ABSTRACT

In this paper, we consider the problem of direction finding in multiple-input multiple-output (MIMO) radar based on focusing the transmitted pulse energy within certain spatial sector(s). We propose a method for designing the transmit weight matrix based on maximizing the energy transmitted within the desired spatial sector and minimizing the energy disseminated in the out-of-sector area. The proposed transmit energy focusing results in the signal-to-noise ratio increase at the receive array which in turn leads to lower Cramer-Rao bound and improved direction of arrival estimation performance. Simulation results show the substantial improvements offered by the proposed transmit energy focusing based MIMO radar as compared to the traditional MIMO radar with receive beamspace post-processing.

Index Terms— MIMO radar, transmit energy focusing, DOA estimation.

1. INTRODUCTION

The emerging technology of multiple-input multiple-output (MIMO) radar has been recently attracting a lot of attention [1], [2]. The main principle behind MIMO radar is to employ multiple antennas to transmit several orthogonal waveforms and to use this orthogonality to decompose the received signal into multiple components. The decomposed components are commonly referred to as the virtual data. A MIMO radar can be either equipped with widely separated antennas [1] or colocated antennas [2]; here we focus on the latter.

Estimating the directions-of-arrival (DOAs) of multiple targets from measurements corrupted by noise at the receiving array of antennas is one of the most important radar applications frequently encountered in practice. Many DOA estimation methods have been developed for traditional single-input multiple-output (SIMO) radar [3]. In particular, subspace-based methods such as MUSIC are the most popular due to their simplicity and high-resolution capabilities. In MIMO radar with colocated antennas, the problem of direction finding involves estimating the DOAs as well as the directions-of-departure (DODs). Recent methods for direction finding in MIMO radar are reported in [4], [5]. However, these methods suffer from low signal-to-noise ratio (SNR) at the receiver as a result of omni-directional radiation at the transmitter.

Several transmit beamforming techniques have been proposed to achieve transmit coherent gain in MIMO radar [6]–[7]. However, the transmit beamforming approach adopted in [6] is based on the assumption that the spatial direction of the target is known a priori. On the other hand, the methods of [7] uses transmit signal cross-correlation to achieve the desired transmit beampattern. However, because the transmitted signals used in [7] are not independent, the data at the receive array can not be simply decomposed into an extended virtual array. In [8], two transmit beams are formed such that an independent waveform is radiated over each beam. By appropriately designing the transmit weight vectors, the received data can be decomposed into two virtual data sets that enjoy phase rotational invariance—a property that enables estimating the DOAs using an arbitrary receive array. However, if the receiver is equipped with one receive antenna, the method of [8] suffers from deterioration in estimation accuracy.

In this paper, we propose to design the transmit weight matrix so that the amount of energy radiated within the desired sector is maximized while the amount of energy wasted in the out-of-sector area is minimized. As a result, the SNR gain improves at each receive antenna, and therefore, the angular resolution of DOA estimation techniques also improves. An expression for the Cramer-Rao bound (CRB) that shows its dependence on the transmit weight matrix is given. The DOA estimation performance of the proposed transmit energy focusing technique is compared to the performance of the traditional MIMO radar technique as well as the performance of the traditional MIMO radar with receive beamspace post-processing. We use MUSIC to estimate the DOAs for all methods. Simulation results show the superiority of the proposed technique over the other two techniques.

1If the relative position of the receive array with respect to the transmit array is known then the problem reduces to either estimating the DODs or DOAs. In this paper, we assume that the MIMO radar system is monostatic and, therefore, we focus only on estimating the DOAs.
2. PROBLEM FORMULATIONS

Let $\Theta$ be the angular sector-of-interest where the targets are likely to be located. We propose to focus the transmitted energy within $\Theta$ by forming $K$ directional beams where an independent waveform is transmitted over each beam. Let $C \triangleq [c_1, \ldots, c_K]^T$ be the transmit weight matrix of dimension $M \times K$ where $c_k$ is the $M \times 1$ unit-norm weight vector used to form the $k$th beam and $(\cdot)^T$ stands for the transpose. Our approach to designing the transmit weight matrix is to maximize the energy transmitted within $\Theta$ and to minimize the energy disseminated in the out-of-sector area.

Let $\phi_K \triangleq [\phi_1(t), \ldots, \phi_K(t)]^T$ be the $K \times 1$ baseband waveform vector. The complex envelope of signal radiated over all beams towards the direction $\theta$ can be modeled as

$$s(t, \theta) = \sqrt{E} \sum_{k=1}^K c_k^H a(\theta) \phi_k(t) = \sqrt{E} \left( C^H a(\theta) \right)^T \phi_K(t)$$

where $(\cdot)^H$ stands for the Hermitian transpose, $a(\theta)$ is the steering vector of the transmit array, and $\sqrt{E/K}$ is a normalization factor used to satisfy the constraint that the total transmit energy over all beams is fixed to $E$.

