ADAPTIVE DETECTION OF MULTIPLE POINT-LIKE TARGETS WITH CONIC ACCEPTANCE

Chengpeng Hao⋆ Francesco Bandiera† Jun Yang⋆ Chaohuan Hou⋆

⋆ Institute of Acoustics, Chinese Academy of Sciences, Beijing, China
† Dipartimento di Ingegneria dell’Innovazione, Universita del Salento, Lecce, Italy
Email: haochengp@mail.ioa.ac.cn, francesco.bandiera@unisalento.it, yangjun@mail.ioa.ac.cn, hch@mail.ioa.ac.cn

ABSTRACT

In this paper we consider the problem of detecting multiple point-like targets in the presence of steering vector mismatches and Gaussian disturbance with unknown covariance matrix. To this end, we first model the actual useful signal as a vector belonging to a proper cone whose axis coincides with the whitened direction of the nominal array response. Then we develop two new robust adaptive detectors resorting to the two-step generalized-likelihood ratio test (GLRT) design procedure without assignment of a distinct set of secondary data. Finally, a performance assessment, conducted by Monte Carlo simulation, show that the proposed detectors achieve a visible performance improvement over their natural counterparts.

Index Terms— Adaptive detection, constant false alarm rate (CFAR), generalized likelihood ratio test (GLRT).

1. INTRODUCTION

Adaptive radar detection of targets embedded in Gaussian disturbance has been an active research field in the last decades. Specifically, adaptive detection of point-like targets embedded in Gaussian disturbance has been addressed in [1, 2], based upon the assumption that a set of secondary data, free of signal components but sharing the spectral properties of the data under test, is available. However, these solutions may experience a performance degradation in practice wherein the actual steering vector is not consistent with the nominal one. A mismatched signal may appear subject to several causes, such as calibration and pointing errors, wavefront distortions and imperfect antenna shape. To handle such mismatched signals, an adaptive beamformer orthogonal rejection test (ABORT) is proposed, which takes the rejection capabilities into account at the design stage [3]. Moreover, some alternative approaches are devised in [4, 5], which depends on constraining the actual signature to span a cone, whose axis coincides with its nominal value. An important point that has emerged from the study of adaptive detection techniques under mismatched signals is the superiority of the so called “tunable detectors,” i.e., algorithms capable of changing their behavior by tuning proper parameters [6].

All of above mentioned papers deal with the case of a single cell under test, while the detection of multiple and/or distributed targets under mismatched signal models has not received much consideration until now. Relevant examples are [7] where tunable detectors for distributed target are proposed. In addition, a customary assumption that all quoted works share is that a set of secondary data is available. Such hypothesis has been removed in [8, 9] where adaptive detection of multiple point-like targets in correlated Gaussian noise, based upon the two-step GLRT-based design procedure, has been addressed. Therein, secondary data are selected as part of the decision process based on a priori knowledge about the maximum number of point-like targets. More precisely, in [8], the authors generalize the GLRT to the case of detecting multiple point-like targets, whereas in [9], the ABORT idea is improved to address detection of multiple point-like targets.

This paper moves a further step: it proposes two tunable radar detectors for multiple point-like targets and without a distinct set of secondary data. More precisely, it deals with the problem of detecting an unknown signal, lying in a conic set whose axis coincides with the nominal steering vector in the whitened observation space. This model is a viable means to address adaptive detection in case of mismatched steering vectors [4]. A preliminary performance assessment, based upon simulated data, confirms the robust behavior of the new detectors with respect to steering vector mismatches.

The paper is organized as follows: Section 2 addresses the problem formulation, Section 3 deals with detector designs, Section 4 is devoted to the performance assessment, and Section 5 contains some concluding remarks.
2. PROBLEM FORMULATION

Assume that data are collected from \( N \) sensors and deal with the problem of detecting the presence of a target across \( K \) range cells \( r_l \in \mathbb{C}^{N \times 1} \), \( l \in \Omega \equiv \{1, \ldots, K\} \). Moreover, denote by \( \Omega_T \subseteq \Omega \) the set indexing the range cells that might contain useful target echoes under the signal-plus-noise hypothesis \((H_1)\); such a set is unknown at the receiver side, but for its cardinality \( H \).

