MULTIPATH MODEL AND EXPLOITATION IN THROUGH-THE-WALL RADAR AND URBAN SENSING

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ABSTRACT

We establish the multipath model which encompasses the target and its multipath “ghosts” in urban and through-the-wall synthetic aperture radar (SAR). The focused downrange and crossrange locations of multipath ghosts are derived and validated using numerical and experimental data. The multipath model permits an implementation of a multipath exploitation algorithm, which maps each target ghost to its corresponding true target location. In doing so, the proposed algorithm improves the radar system performance by aiding in ameliorating the false positives in the original SAR image as well as increasing the SNR at the target locations, culminating in enhanced behind the wall target detection and localization.

Index Terms—Through-the-wall radar, multipath exploitation, eigenrays, least squares, SAR

1. INTRODUCTION

Through-the-wall (TTW) radar and urban sensing address the desire to obtain the electromagnetic (EM) blueprint and the layout of the building along with traditional radar capabilities, such as detection, localization, and classification of one or more targets inside rooms or buildings [1, 2]. The existence of targets inside a room or an enclosed structure is deemed to introduce ghosts in the beamformed SAR images. Without a reasonable TTW multipath model, it becomes difficult to associate a ghost to a particular target.

In this paper, we derive a multipath model, as well as present a multipath exploitation technique which associates and then maps the multipath ghosts to their true target locations. The proposed multipath exploitation algorithm is neither a vehicle to compensate for the lack of line-of-sight, as in [3], nor is it attempting to reveal shadowed regions, as in [4]. The goal in this paper is twofold; reducing clutter and false positives caused by target multipath ghosts and increasing the SNR by folding these ghosts over to the true target image. This is achieved by proper utilization of the prior known locations of major reflectors in the scene, such as interior walls in through wall radar applications.

The multipath model assumes SAR or multiple antenna systems, whereas the exploitation algorithm is based on incorporation of higher order specular scatterings in backprojection imaging. The problem is cast for stationary targets, but can be readily extended to moving targets via the application of Doppler filters. Although the presented multipath model deals with walls, reflections from the ceiling and floor can be easily incorporated by using a 3D model, similar to the work in [2, 5, 6].

The paper is organized as follows. In Section 2, we derive the multipath model and analyze the locations of the ghosts in a rigorous fashion. In Section 3, the multipath exploitation algorithm is presented. Section 4 provides supporting results based on simulated and experimental data, and conclusions are drawn in Section 5.

2. MODEL

Consider a room under surveillance using a SAR system. Prior knowledge of the room layout, i.e., wall locations and material properties, is assumed. The scene of observation relative to the $n$-th sensor location is shown in Fig. 1. The front wall has a thickness and dielectric constant given as $d_i$ and $\varepsilon_i$, respectively. For simplicity of notation, we assume that the back and the side walls also have $\varepsilon_i$ as their dielectric constants. The walls have been numbered as shown in the figure, and are self explanatory. The $n$-th sensor location is denoted as $R_n = [-D_{sn}, 0]^T$. The target is at position $A = [-x_t, y_t]^T$. The standoff distance from the front wall is denoted as $D_s$. In the figure, we consider the direct path, referred to as path-A, and three additional paths, namely, paths-B, C, and D, which correspond to single-bounce multipath. Other and higher order paths are ignored for their relatively weaker powers.

As dictated by Snell’s law, and as seen in Fig. 1, each path has associated angles of incidence and refraction. For example, the angles of incidence and refraction for path-B are denoted as $\psi_{ib}^{(n)}$ and $\psi_{ob}^{(n)}$, respectively. Let the reflection points on wall-1, 2, and 3 be denoted by $B_s$, $C_n$, and $D_n$, with respective position vectors $B_s = [y_{3n}, 0]^T$, $C_n = [x_{cn}, D_1 + D_2]^T$, and $D_n = [-D_2, y_{3n}]^T$. These vectors
can be obtained from geometry. The one-way path delays for the four paths with the antenna at the n-th position can be easily derived and are denoted as $\tau_{p}^{(n)}$, $p \in \{A, B, C, D\}$. For example, the one way delay for path-C can be readily shown to be

$$\tau_{C}^{(n)} = \left( \frac{d_{\text{tot}}}{c} \right) \left( \sec(\gamma_{C}) \right)$$