At the receive array, the $N \times 1$ complex vector of array observations can be expressed as

$$x(t, \tau) = \sqrt{E} \sum_{l=1}^L \alpha_l(\tau) \left( \left( C^H a(\theta_l) \right)^T \phi_K(t) \right) b(\theta_l) + z(t, \tau)$$

where $\tau$ is the slow time index (pulse number), $\alpha_l(\tau)$ is the reflection coefficient associated with the $l$th target, $L$ is the number of targets, $b(\theta)$ is the steering vector of the receive array, and $z(t, \tau)$ is the $N \times 1$ zero-mean white Gaussian noise term with covariance $\sigma_w^2 I_N$.

By matched filtering $x(t, \tau)$ to each of the waveforms $\phi_k(t), k = 1, \ldots, K$, the received signal component associated with the $k$th transmit waveform can be obtained as

$$x_k(\tau) \triangleq \int x(t, \tau) \phi_k^*(t) dt = \sqrt{E} \sum_{l=1}^L \alpha_l(\tau) (c_k^H a(\theta_l)) b(\theta_l) + z_k(\tau)$$

where $z_k(\tau) \triangleq \int z(t, \tau) \phi_k^*(t) dt$ is the $N \times 1$ noise term.

Stacking the individual vector components (3) in one column vector, the $KN \times 1$ virtual data vector is obtained as

$$y(\tau) \triangleq [x_1^T(\tau) \cdots x_K^T(\tau)]^T = \sqrt{E} \sum_{l=1}^L \alpha_l(\tau) \left( C^H a(\theta_l) \right) \otimes b(\theta_l) + z_K(\tau)$$

where $z_K(\tau) \triangleq [z_1^T(\tau), \ldots, z_K^T(\tau)]^T$ is the $KN \times 1$ noise term whose covariance is given by $\sigma_w^2 I_{KN}$.

The transmit beamspace signal model given by (4) provides the basis for optimizing a general-shape transmit beam-pattern over the transmit beamspace weight matrix $C$. By carefully designing $C$, the transmitted energy can be focussed in a certain spatial sector, or divided between several disjoint sectors in space. As compared to traditional MIMO radar, the benefit of using transmit energy focusing is the possibility to increase the signal power at each virtual array element. This increase in signal power is attributed to two factors:

(i) transmit beamforming gain, i.e., the signal power associated with the $k$th waveform reflected from a target at direction $\theta$ is magnified by factor $|c_k^H a(\theta)|^2$;
(ii) the signal power associated with the $k$th waveform is magnified by factor $E/K$ due to dividing the fixed total transmit power $E$ over $K < M$ waveforms instead of $M$ waveforms.

3. TRANSMIT WEIGHT MATRIX DESIGN

The transmit weight matrix $C$ can be designed such that the following two main requirement are satisfied: (i) The amount of energy transmitted within the desired sector is maximized; (ii) The amount of energy that is inevitably disseminated in the out-of-sector area is minimized. One meaningful approach for satisfying these two requirements is to maximize the ratio of the energy that comes from within the desired sector $\Theta$ to the total energy used, i.e., the energy within the total spatial domain $[-\pi, \pi]$. Following this principle, we propose to design the transmit weight matrix such that the ratio of the energy radiated within the desired spatial sector to the total radiated energy is maximized. That is, the transmit weight matrix $C$ is designed based on maximizing the following ratio

$$\Gamma_k \triangleq \frac{\int_{\Theta} \int_T |c_k^H a(\theta) \phi_k(t)|^2 d\theta dt}{\int_T \int_{-\pi}^{\pi} |c_k^H a(\theta) \phi_k(t)|^2 d\theta dt}$$

$$= \frac{c_k^H \int_{\Theta} (a(\theta) a^H(\theta) d\theta) c_k}{\int_{-\pi}^{\pi} |c_k^H a(\theta)|^2 d\theta} = \frac{c_k^H A c_k}{\int_{-\pi}^{\pi} |c_k^H a(\theta)|^2 d\theta}$$

(5)

where $A \triangleq \int_{\Theta} a(\theta) a^H(\theta) d\theta$ is a positive definite matrix and $T$ is the pulse width. In (5), the fact that $\int_T \phi(t) \phi^*(t) dt = 1$ is used. It can be easily shown that [3]

$$\int_{-\pi}^{\pi} |c_k^H a(\theta)|^2 d\theta = 2\pi c_k^H A c_k$$

(6)