The detection problem at hand can be formulated in terms of the following binary hypotheses test

\[
H_0 : \quad r_l = n_l, \quad l \in \Omega, \\
H_1 : \quad \begin{cases} 
   r_l = n_l, \quad l \in \Omega \setminus \Omega_T, \\
   r_l = v_l + n_l, \quad l \in \Omega_T \end{cases} \tag{1}
\]

where

- the \( n_l \)'s \( \in \mathbb{C}^{N \times 1} \), \( l \in \Omega \), are independent, zero-mean, complex Gaussian vectors with covariance matrix given by \( E[n_l n_l^\dagger] = M \), \( l \in \Omega \), where \( E[\cdot] \) denotes expectation and \( \dagger \) conjugate transposition;
- \( \Omega_T \) is an unknown subset of \( \Omega \) of cardinality \( H (H \leq K) \);
- \( \Omega \setminus \Omega_T \) denotes the difference between \( \Omega \) and \( \Omega_T \);
- \( v_l, l \in \Omega_T \), denotes the actual steering vector which might not be aligned with the nominal steering vector \( v_0 \).

In order to facilitate the study of the problem (1), we apply a unitary transformation to bring the signal representation into a simplified form, which does not change the problem as it is equivalent to a change of coordinates. Precisely, we denote by \( \mathbf{U} \) a unitary transformation such that \( \mathbf{U} \mathbf{M}^{-1/2} v_l \) is parallel to \( e_N = [0, 0, \ldots, 1]^T \) (\( \mathbf{U}^T \) denotes the transpose operator). In the perfect matching case, it is assumed that \( v_{wl} = \mathbf{U} \mathbf{M}^{-1/2} v_l, l \in \Omega_T \), with \( v_l \) an unknown complex parameter accounting for the channel propagation effects as well as the target reflectivity. In practical situations, mismatches may occur which cause deviations from the nominal direction \( e_N \). A possible way to cope with these scenarios is to assume more uncertainty about \( v_l \) than the case of perfect matching. According to this guideline, direction mismatches can be accounted for, at the design stage, assuming that \( v_{wl} \) belongs to the set \( \Gamma \), which is defined as follows [4]:

**Definition:** Let \( \mathbf{x} = (x_1^T, x_N)^T \) be an \( N \)-dimensional complex vector with last component \( x_N \), then

\[
\Gamma = \left\{ \mathbf{x} = (x_1^T, x_N)^T \in \mathbb{C}^{N \times 1} : ||x_1|| \leq \gamma ||x_N|| \right\},
\]

where \( \cdot \) \( \cdot \) denotes the Euclidean norm of a complex vector, and \( | \cdot | \) is the modulus of a complex number. Observe that \( \gamma \) is the design parameter and that it should be set in order to reflect the possible a priori knowledge about the mismatch.

3. DETECTOR DESIGNS

In order to solve the test (1), a possible way is to resort to the two-step GLRT design criterion. In the first step, derive the GLRT over the \( r_l \)'s, \( l \in \Omega \), assuming that \( M \) is known. Fully adaptive detectors are then obtained by substituting the unknown matrix by the sample covariance matrix based on all of range cells. Following this guideline in the sequel we design two robust detectors for the problem at hand.

3.1. Conic-Acceptance GLRT

The GLRT for the test (1), under the assumption that \( M \) is known, is given by

\[
\max_{\Omega_T} \max_{\{w_l \in \Gamma\} \in \Omega_T} \frac{f(r_1, \ldots, r_K | \Omega_T, \{v_l\} \in \Omega_T, M, H_1)}{f(r_1, \ldots, r_K | M, H_0)} \geq \eta, \tag{2}
\]

where \( f(r_1, \ldots, r_K | \Omega_T, \{v_l\} \in \Omega_T, M, H_1) \) and \( f(r_1, \ldots, r_K | M, H_0) \) denote the complex multivariate probability density functions (pdfs) of the vectors \( r_1, \ldots, r_K \) under the \( H_1 \) and the \( H_0 \) hypotheses, respectively, and \( \eta \) is the threshold value to be set in order to ensure the desired probability of false alarm \((P_{fa})\).

After some algebraic manipulations, the GLRT can be recast as

\[
\max_{\Omega_T} \left[ -\min_{\{w_l \in \Gamma\} \in \Omega_T} \sum_{l \in \Omega_T} (r_l - v_l)^\dagger M^{-1}(r_l - v_l) + \sum_{l \in \Omega_T} r_l^\dagger M^{-1} r_l \right]_{H_1 \geq \eta_{H_0}} \tag{3}
\]

where \( \eta \) is a suitable modification of the original threshold.