The signal returns are a superposition of the direct path and the multipath returns. For the n-th sensor position, we obtain

$$r_{n}(t) = \sum_{p=\{A, B, C, D\}} \Gamma_{mp}^{2} s(t - 2\tau_{p}^{(n)}) + \sum_{(p,q)\in\{(A, B, C, D)\}, p\neq q} \Gamma_{pq} \omega_{pq} s(t - \tau_{p}^{(n)} - \tau_{q}^{(n)})$$

where $\Gamma_{pq}$ is the complex amplitude associated with reflection and transmission coefficients for the one-way propagation along path-p and returning via path-q, $\omega_{pq}$ is the second sum that comprises of the signal traveling via path-p returning via path-q, referred to here as the combination paths. Equation (1) depends on the angles of incidence and refraction. These angles maybe computed from a numerical optimization, see for example [5, 6]. The coordinates of the reflections at the back and side walls being sensor dependent implies that the multipath presents itself at different locations to the different sensors.

### 2.1 Location of multipath ghosts

Consider Fig. 2 and the combination path comprising of path-A and path-B, i.e., the ray travels to the target via path-A, and follows path-B back to the radar or vice versa. We wish to find the location of this path as seen by the N sensors. For ease of exposition, we consider the scenario with no front wall and only wall-1 as shown in Fig. 2. It can then be readily derived that multipath ghost denoted by $P_{wn} = [-x_{wn}, y_{wn}]^T$, lies on the intersection of an ellipse and a circle. The ellipse is the bistatic locus of the multipath range, whereas the circle is the classical monostatic locus of the multipath. The superscript 'wn' stresses that the ghost is associated with the reflection at wall-1. In Fig. 2, the virtual target is denoted as $A'$ given by coordinates $[x_{1}, y_{1}]$. The equations for the ellipse and circle are given by,

$$\frac{x_{wn}^2}{R_{wn}^2} + \frac{y_{wn}^2}{D_{wn}^2} = 1$$

$$R_{wn} = A'R_{ wn}$$

The solution of (3a), (3b) sets $x_{wn} = 0$ as the only possibility, i.e., the multipath always falls on the wall. The y-coordinate of $P_{wn}$ can then be derived and is sensor dependent. In the presence of the front wall, the solution is unchanged, but the ellipse and circle are different than those presented in (3a) and (3b). The above analysis can be extended to the other walls in a straightforward manner. Since the multipath location is sensor dependent, the multipath may be regarded as a moving target. As a result, during beamforming, the multipath ghost focuses (appears) at a different pixel in the vicinity of the true multipath locations.

### 2.2 Multipath focusing analysis

Consider the multipath locations associated with wall-1. Assume that the focused multipath ghost appears at pixel $x_{wn} = [-\Delta x_{1}, x_{wn}, \Delta y_{1} + y_{wn}]^T$, where $x_{wn} = [x_{wn}, y_{wn}]^T$ is the true multipath location corresponding to the first sensor. Then, similar to the work in [8], and using a first order Taylor series expansion, we can obtain the difference in propagation delay between the focused multipath and the true multipath w.r.t the sensors as

$$\Delta \tau_{n}^{(n)} = \Delta \tau_{n} \sin(\psi_{wp}^{(n)}) + (\Delta y_{1} - (y_{wn} - y_{wn})) \cos(\psi_{wp}^{(n)}), \forall n$$
For the multipath to focus at $x_n^m$, we must have
\[ \Delta r_n^m = 0, \forall n = 1, \ldots, N. \]
A least squares (LS) problem may then be formulated as,
\[
A_i e_i = b_i, 
A_i = [\Delta x_i, \Delta y_i]^T, A_i = [a_i^n; a_i^m]^T
\]
\[
a_i^n = [\sin(\psi_{1n}^m), \ldots; \sin(\psi_{Nn}^m)], a_i^m = [\cos(\psi_{1n}^m), \ldots; \cos(\psi_{Nn}^m)]^T
\]
\[
b_i = [0; y_{1n}^m - y_{i1}^m, \ldots; y_{Nn}^m - y_{iN}^m]^T \odot a_i^n
\]
where '$\odot$' denotes the Hadamard product. The LS solution is $(A_i^T A_i)^{-1} A_i^T b_i$. A similar LS formulation can be derived and solved for the other walls as well.