Substituting (6) in (5), we obtain

$$\Gamma_k = \frac{c_k^H A c_k}{2\pi c_k^H c_k}, \quad k = 1, \ldots, K.$$  

(7)

The maximization of the expression (7) is equivalent to maximizing its numerator while fixing its denominator by imposing the constraint $\|c_k\| = 1$. It is easy to see that $\Gamma_1$ attains
its maximum value when \( c_1 \) is taken as the eigenvector associated with the maximum eigenvalue of the matrix \( A \). It is worth noting that intuitively one might think that choosing \( c_1 = \ldots = c_K \) would be an optimal solution to the problem of maximizing (5). However, this choice is equivalent to forming one single beam that radiates the waveform \( \varphi(t) = \sum_{k=1}^{K} \phi(t) \) which means that the concept of waveform diversity is not utilized. To avoid this trivial solution, the orthogonality constraint \( c_k^H c_j = 0, \ k \neq j \) has to be also imposed. Then, the maximization of \((\Gamma_k)_{k=1}^K \) subject to the constraint \( C^H C = I \) corresponds to finding the eigenvectors of \( A \) that are associated with the \( K \) largest eigenvalues of \( A \). That is, the transmit beamspace matrix is given as

\[
C = [u_1, u_2, \ldots, u_K]
\]

where \( \{u_i\}_{i=1}^K \) are \( K \) principal eigenvectors of \( A \). It is worth noting that the idea of using the principle eigenvectors for transmit beamforming has been also exploited in [9] to control the transmit antenna quality factor, i.e., to minimize the reactive power.

\section{Transmit Beamspace Based MUSIC}

The virtual data model (4) can be rewritten as

\[
y(\tau) = \sqrt{\frac{E}{K}} V \alpha(\tau) + \tilde{z}_{KN}(\tau)
\]

where \( \alpha(\tau) \triangleq [\alpha_1(\tau), \ldots, \alpha_L(\tau)]^T, \ V \triangleq [v(\theta_1), \ldots, v(\theta_L)], \) and \( v(\theta) = (C^H a(\theta)) \otimes b(\theta). \)

The \( KN \times KN \) transmit energy focusing based covariance matrix is then given by

\[
R \triangleq E \{y(\tau)y^H(\tau)\} = \frac{E}{K} VS V^H + \sigma^2_d I_{KN}
\]

where \( S \triangleq E[\alpha(\tau)\alpha^H(\tau)] \) is the covariance matrix of the vector of reflection coefficients. The sample estimate of (10) takes the form \( \hat{R} = \frac{1}{Q} \sum_{\tau=1}^Q y(\tau)y^H(\tau) \) where \( Q \) is the total number of samples available.

The eigendecomposition of \( \hat{R} \) can be written as

\[
\hat{R} = E_n \Lambda_n E_n^H + E_n \Lambda_n E_n^H
\]

where the \( L \times L \) diagonal matrix \( \Lambda_n \) contains the largest (signal-subspace) eigenvalues and the columns of the \( KN \times L \) matrix \( E_n \) are the corresponding eigenvectors. Similarly, the \( (KN-L) \times (KN-L) \) diagonal matrix \( \Lambda_n \) contains the smallest (noise-subspace) eigenvalues, while the \( KN \times (KN-L) \) matrix \( E_n \) is built from the corresponding eigenvectors.

Applying the principle of MUSIC [3], the transmit energy focusing based spectral-MUSIC estimator can be expressed as

\[
f(\theta) = \frac{\nu^H(\theta) v(\theta)}{\nu^H(\theta) Q v(\theta)}
\]

where \( Q = E_n E_n^H = I - E_n E_n^H \) is the projection matrix onto the noise subspace. Substituting \( v(\theta) \) into (12), we obtain

\[
f(\theta) = \frac{N a^H(\theta) C C^H a(\theta)}{[(C^H a(\theta)) \otimes b(\theta)]^H Q [(C^H a(\theta)) \otimes b(\theta)]}, \quad (13)
\]

Then, the DOAs of the targets can be obtained by searching for the highest \( L \) peaks of the spectrum (13).

The CRB for estimating the DOAs can be expressed as

\[
\text{CRB}(\theta) = \frac{\sigma^2_d}{2QEB} \left( \text{Re} \left( D^H P_V D \otimes G^T \right) \right)^{-1}
\]

where \( P_V \triangleq V(V^H V)^{-1} V^H, \ G \triangleq (SV^H R^{-1} VS), \) and \( D \triangleq [d(\theta_1), \ldots, d(\theta_L)] \) is the matrix whose \( l \)-th column is given by the derivative of the \( l \)-th column of \( V \) with respect to \( \theta_l \). The full derivation is given in [10].

