WIDELY DISTRIBUTED MIMO RADAR BEAMFORMING FOR DETECTING TARGETS WITH SLOW RCS FLUCTUATIONS

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ABSTRACT
In a widely distributed MIMO radar, the transmitters and the receivers are distributed so that they see a target from different aspects. The scattering from the target is therefore different for each transmitter-receiver pair. It is commonly assumed that a set of orthogonal waveforms are transmitted and the scattered waveforms are then separated using a matched filter to achieve diversity. In this paper, we propose a beamforming method for the widely distributed MIMO radar. Orthogonal waveforms are used initially for probing the target, i.e. to obtain information about the the target and the channel and a single waveform is then transmitted so that the SNR in the receiver is increased. Numerical simulations demonstrate that this will increase the probability of detecting the target.

Index Terms— MIMO radar, Radar detection, Jamming, beamforming

1. INTRODUCTION
In a multiple-input multiple-output radar, multiple waveforms are used to improve the performance of the radar. There exist several different approaches to take advantage of the diversity provided by such systems. In MIMO radars with colocated antennas, the transmitters or receivers function as a coherent array. This approach is taken for example in [1, 2]. A different approach is the MIMO radar with widely separated antennas[3], which is also referred to as the statistical MIMO radar[4]. Using widely distributed transmitters or receivers provides angular diversity, as the target is seen from several different angles.

MIMO radars with colocated antennas typically employ beamforming to increase the power radiated towards the target or received from it. In widely distributed MIMO radars, array geometry may be arbitrary and array manifold is not commonly known, so typical beamforming cannot be used.

In this paper, however, we propose a coding scheme similar to the beamforming in MIMO communication systems. This beamforming method requires channel state information so using it is possible only when the fluctuations of the target RCS are sufficiently slow making channel estimation possible and ensuring that the feedback is not outdated. Suitable targets for this method include those that follow Swerling 1 or 3 scattering models[5], for example.

Obtaining the channel state information is a difficult task especially in the presence of strong interference or jamming. We alleviate this problem by using a projection that is orthogonal to the interference subspace. Some of the target information is lost in the process, but the beamforming method will minimize such losses. It will be seen in the numerical examples that the probability of detection is improved significantly by using the projection.

This paper is organized as follows: The signal model is discussed in Section 2 and target detection in Section 3. The beamforming method is described in Section 4. Numerical results will be provided in Section 5. Finally, Section 6 gives the concluding remarks.

2. SIGNAL MODEL
In a system with $M$ transmitters and $N$ receivers, the signal received by the $k$-th element can be written as

$$r_k(t) = \sum_{m=1}^{M} \sqrt{\frac{P}{M}} c_{km} s_m(t-\tau) e^{j2\pi f_{km} t + j\phi_m} + n_k(t),$$

(1)

where $P$ is a power parameter, $c_{km}$ a scattering amplitude, $s_m$ the signal transmitted by the $m$-th transmitter, $\tau$ the time delay, $f_{km}$ the Doppler shift, $\phi_m$ as well as $\phi_k$ are phase terms, and $n_k$ is noise and interference term. We can write the model as

$$r(t) = (H \odot F)s(t-\tau) + n(t),$$

(2)

where $H$ is the channel matrix, $\odot$ denotes element-wise (Hadamard) matrix multiplication, $F$ is a matrix containing the Doppler shifts, and $n(t)$ is the noise plus interference vector.
We assume that the transmitted signals have sufficient orthogonality properties so that the time delay $\tau$ and the Doppler shifts can be efficiently estimated and removed using a bank of matched filters. The resulting matched filter output can be written as

$$ y = \text{vec}(H) + \tilde{n}(t), $$

where $\tilde{n}$ is the filtered noise plus interference vector. If $n$ is white and the waveforms are orthonormal, the filtered noise will be white as well.

We assume that the scattering amplitudes $c_{km}$ are i.i.d. zero-mean circular complex Gaussian random variables with unit variance corresponding the Case 1 in the Swerling scattering model[6]. As the phase terms do not affect the distribution in this case, the elements of the channel matrix $H$ are similarly distributed. Furthermore, using the assumption that the change in the scattering amplitudes is small between pulses, we can obtain an estimate of the channel matrix by averaging the matched filter output over several pulses.