3. MULTIPATH EXPLOITATION

Noting that the multipath ghosts exist due to the presence of the target, we state our objective as follows. Given the original beamformed image, $I_i()$, our aim is to tag the multipath ghosts to the associated target via the model developed in Section 2. For ease of exposition, the technique is explained considering the focused multipath ghost due to wall-1. The technique for exploiting the ghosts w.r.t all three walls is enumerated subsequently. Let $I(x, y), x \in X, y \in Y$ be the complex amplitude of the pixel at $(x, y)$. Here, $X$ and $Y$ index the crossrange and downrange, respectively. The image $I(\cdot)$ in practice consists of $N_x \times N_y$ pixels, representing the dimensions of $X$ and $Y$, respectively. Consider an arbitrary location indexed by $x_i \in X, y_i \in Y$, whose focused multipath ghost w.r.t wall-1 presents itself at $[x_i^n, y_i^n], x_i^n \in X, y_i^n \in Y$. Consider an intermediate image, wherein the association and mapping of the focused multipath ghosts is performed using simple 2D weighting functions, $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$. That is,
\[
I_i(x_i, y_i) \in \mathbb{C}^{N_x \times N_y} = \sum_{x, y} \sum_{x', y'} ||I(x, y)||^2 \Phi_1(x_i^n, x, \sigma^2) \Phi_2(x_i^n, x, \sigma^2)
\]
\[
x := [x, y]^T, x_i := [x_i, y_i]^T, x_i^n := [x_i^n, y_i^n]^T
\]
\[
\Phi_1(x_i^n, x, \sigma^2) = e^{-|x - x_i^n|/\sigma^2}, \Phi_2(x_i^n, x, \sigma^2) = 1 - e^{-|x - x_i^n|/\sigma^2}
\]
(6)

where $\sigma^2$ is an arbitrary variance, which can assume different values. It is noted that in (6) these weighting functions are related to the 2D real Gaussian distributions. The role of $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$ is explained as follows. Consider $\Phi_1(\cdot)$; if the pixel $x$ is in the vicinity of the focused multipath ghost pixel $x_i^n$ associated with the target location $x_i$, then $\Phi_1(\cdot)$ assumes a large weight. On the other hand, if $x$ is not in the vicinity of $x_i^n$ then $\Phi_1(\cdot)$ assumes a low weight. The vicinity is controlled by $\sigma^2$. Since an exponential function is used, the weights are always between 0 and 1.

The weighting function, $\Phi_2(\cdot)$, serves two purposes. First, it rejects pixel locations in the vicinity of the imaged pixel, i.e., the location corresponding to the true target $x = x_i$. Although the focus point is singular, but due to the system’s point spread function, some of the energy is spread to the neighboring pixels. Second, it disallows energy to be focused back to the multipath ghost locations.

Considering the ghosts w.r.t all three walls, we can readily see that (6) is modified as,
\[
I_f(x_i, y_i) = \sum_{n=1}^{N} \sum_{w} \sum_{y} ||I(x, y)||^2 \Phi_1(x_i^n, x, \sigma^2) \Phi_2(x_i^n, x, \sigma^2)
\]
(7)

Image $I_f(\cdot)$ will have no intensity at and near the vicinity of the focused multipath pixels. Hence, consider the following composite image obtained by simple pixel-wise multiplication.
\[
I_f(x_i, y_i) = I_i(x_i, y_i) \times |I_i(x_i, y_i)|
\]
(8)

The composite image $I_f(\cdot)$ will alleviate the false positives, i.e., the multipath ghosts, and increase the intensity of the true target pixel. In the approach described, the multipath ghost locations are readily obtained using the multipath focusing analysis in Section 2.

4. SIMULATION AND EXPERIMENTAL RESULTS

A SAR system with a carrier frequency of 1.8 GHz and 400 MHz bandwidth, was simulated in Matlab. There are 12 antenna locations starting at (-10, 0)m, with an inter-element spacing of 4.17cm. The length and width of the room are given by $D_1 = 20$ m, $i = 1, 2$. Array standoff distance is $D_2 = 4$ m and the 0.2m thick front wall has a dielectric constant of 7.6, which is typical of concrete. The other three walls have similar properties.