\section{Comparison to Existing Techniques}

To clearly show the benefits of the proposed energy focusing technique, we compare it to the following two techniques.

(i) Traditional MIMO radar.

Choosing \( C = I_M \), the transmit energy focusing based signal model (4) simplifies to the traditional MIMO radar signal model, that is,

\[
y_{\text{MIMO}}(\tau) = \sqrt{\frac{E}{M}} \sum_{l=1}^L a_l(\tau) b(\theta_l) + z_{BS}(\tau)
\]

where \( y_{\text{MIMO}}(\tau) \) is the \( MN \times 1 \) virtual data vector associated with the traditional MIMO radar.

(ii) MIMO radar with receive beamspace post-processing.

The well known beamspace dimension reduction technique widely used in passive arrays can be applied to the traditional MIMO radar data model (14). For the sake of fair comparison, we employ the same weight matrix \( C \) designed for transmit energy focusing but use it for receive beamspace post-processing, yielding

\[
y_{\text{BS}}(\tau) = (C \otimes I_N)^H y_{\text{MIMO}}(\tau)
\]

\[
= \sqrt{\frac{E}{M}} \sum_{l=1}^L a_l(\tau) (C^H a(\theta)) \otimes b(\theta_l) + z_{BS}(\tau)
\]

where \( z_{BS}(\tau) \) is the \( KN \times 1 \) noise term whose covariance is given by \( C^H (\sigma^2_d I_M) C = \sigma^2_d I_{KN} \). In (15), the fact that \( (C \otimes I_N)^H (a(\theta) \otimes b(\theta)) = (C^H a(\theta)) \otimes b(\theta) \) is used.

By comparing (4), (14), and (15), it is easy to observe that the SNR gains for the transmit energy focusing based MIMO radar, the MIMO radar with receive beamspace post-processing, and the traditional MIMO radar are proportional to \( (E/K) |C^H a(\theta)|^2, (E/M) |C^H a(\theta)|^2, \) and \( E/M, \) respectively. Noting that \( K < M \), it can be readily proved that the transmit energy focusing based MIMO radar has the highest SNR per virtual element. This observation is confirmed also by simulation examples.
6. SIMULATION RESULTS
A uniform linear transmit array of $M = 10$ antennas spaced half a wavelength apart from each other is assumed. Two types of receivers are considered (i) a single receive antenna; (ii) an arbitrary linear receive array of $N = 10$ antennas whose locations are randomly drawn. The additive noise is Gaussian zero-mean unit-variance spatially and temporally white. The sector of interest $\Theta = [-10^\circ, 10^\circ]$ is taken. Two targets are located at directions $2^\circ$ and $4^\circ$, respectively. We compare the proposed energy focusing based MIMO radar (4) to the traditional MIMO radar (14) and the MIMO radar with receive beamspace post-processing (15). The matrix $C$ of dimension $M \times 4$ is used and the MUSIC algorithm is applied to estimate the DOAs for all methods tested based on 100 data snapshots. The probability of source resolution and the estimation root-mean-square error (RMSE) are computed based on 500 independent runs.

Fig. 1 shows the probability of source resolution versus SNR for all methods tested. It can be seen from this figure that the MIMO radar with receive beamspace post-processing has better probability of source resolution than the traditional MIMO radar for both cases of $N = 1$ and $N = 10$. Also, it can be seen from the figure that the proposed transmit energy focusing based MIMO radar has the best probability of source resolution as compared to the other two methods. Fig. 2 shows the RMSEs for all methods tested. It can be seen from this figure that the MIMO radar with receive beamspace post-processing has better RMSE than the traditional MIMO radar, while the proposed transmit energy focusing based MIMO radar outperforms all aforementioned methods for both cases of $N = 1$ and $N = 10$.

7. CONCLUSIONS
The problem of DOA estimation in MIMO radar based on focusing the energy of the transmitted pulses within a certain spatial sector has been considered. A method for designing the transmit weight matrix, which is based on maximizing the energy transmitted within the desired spatial sector and minimizing the energy disseminated in the out-of-sector area, has been proposed. The proposed transmit energy focusing results in SNR improvement at the receive array which in turn leads to improved DOA estimation performance. Simulation results have confirmed the superiority of the proposed method over the traditional MIMO radar and the MIMO radar with receive beamspace post-processing.

8. REFERENCES