### 3. TARGET DETECTION

Target detection is typically formulated as hypothesis testing problem with the hypotheses

$$ H_0 : \text{no target present} $$

$$ H_1 : \text{a target is present}. $$

The likelihood ratio test (LRT) is known to be an optimal testing procedure in the Neyman–Pearson sense.

It was shown in [4] that the optimal test statistic for independent Swerling 1 or 2 scattering from the target and white noise is the norm of the matched filter output vector. We showed in [7] that the LRT in colored noise and correlated scattering is equivalent to

$$ S = y^H \Gamma^{-1} y, $$

where matched filter output vector is

$$ y = \sum_i s^*(t_i - \tau) \otimes R_n^{-1} r(t_i) $$

and the weighting matrix defined as

$$ \Gamma = \frac{P}{M} \mathbb{I} \otimes R_n^{-1} + R_c^{-1} $$

for a MIMO radar configuration with both transmitters and receivers widely distributed, where $R_n$ is the covariance matrix of interference plus noise, $R_c$ is the covariance matrix of scattering, and $\otimes$ stands for Kronecker product.

Using the result in [6], the optimal test statistic for multiple pulses from a target with slow RCS fluctuations can be seen to be the norm of the averaged matched filter output vector.

### 4. BEAMFORMING

It has been previously suggested that the transmitters of a widely distributed MIMO radar should transmit orthonormal waveforms independent of each other. We show that after obtaining information about the channel, the probability detecting the target can be increased by changing the transmission scheme.

In order to improve the probability of detection, we take a similar approach as in MIMO communication systems and use precoding writing the signal vector as

$$ \tilde{s} = Ws(t), $$

where $\tilde{s}$ is the new signal vector and $W$ is the coding matrix. However, the goal is to maximize the probability of detection instead of improving the spectral efficiency or robustness of a radio link. Since the optimal detector is essentially the power of the matched filter output with suitable weighting, we can increase the probability of detection by increasing the power of the received signal.

The maximization of the total received signal power after precoding can be rewritten as a following constrained optimization problem:

$$ \max_W E[\|HWs\|^2] \quad \text{s.t.} \quad \text{tr}(WHW) = 1. $$

To simplify the optimization, we note that

$$ HWs = \text{vec}(HWs) = (s^T \otimes H)\text{vec}(W), $$

so by letting $w = \text{vec}(W)$, the total received signal power can be written as

$$ E[\|HWs\|^2] = E[w^H (s^T \otimes H)^H (s^T \otimes H)w] = w^H (I \otimes H^H H)w, $$

where $H$ was assumed to be constant due to slow fluctuation. Thus, the optimization problem can be written as

$$ \max_w \frac{w^H (I \otimes H^H H)w}{w^H w}, $$

the solution of which is simply the eigenvector corresponding the maximum eigenvalue of $I \otimes H^H H$. The optimal coding matrix $W$ has therefore the right singular vector corresponding the largest singular value of $H$ in any of its columns and zeroes elsewhere.

The resulting transmitted signal can be written as

$$ \tilde{s} = \hat{v}_1 s(t), $$

where $\hat{v}_1$ is the estimate of the right singular vector corresponding the largest singular value and $s(t)$ is any one of the available waveforms.

Obtaining an estimate of the channel matrix $H$ is challenging in presence of strong interference or jamming. It can
be shown that the covariance matrix of noise plus interference vector after matched filtering is

$$R\tilde{n} = W^*W^T \otimes R_n,$$  \hspace{1cm} (10)

so it is not possible to improve SINR using the precoder. However, assuming that the interference or jamming signals exhibit spatial correlation across the receivers, the interference can be cancelled by projecting the data to subspace orthogonal to the interference subspace. The projection matrix can be written as

$$P = I - V_i V_i^+,$$  \hspace{1cm} (11)

where $V_i$ is the set of eigenvectors spanning the interference subspace and $(\cdot)^+$ denotes the Moore-Penrose pseudo-inverse. Some of the target information is subsequently lost in the projection, but the improvement in the SINR outweighs such losses significantly. Furthermore, the beamforming is done based on the projected estimate of the channel matrix, so the beamforming will maximize the power of the projected signal.