The minimization over \( \{v_l \in \Gamma\} \subseteq \Omega_T \) can be accomplished in closed form exploiting the results in [6, 7], i.e.,

\[
\min_{\{w_l \in \Gamma\} \subseteq \Omega_T} \sum_{l \in \Omega_T} (r_l - v_l)^\dagger M^{-1}(r_l - v_l) = \sum_{l \in \Omega_T} a_l^2 u(a_l) \frac{1}{1 + \gamma^2},
\]

where \( u(\cdot) \) is the unit step function, and

\[
a_l = \sqrt{r_l^\dagger M^{-1} r_l - \frac{|v_l^0 M^{-1} r_l|^2}{v_l^0 M^{-1} v_l}} - \gamma \sqrt{\frac{|v_l^0 M^{-1} r_l|^2}{v_l^0 M^{-1} v_l}}.
\]

This implies that the GLRT can be written as

\[
\max_{\Omega_T} \sum_{l \in \Omega_T} \left[ r_l^\dagger M^{-1} r_l - \frac{1}{1 + \gamma^2} a_l^2 u(a_l) \right]_{H_1 \geq \eta_{H_0}} \tag{4}
\]

Finally, maximization over the unknown set \( \Omega_T \) can be easily obtained selecting \( \Omega_T \) as the set of integers indexing the range cells in \( \Omega \), which correspond to the greatest values of \( r_l^\dagger M^{-1} r_l - a_l^2 u(a_l)/(1 + \gamma^2) \), \( l \in \Omega \).
The GLRT is thus given by
\[
\sum_{l \in \hat{\Omega}} \left[ r_l^\dagger M^{-1} r_l - \frac{1}{1 + \gamma^2} a_l^2 u(a_l) \right] H_1 \gtrsim H_0 \eta. \tag{5}
\]
Plugging the sample covariance matrix \( S = \sum_{l \in \Omega} r_l r_l^\dagger / K \) in place of \( M \) in (5), the GLRT, referred to in the following as Robust-Conic-Acceptance-GLRT (RCA-GLRT), can be written as
\[
\sum_{l \in \hat{\Omega}} \left[ r_l^\dagger S^{-1} r_l - \frac{1}{1 + \gamma^2} b_l^2 u(b_l) \right] H_1 \gtrsim H_0 \eta, \tag{6}
\]
where
\[
b_l = \sqrt{r_l^\dagger S^{-1} r_l - \frac{|v_0^\dagger S^{-1} r_l|^2}{v_0^\dagger v_0} - \gamma \sqrt{|v_0^\dagger S^{-1} r_l|^2}} \]
and \( \hat{\Omega} \) denotes the set of integers indexing the range cells in \( \Omega \) which correspond to the greatest values of \( r_l^\dagger S^{-1} r_l - b_l^2 u(b_l)/(1 + \gamma^2) \), \( l \in \Omega \).

In particular, for \( \gamma = 0 \), we obtain the GLRT for multiple point-like targets [8].

3.2. Conic-Acceptance ABORT

According to the orthogonal rejection criterion [3], we assume that under \( H_0 \) the received signal contains a fictitious signal which is orthogonal to the nominal steering vector \( v_0 \) in the whitened space. Otherwise stated the \( H_0 \) hypothesis of the test (1) can be formulated as follows
\[
H_0 : \begin{cases} r_l = n_l, \\ r_l = x_l + n_l, \end{cases} \quad l \in \Omega \setminus \hat{\Omega}, \tag{7}
\]
where \( x_l \)'s \( \in \mathbb{C}^{N \times 1} \), \( l \in \Omega \), are fictitious vectors, which can be expressed as \( x_l = W p_l \) with \( p_l \in \mathbb{C}^{(N-1) \times 1} \), \( W \in \mathbb{C}^{N \times (N-1)} \) such that \( (M^{-1/2}W)^\dagger = (M^{-1/2}v_0) \), where \( \langle \cdot \rangle \) denotes the range space spanned by the columns of the matrix argument, and \( \langle \cdot \rangle^\perp \) its orthogonal complement. Moreover the hypothesis \( H_1 \) of (1) remains unaltered.

