<td>Solve problem (14) with (\tilde{h}<em>{d,m} = \tilde{h}</em>{d,m}^{(i)}).</td>
<td></td>
</tr>
<tr>
<td>Perform the rotation of (15) with (\alpha_{d,m}^i = \angle(w_{d,m}^{(i)}H\tilde{h}_{d,m}^{(i)})).</td>
<td></td>
</tr>
</tbody>
</table>

### 5. ITERATIVE FEASIBILITY SEARCH

The proposed iterative algorithm is based on the assumption that (14) is feasible. As mentioned earlier, this assumption may be wrong even if the original problem (11) is feasible. However, it is clear from the discussion above that if (14) is feasible in the first iteration, then it remains feasible in the following iterations. Even if the problem (14) is infeasible for initialization \(\tilde{h}_{d,m} = \tilde{h}_{d,m}\), there may exist initial rotations of the channel vectors for which (14) is feasible. These can be obtained from a reformulation of (14) as a feasibility problem and applying a similar iterative procedure as described in the previous section.

Introducing the real-valued vector \(z = [z_1, \ldots, z_M]^T\), where \(z_m\) is a measure of how far the \(m\)th QoS constraint in (14) from being satisfied, we can formulate the following feasibility problem

\[
\min_{\tilde{u} \in \mathbb{C}^L} \tilde{u}^Tz
\]

s. t. \(\sqrt{\gamma_m/P_d}||u_m\tilde{w}|| - \Re\{\tilde{w}^H\tilde{h}_{d,m}\} \leq z_m, \forall m \in G_d, \forall d \in K\)

where \(\tilde{u}\) is the \(M \times 1\) vector with all the elements equal to one. The SOCP problem (17) is always feasible and yields the optimal values \(z_{opt}\) and \(\tilde{w}_{opt}\). Note that \(\tilde{u}^Tz_{opt}\) can be viewed as a measure of how far the solution \(\tilde{w}_{opt}\) of (17) is from being feasible for (14). \(z_{m} = 0\) indicates that the \(m\)th QoS constraint in (14) is satis-

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fied, whereas \( z_m > 0 \) indicates the contrary. Hence, if \( 1^T \mathbf{z}_{\text{opt}} = 0 \), then all QoS constraints are satisfied and \( \mathbf{w}_{\text{opt}} \) is a feasible point of (14). However, if \( 1^T \mathbf{z}_{\text{opt}} > 0 \), then (14) is infeasible. In this case, we propose a similar iterative procedure as described above, where we successively improve the approximation around the current \( \mathbf{w}_{\text{opt}} \). Hence, we can perform the iterative procedure described in Table 1 for problem (17) instead of (14). Using similar considerations as above, it can be seen from the inequality in (16) and (17) that \( 1^T \mathbf{z}^{(i+1)}_{\text{opt}} \leq 1^T \mathbf{z}^{(i)}_{\text{opt}} \). Once the stopping criterion \( 1^T \mathbf{z}^{(i)}_{\text{opt}} = 0 \) is reached, a feasible approximation of problem (11) is found. Then, we can proceed with the iterative procedure of Table 1 performed for problem (14), using the previously obtained set of rotated channel vectors as initializations.

We remark that this feasibility search also facilitates the procedure of admission control [10]. A large value of \( z_m \) at the solution of (17) reveals that the QoS constraint corresponding to the \( m \)-th user is far from being satisfied. In the case when no feasible SOCP approximation of (11) could be found after \( I \) iterations, we can exclude the users yielding the largest \( z_m \) and run the feasibility search algorithm again with this reduced set of users. The probability that a feasible approximation can then be found for the reduced set of users is increased.

### 6. SIMULATION RESULTS

Throughout our simulations, we assume flat-fading Rayleigh channels with unit variance. We also assume that the relay and receiver noise powers are equal to each other and that the power of each transmitter is 10 dB above the noise power. For our algorithm, the number of iterations has been chosen as \( I = 5 \). For each SDP-based solution, 5000 randomization vectors were generated (see [7]). The proposed iterative SOCP method (without admission control) is compared to the non-iterative approach of [6] and to the SDP approach of [5] with iterative SOCP method (without admission control) is compared to 5000 randomization vectors were generated (see [7]). The proposed iterative method of [6] proves the performance of the non-iterative SOCP approach of [6] and also substantially outperforms the SDP approach of [5].

In Fig. 3, the relative number of infeasible Monte Carlo runs versus the SINR threshold is displayed for all the methods tested. It can be observed that the probability of problem infeasibility could be reduced significantly as compared to the methods [5] and [6].

### 7. REFERENCES