An example of the exploitation technique for a two-target scenario is shown in Fig. 3. The variances of the weighting functions were assumed to be equal to the system range resolution. We can clearly see from the original beamformed image, Fig. 3(a), the two targets at (-4m, 14m) and (-12m, 12m), and their associated multipath ghosts. The image after using (7-8) is shown in Fig. 3(b). We can still see the remnants of the ghost w.r.t wall-1 of the second target. After careful analysis, we found that a part of the ghost w.r.t wall-2 of the first target is being mapped back to the ghost w.r.t wall-1 of the second target. This mapping is understood by examining Fig. 3(c), which shows the weighting when the imaged pixel is located at the ghost w.r.t wall-1 of the second target. A simple technique to get rid of such remnants is to use a pre-processing mask [4].

The final image is then obtained as,
\[
I'_f(x_i, y_i) = I_{ haut}(x_i, y_i) \times I_i(x_i, y_i) \times |I_i(x_i, y_i)|
\]
\[
I_{ haut}(x_i, y_i) = \begin{cases} 1, & I_i(x_i, y_i) > \lambda_{ haut} \\ 0, & \text{otherwise} \end{cases}
\]
(9)

The weighting function, $\Phi_2(\cdot)$, serves two purposes. First, it rejects pixel locations in the vicinity of the imaged pixel, i.e., the location corresponding to the true target $x = x_i$. Although the focus point is singular, but due to the system’s point spread function, some of the energy is spread to the neighboring pixels. Second, it disallows energy to be focused back to the multipath ghost locations.

Considering the ghosts w.r.t all three walls, we can readily see that (6) is modified as,
\[
I_i(x_i, y_i) = \sum_{n=1}^{N} \sum_{w} \sum_{y} ||I(x, y)||^2 \Phi_1(x_i^n, x, \sigma^2) \Phi_2(x_i^n, x, \sigma^2)
\]
(7)

The composite image $I_f(\cdot)$ will have no intensity at and near the vicinity of the focused multipath pixels. Hence, consider the following composite image obtained by simple pixel-wise multiplication.
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I_f(x_i, y_i) = I_i(x_i, y_i) \times |I_i(x_i, y_i)|
\]
(8)
where $\lambda_{th}$ is a predefined threshold. Since detection strategies are not the focus of the paper, and noting that $I_{j}(x_{j}, y_{j})/\max_{(x_{j}, y_{j})} I_{j}(x_{j}, y_{j}) \in [0, 1]$, a value of $\lambda_{th} = 0.5$ proves effective. The result after pre-processing is shown in Fig. 3(d), wherein the remnant completely disappears.

To validate the technique experimentally, a through-the-wall SAR system was set up in the Radar Imaging Lab at Villanova University. A stepped-frequency signal with 696 steps covering 0.7-3GHz was used. The room dimensions are $D_{1} = 5.09\text{m}$ and $D_{2} = 3.78\text{m}$. The side walls were covered with absorbers and only multipath due to the back wall was considered. The front concrete wall is 0.15m thick with a dielectric constant of 7.66. The synthetic aperture consisted of an 81-element monostatic linear array with an inter-element spacing of 2.22cm. The standoff distance of the array from the front wall is 1.52m. A 0.35m diameter sphere was used as the target. The target coordinates can be inferred from Fig. 4(a), which shows the original beamformed image (after background subtraction). The target and its ghost w.r.t to the back wall are marked in the figure. The result after multipath exploitation is provided in Fig. 4(b). One can readily see that the ghost has been mapped back to the target.

5. CONCLUSIONS

A mathematically rigorous multipath model for radar imaging based on ray tracing was introduced. The model was derived considering target reflections inside an enclosed room comprised of four walls. The walls were assumed to be homogenous and smooth, yielding specular reflections. Using the model, we illustrated that the multipath corresponding to each sensor appears on the wall but changes position from one sensor to another. Hence, a least squares technique was applied to estimate its actual focusing location in both downrange and crossrange. The model was utilized to develop a multipath exploitation technique which associates multipath ghosts with their respective targets and maps them back to their true target locations. This technique reduces the false positives in the original beamformed image as well as increases the SNR at the true target locations. Numerical simulations and experimental data were used to validate the findings.

6. REFERENCES