Target motion and fading that is faster than what Swerling model assumes would cause uncertainty in the singular vector of estimated channel matrix. Taking these into account is considered in future work.

### 5. NUMERICAL EXAMPLES

In this section, we demonstrate that the beamforming method for distributed MIMO radar can be used to increase the probability of detecting the target. We compare numerically evaluated receiver operating characteristic (ROC) curves of the diversity method, in which an independent signal is transmitted from each transmitter, and the beamforming method proposed in this paper. We also compare the results with a SISO system with colocated antenna arrays capable of beamforming with equal number of antennas as in the MIMO system or a distributed MIMO radar with reduced number of transmitters.

In the following examples, six transmitters and six receivers are used in a widely distributed MIMO radar configuration. It was assumed that there was 32 pulses available for detecting a single target. The scattering from the target was assumed to follow the Swerling case I. The beamforming method used initially eight pulses to obtain an estimate of the singular vector and then the rest of the pulses this vector was used for transmitting with the proposed precoding method. The SNR was defined as $P/\sigma_n^2 = -6\text{dB}$. The ROC curves where evaluated for each combination of the parameter values by computing the empirical distributions of the test statistics by the method of Monte Carlo using $4 \times 10^7$ samples.

Fig. 1 shows the ROC curves of the compared systems with no interference or jamming. The MIMO radar beamforming method has significantly higher probability of detecting the target than the diversity scheme transmitting orthogonal waveforms from each transmitter. If SNR was increased enough or the probability of false alarm was close to one, the diversity scheme would be eventually better than the beamforming. Also if the probability of false alarm or SNR are decreased enough, the SISO system with phased array would be best. However, the beamforming method is the clearly the most useful in this example scenario.

In Fig. 2, the ROC curve of the beamforming method is compared to the diversity transmission method with different number of active transmitters. The number of transmitters is denoted by $M$. The optimal number of transmitters depends again on the SNR and the chosen probability of false alarm, but $M = 2$ is the best choice for wide range of $pf$ in this example. However, even higher probability of detection is achieved by using the proposed beamforming method.

The performance of the beamforming method was then tested in the presence of strong jamming. There were three jammers with jammer to signal ratios equal to 20 dB, 10 dB and 6 dB. There were 256 signal-free samples for estimation of the covariance matrix of jamming and noise. The projection matrix was formed by taking the eigenvectors corresponding the eigenvalues that were 2 dB above the noise eigenvalues. The setup was otherwise the same as in the previous examples.

The ROC curves shown in Fig. 3 where evaluated with and without the projection using $10^6$ Monte Carlo samples. Without the projection to orthogonal subspace to cancel jamming, both the beamforming and the diversity methods perform poorly because of the low signal power compared to the power of the jammers. The performance can be improved with the projection, but the drawback is reduced diversity due to the lower dimension of the remaining signal subspace. This
Angular diversity is achieved in a widely distributed MIMO radar by positioning the antennas apart from each other. The system can seldom be calibrated making typical beamforming approaches impossible. We have proposed a novel transmit beamforming scheme for the widely distributed MIMO radar in this paper. Using this method, it is possible to increase the SNR of the received signal and thus improve the probability of detecting a target.

In order to use the proposed beamforming method, an estimate of the channel matrix is required. An estimate may be obtained by transmitting orthogonal waveforms from all the transmitters. The proposed beamforming scheme is applicable only to scenarios in which the target RCS fluctuations are very slow from pulse to pulse and velocity of the target is not too high. The beamforming method can be applied when strong interference or jamming is present if the received signal is projected into a subspace that is orthogonal to the interference subspace. It was seen in the numerical examples that even though some target information is lost in the projection, the target detection performance is improved remarkably.

6. CONCLUSIONS

7. REFERENCES