The GLRT for this modified hypotheses test, under the assumption that \( M \) is known, can be written as
\[
\max_{\hat{\Omega}} \left[ \sum_{l \in \hat{\Omega}} \left( r_l^\dagger M^{-1} r_l - \frac{1}{1 + \gamma^2} a_l^2 u(a_l) \right) \right] \tag{8}
\]
where \( f(\{r_l\}, \{p_l\}) \) denotes the complex multivariate pdf of the vectors \( r_1, \ldots, r_K \) under the modified \( H_0 \) hypothesis.

After some algebraic manipulations, test (7) can be recast as
\[
\max_{\hat{\Omega}} \left[ \sum_{l \in \hat{\Omega}} \left( r_l^\dagger M^{-1} r_l - \frac{1}{1 + \gamma^2} a_l^2 u(a_l) \right) \right] + \tag{9}
\]
and the greatest values of
\[
\max_{\hat{\Omega}} \left[ \sum_{l \in \hat{\Omega}} \left( r_l^\dagger M^{-1} r_l - \frac{1}{1 + \gamma^2} a_l^2 u(a_l) \right) \right] \gtrsim H_0 \eta, \tag{10}
\]
where, of course, \( \hat{\Omega} \) and \( \hat{\Omega}^\perp \) have to be computed using \( S \) in place of \( M \). This detector reduces to the ABORT for multiple point-like targets when \( \gamma = 0 \), which can be straightforwardly derived using results in [9].

Finally, it is worth pointing out that the Rao test and the Wald test guarantee the CFAR property with respect to \( M \). Proofs of such statements, not reported here for the sake of brevity, follow the lead of [7] and references therein.

4. PERFORMANCE ASSESSMENT

This section is devoted to the performance assessment of the proposed receivers, also in comparison with the GLRT and the ABORT. We assume that the mainlobe (sidelobe) signals in different range cells have the same direction \( \phi \), and denote \( \phi \) the angle between the actual steering vector and the nominal one in the whitened-dimensional data space, i.e.,
\[
\cos^2 \phi = |v_0^\dagger M^{-1} v_0|^2 / [(v_0^\dagger M^{-1} v)(v_0^\dagger M^{-1} v_0)].
\]
As to the noise, it is modeled as an exponentially-correlated complex normal vector with one-lag correlation coefficient \( \rho \), namely the \((i,j)\)-th element of the covariance matrix \( M \) is given by \( \rho^{i-j} \). The signal-to-noise power ratio (SNR) is defined as
\[
SNR = \sum_{l \in \Omega} |v_l^\dagger M^{-1} v_l|.
\]

In Fig. 1 and Fig. 2 we show the performances of the RCA-GLRT and the RCA-ABORT, respectively. More precisely, in the upper subplot of each figure we plot \( P_t \) versus SNR for \( \cos^2 \phi = 1 \) and several values of \( \gamma \), while in the lower subplot we report \( P_t \) versus \( \cos^2 \phi \) for the same values of \( \gamma \) as in Fig. 1 and SNR = 30 dB. Moreover, both figures refer to \( N = 20, K = 60, H = 5, P_{fa} = 10^{-4}, \) and \( \rho = 0.9 \).

As it can be seen, the RCA-GLRT and the RCA-ABORT are more robust than their counterparts, although at the price
of a certain loss in terms of detection of matched signals. More precisely, the higher $\gamma$, the higher the performance loss in matched signals case but, at the same time, the higher the capability of the receivers to detect severely mismatched signals. Otherwise stated, varying $\gamma$, we can trade off target sensitivity with sidelobes energy rejection. Finally, the curves also show that for a given $\gamma$, the RCA-GLRT is more robust (less selective) than the RCA-ABORT.

5. CONCLUSION

In this paper, we have proposed tunable detectors for multiple point-like targets without a preassigned set of secondary data. First of all we have assumed that the useful signal belongs to a conic region and thus we have devised two adaptive GLRT-based detectors relying on the two-step GLRT design procedure. Interestingly, the proposed detectors ensure CFAR property. Computer simulations show that the new detectors can providing a wider range of performances with respect to their counterparts and with a limited loss in terms of $P_d$ for matched signals. Further work will involve the analysis of the proposed detectors in a clutter-dominated non-Gaussian scenario.

6. REFERENCES


